AFGROW User Workshop 2024

Working Group on
Engineered Residual
Stress Implementation

Analysis Methods and Testing

Comparison of 3D FEA-based Solutions against Single Edge Notch Handbook Formulations for Fatigue Crack Growth Rate Assessment using K_b Specimens

Adrian Loghin, Simmetrix Inc.

Motivation & References

- **This work is related to my action item from ERSI's Workshop 2024 (Kevin Walker's notes)**

Actions from Breakout session at 2024 Workshop

3. Investigate and provide recommendations on issues around material characterisation fatigue crack growth rate test data and the stress-intensity factor solutions used at the time to generate those data. Quantify the impact of using more accurate solutions which are now possible. Action Leader – ADRIAN LOGHIN to revise and tighten this description and circulate to our committee. Aim is then to seek agreement/concurrence on how we work this problem area. Is it part of our focus areas?

- **Introduction of K_b specimen for fatigue crack growth rate (FCGR) measurement**

[1] Coles, A., Johnson, R.E., and Popp, H.G., 1976, "Utility of Surface-Flawed Tensile Bars in Cyclic Life Studies," Journal of Engineering Materials and Technology, 98, pp. 305–315. .

- **References about limitations of SEN handbook solutions usage for K_b specimen FCGR measurements**

[2] S. B. Narasimhachary, K. S. Bhachu, Sachin R. Shinde, P. W. Gravett, J. C. Newman Jr., A single edge notch specimen for fatigue, creep-fatigue and thermo-mechanical fatigue crack growth testing, Engineering Fracture Mechanics, v.199, 2018.
[3] Harris, Z.D., Cochran, J.W., Gangloff, R.P., Hochhalter, J.D., Burns, J.T., On the validity of the Tada stress intensity factor solution for the single edge notch tension specimen with pinned ends, Engineering Fracture Mechanics, v.301, 2024.

- **Single Edge Notch (SEN) K_I closed-form solutions used in this presentation**

[4] Tada H., Paris, P.C., Irwin, G.R., The Stress Analysis of Cracks Handbook, 2000

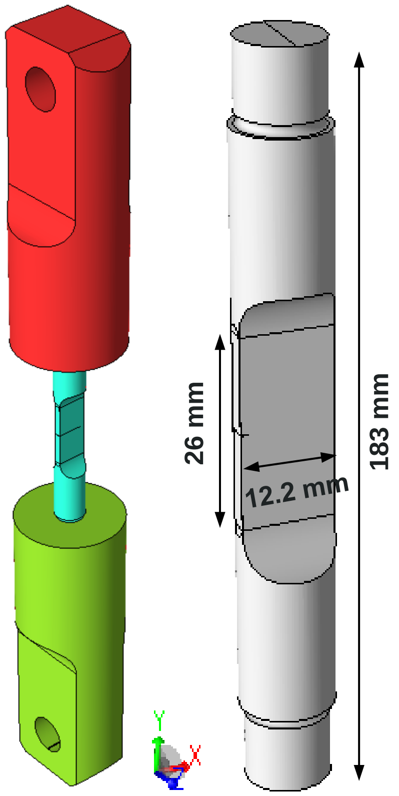
- **Crack growth solution comparison using 3D FEA-based vs. handbook solution K_I solutions (Feddersen) to establish FCGR for Al 2024-T3 using M(T) specimen geometry**

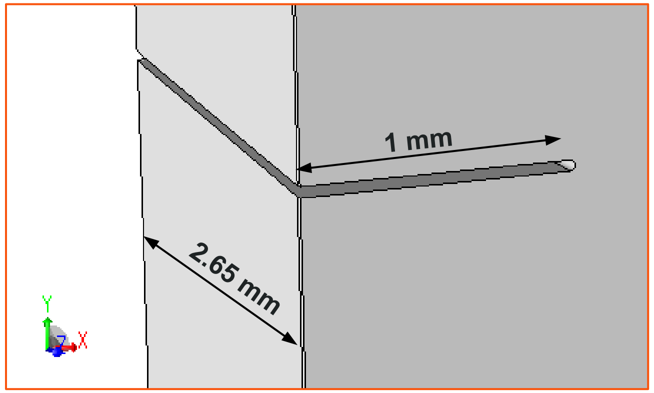
[5] Loghin A., Fatigue Crack Growth Modeling Validation and Sensitivity Study: Corner Crack Round Robin Challenge, ASIP, 2021.

Comparison of 3D FEA based solutions against analytical SEN solutions

Grips & K_b specimen geometry

Provided by Zach Harris (U. Pitt) & Jake Hochhalter (U. Utah)

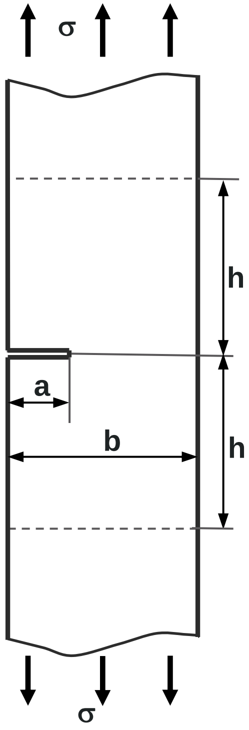




Reference handbook solutions used in this study

Single Edge Notch Test Specimen

$$K_I = \sigma\sqrt{\pi a}\, F\left(\frac{a}{b}\right)$$



- Gross 1964 & Brown 1966
  - least squares fitting
  - accuracy (from [4]): 0.5% for a/b ≤ 0.6

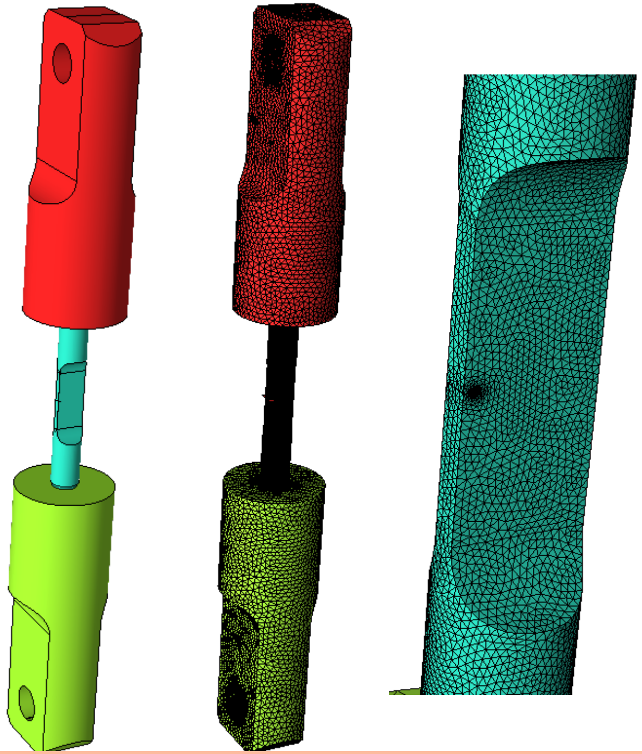
$$F\left(\frac{a}{b}\right) = 1.122 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.71\left(\frac{a}{b}\right)^3 + 30.382\left(\frac{a}{b}\right)^4$$

- Tada 1973
  - 2D plane strain model
  - accuracy (from [4]): better than 0.5% for any a/b

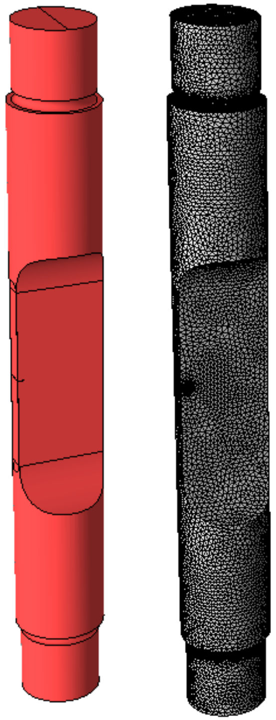
$$F\left(\frac{a}{b}\right) = \sqrt{\frac{2b}{\pi a}\tan\left(\frac{\pi a}{2b}\right)}\,\frac{0.752 + 2.02\left(\frac{a}{b}\right) + 0.37\left(1 - \sin\left(\frac{\pi a}{2b}\right)\right)^3}{\cos\left(\frac{\pi a}{2b}\right)}$$

Models that are considered in this assessment

FE Model of the K_b specimen and the attachments



FE Model of the K_b specimen

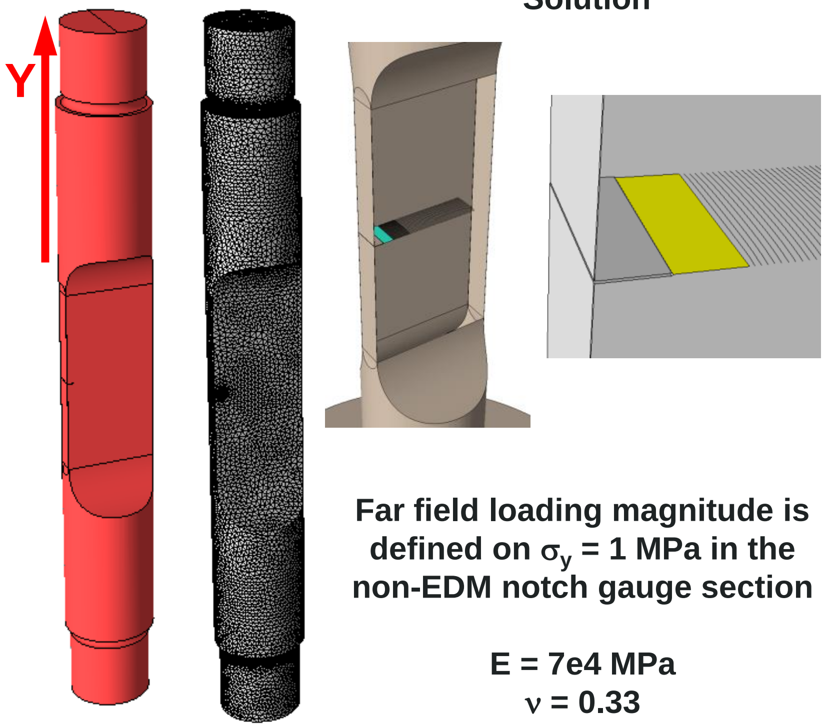


- There are different ways a finite element model representative of the K_b specimen in the testing rig can be created.
- In this study I will use FE models that capture only the K_b geometry and, K_b specimen with the attached tangs.
- Assumptions related to the boundary conditions will be made to represent conditions at locations (pin holes in the tangs or threaded grips of the specimen) where the adjacent structure is not represented.

FE Model of the K_b specimen

Overall FEM Setup

CAD model | Mesh | Automatic Crack Propagation Solution



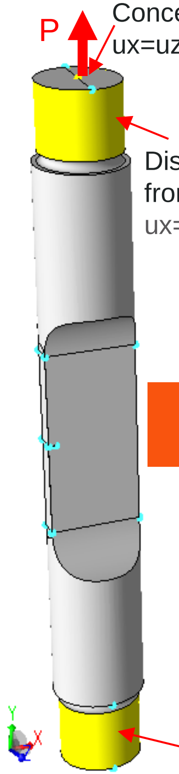
Far field loading magnitude is defined on σ_y = 1 MPa in the non-EDM notch gauge section

E = 7e4 MPa
ν = 0.33

Model preparation (geometry, overall mesh, crack front mesh pattern for each increment) is the same for all boundary conditions considered in the study.

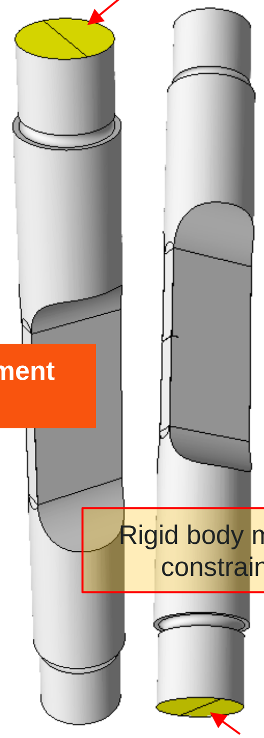
Boundary Conditions & Loading

Configuration #1

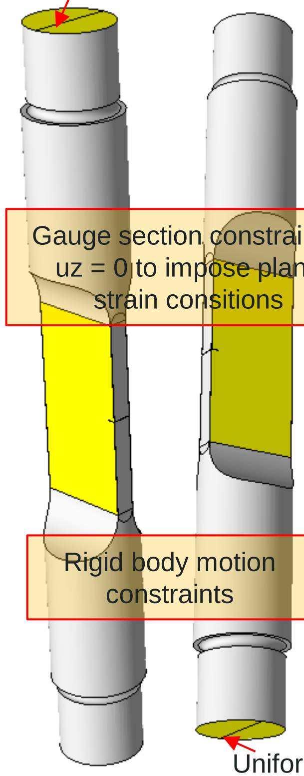


Show stress and displacement field comparison

Configuration #2



Configuration #3

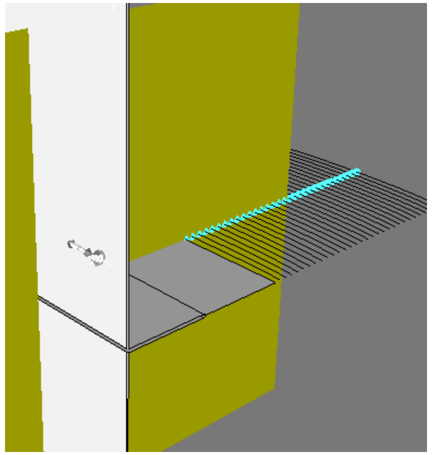


Boundary conditions more representative of the test conditions

Boundary conditions more representative of model used to generate handbook solutions

K_b: comparison of FEA solutions against SEN handbook solutions

K_I post-processing

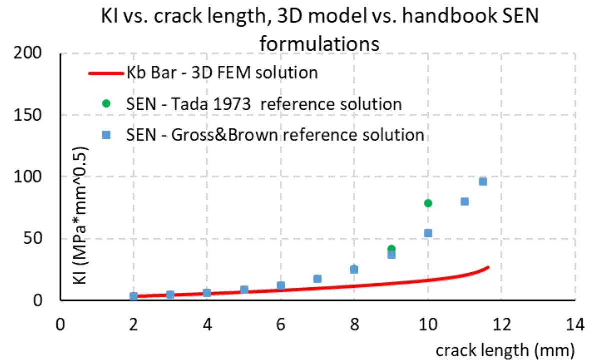


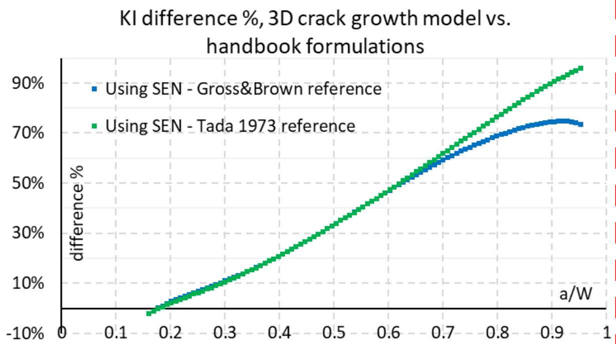
Automatic crack growth is performed to generate edge crack fronts and extract K_I values along the midplane of the specimen geometry.

Similar to Harris [3],
% difference relative to the handbook solution reference = $(K_{I\,FE} - K_{I\,handbook})/K_{I\,handbook}$

Comparison of the four FE based K_I solutions against handbook references

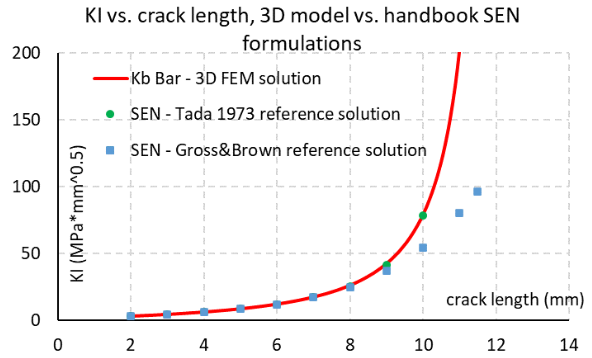
Configuration #1

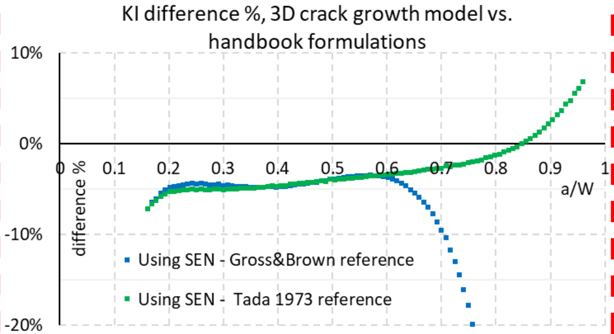




SEN handbook solutions are not a good approximation of K_I values of edge cracks in K_b specimen

Configuration #2

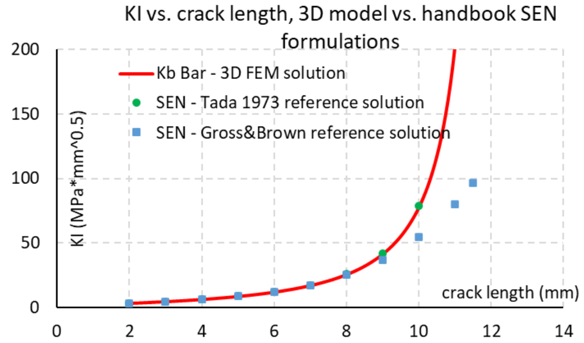


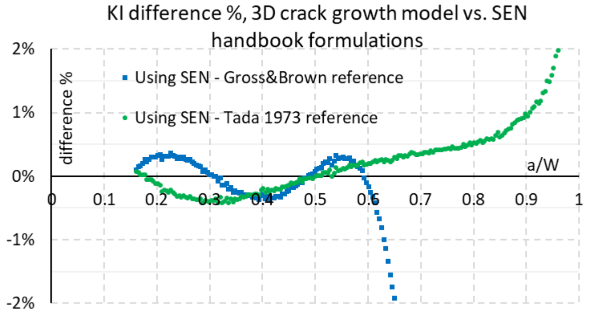


For both handbook SEN references, -7.2% < solution difference < 0.0 % for a/b < 0.6

Using Tada's handbook solution, -7.2% < solution difference < 4 % for a/b < 0.9

Configuration #3





For both handbook SEN references, -0.5% < solution difference < 0.5% for a/b < 0.6

Using Tada's handbook solution, -0.5% < solution difference < 1% for a/b < 0.9

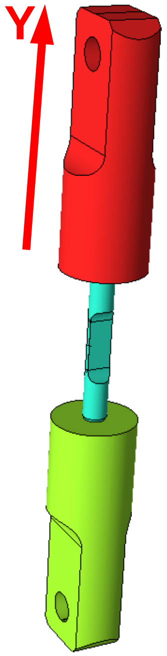
Boundary conditions more representative of the test conditions

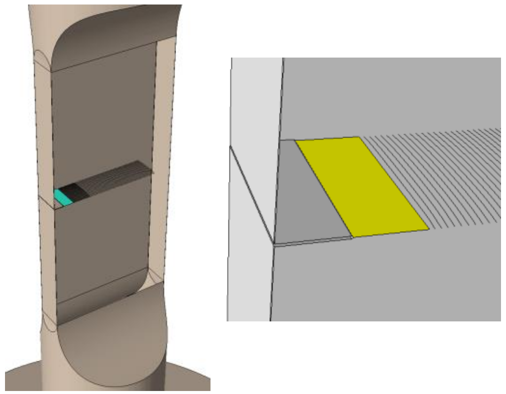
Boundary conditions more representative of model used to generate handbook solutions

FE Model of the K_b specimen with grips

Overall FEM Setup

CAD model | **Mesh** | **Automatic Crack Propagation Solution**





Far field loading magnitude is defined on σ_{yy} = 1 MPa in the non-EDM notch gauge section

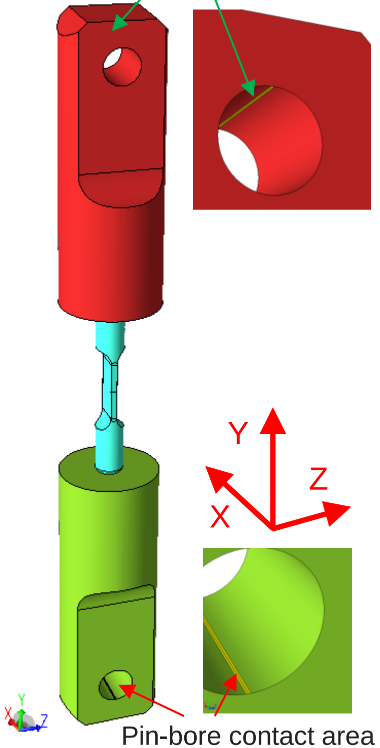
K_b: E = 7e4 MPa, ν = 0.33
Grips: E = 21e4 MPa, ν = 0.3

Model preparation (geometry, overall mesh, crack front mesh pattern for each increment) is the same for all boundary conditions considered in the study. Volumes are glued together to create a continuous mesh.

Boundary Conditions & Loading

Configuration #1

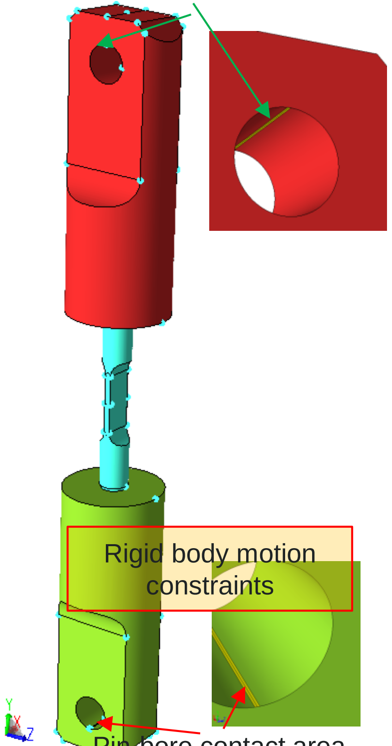
Pin-bore contact area: distributed uniform load and ux=uz=0



Pin-bore contact area ux=uy=uz=0

Configuration #2

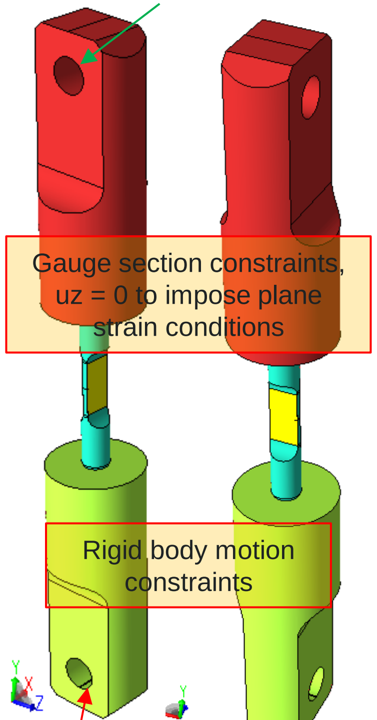
Pin-bore contact area Distributed uniform load



Pin-bore contact area Distributed uniform load

Configuration #3

Pin-bore contact area Distributed uniform load



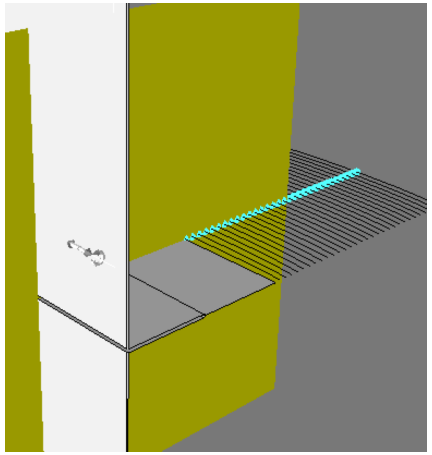
Pin-bore contact area Distributed uniform load

Boundary conditions more representative of the test conditions

Boundary conditions more representative of model used to generate handbook solutions

K_b with grips: comparison of FEA solutions against SEN handbook solutions

K_I post-processing

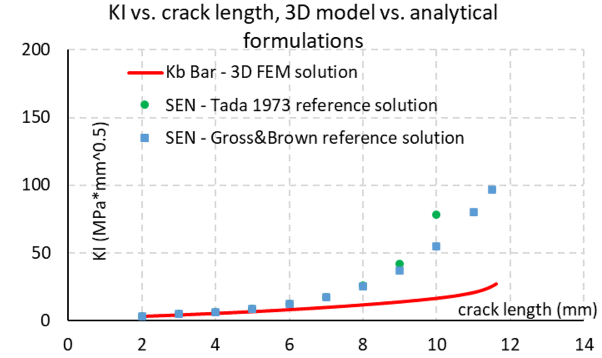


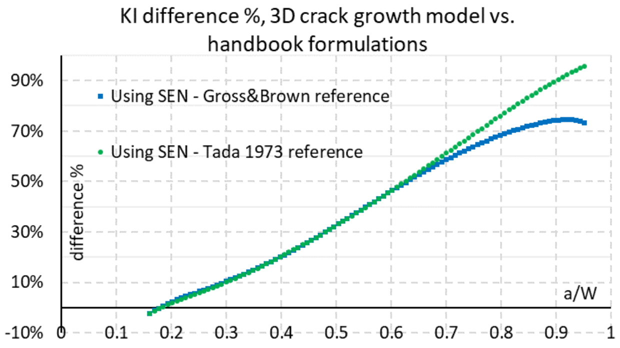
Automatic crack growth is performed to generate edge crack fronts and extract KI values along the midplane of the specimen geometry.

Similar to Harris [3],
% difference relative to the handbook solution reference = $(K_{I\ FE} - K_{I\ handbook})/K_{I\ handbook}$

Comparison of the four FE based K_I solutions against handbook references

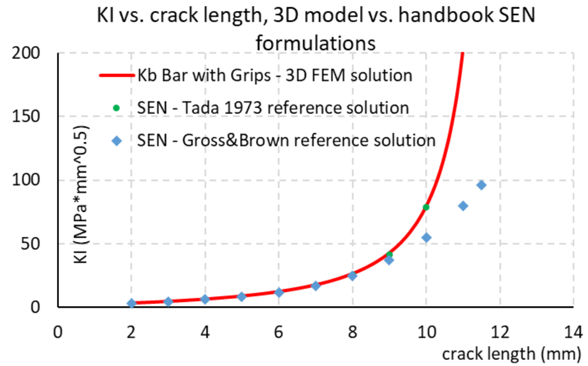
Configuration #1

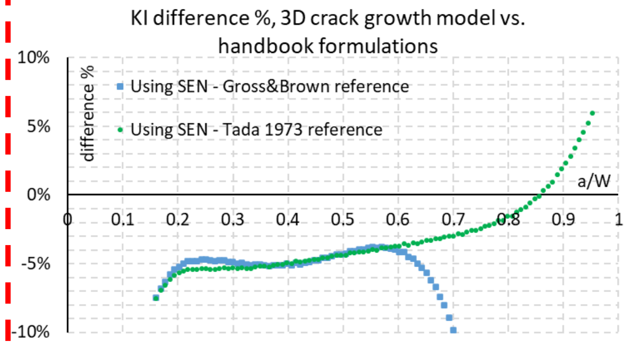




SEN handbook solutions are not a good approximation of K_I values of edge cracks in K_b specimen

Configuration #2

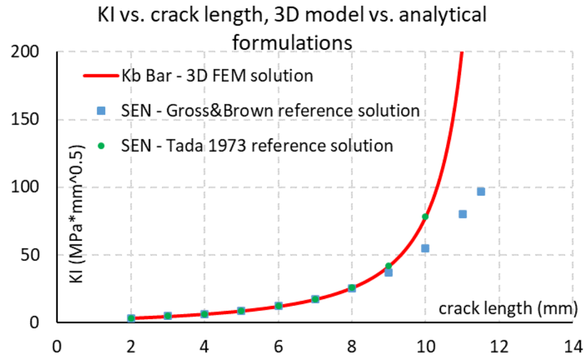


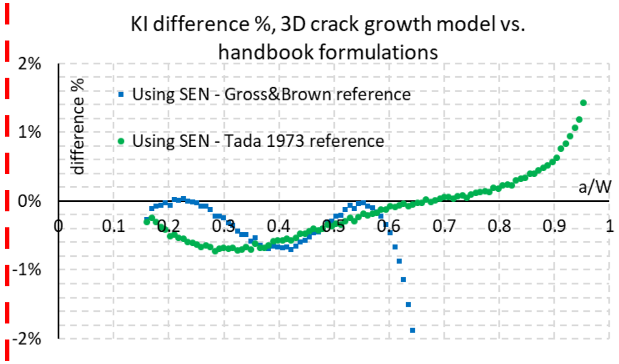


For both handbook SEN references, -7.2% < solution difference < 0.0% for a/b < 0.6

Using Tada's handbook solution, -7.2% < solution difference < 2% for a/b < 0.9

Configuration #3





For both handbook SEN references, -0.8% < solution difference < 0.0% for a/b < 0.6

Using Tada's handbook solution, -0.8% < solution difference < 0.6% for a/b < 0.9

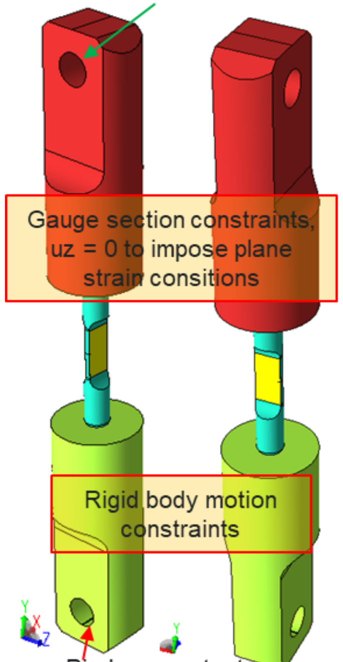
Boundary conditions more representative of the test conditions

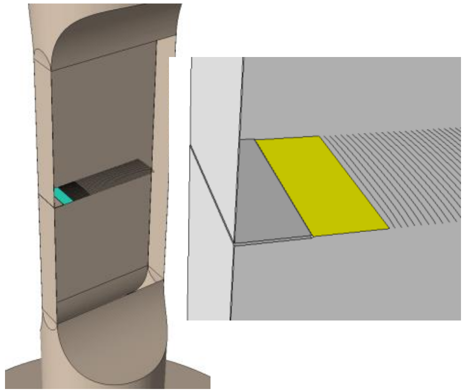
Boundary conditions more representative of model used to generate handbook solutions

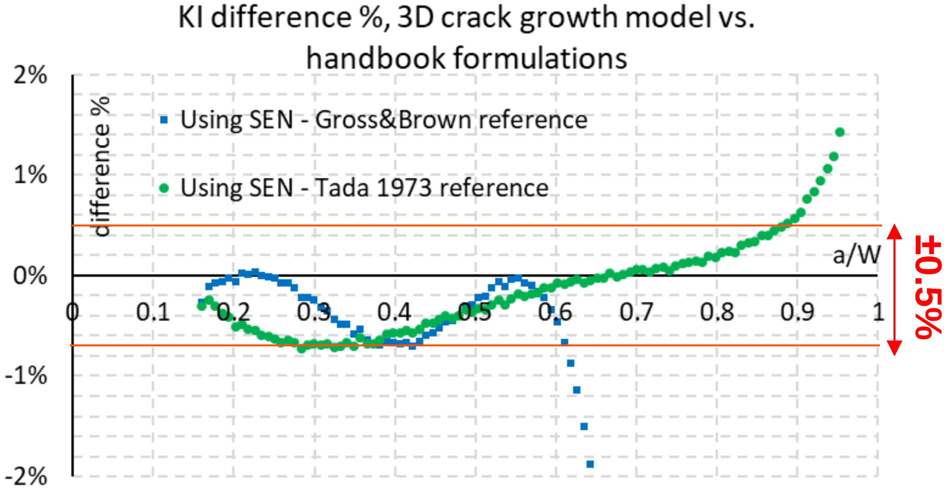
3DFEA (plane strain on gauge section) vs. Tada's Handbook solutions (1973)

3D FEA solution

Configuration #3







Tada's Handbook solutions

$$K_I = \sigma\sqrt{\pi a}\, F\left(\frac{a}{b}\right)$$

- Gross 1964 & Brown 1966
  - least squares fitting
  - accuracy (from [4]): 0.5% for a/b ≤ 0.6

$$F\left(\frac{a}{b}\right) = 1.122 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.71\left(\frac{a}{b}\right)^3 + 30.382\left(\frac{a}{b}\right)^4$$

- Tada 1973
  - 2D plane strain model
  - accuracy (from [4]): better than 0.5% for any a/b

$$F\left(\frac{a}{b}\right) = \sqrt{\frac{2b}{\pi a}\tan\left(\frac{\pi a}{2b}\right)}\,\frac{0.752 + 2.02\left(\frac{a}{b}\right) + 0.37\left(1 - \sin\left(\frac{\pi a}{2b}\right)\right)^3}{\cos\left(\frac{\pi a}{2b}\right)}$$

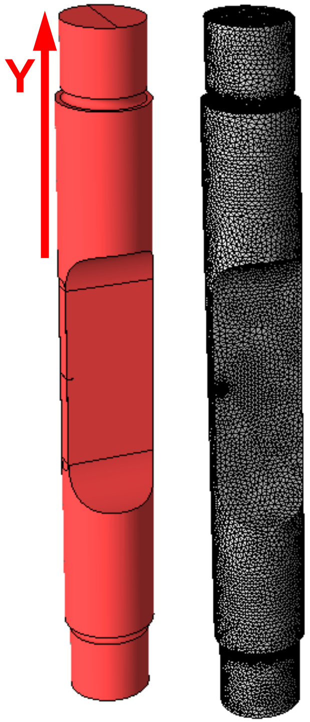
- ✓ **The 3D FE model that resembles the 2D plane strain conditions that was used to determine the closed-form SEN K_I solution confirms the accuracy stated with Tada's solution (0.5%) up to a/b = 0.9.**
- ✓ **The accuracy of the other Handbook solution, Gross&Brown, is also confirmed by the 3D FE model solution: the closed form SEN K_I solution has an accuracy better than 0.5% for a/b < 0.6.**

K_b with grips: partial conclusions

- ✓ **All solutions are correspondent to 1 MPa load relative to the un-notched gauge section**
- ✓ **The two 3D FE models (specimen only and specimen with attached grips) provide very similar K_I solutions.**
- ✓ **The 3D FEA model is verified against the two SEN K_I closed-formulations for boundary conditions that match with the model that was used to develop the analytical K_I**
- ✓ **For boundary conditions that are more representative to the stiff testing frame, the 3D FEA K_I values deviate from the closed-form solutions considerably**
- ✓ **The question that is raised is: what is the impact of using Handbook closed-form solutions in assessing fatigue crack growth rate instead of a 3D FEA based solution that is more representative of the testing conditions (frame stiffness)?**

Usage of 3D FEA against SEN handbook solutions in FCGR assessment

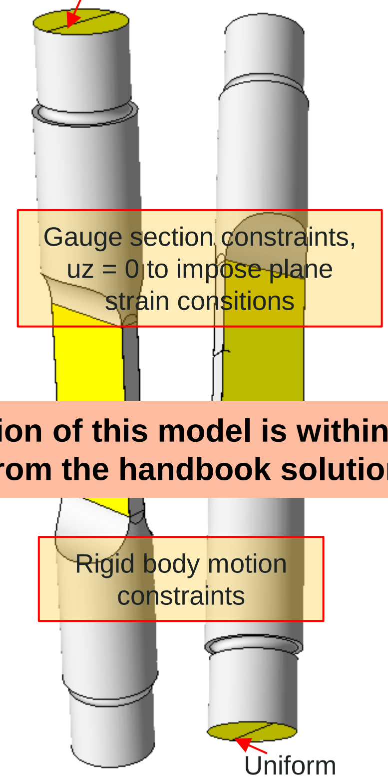
Geometry & Mesh



Far field loading magnitude is defined on σ_y = 200 MPa in the non-EDM notch gauge section
E = 7e4 MPa
ν = 0.33

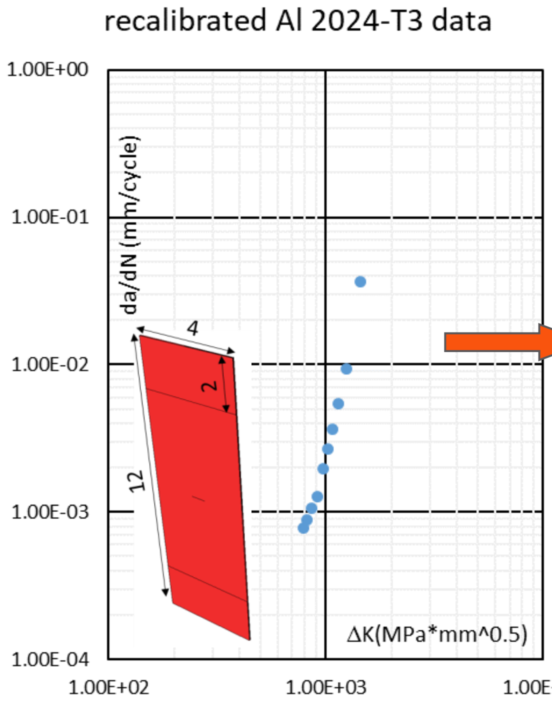
Boundary Conditions & Loading

Configuration #3

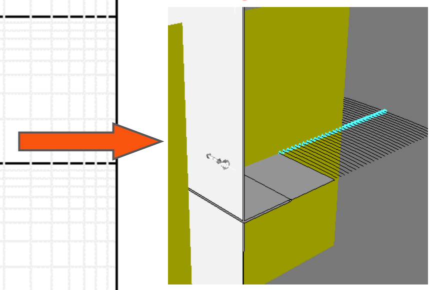


Solution of this model is within 0.5% from the handbook solution

FCGR M(T) data: Al 2024-T3 using 3D FEA instead of handbook solution (Feddersen)

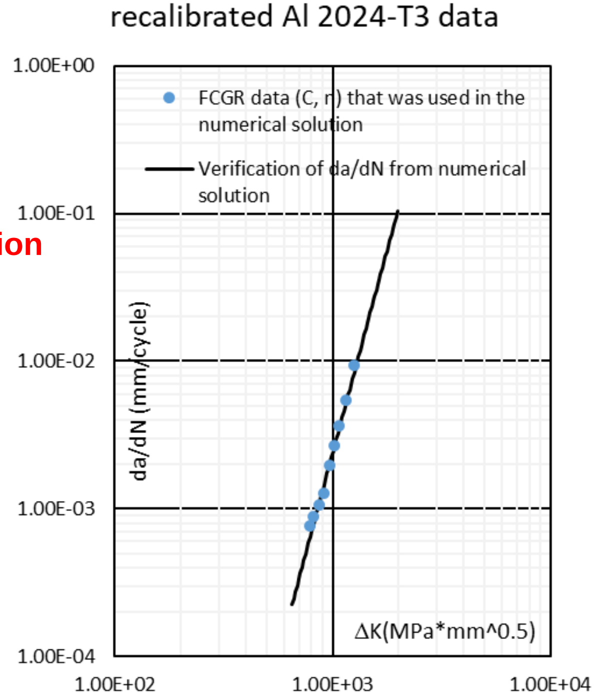


3D FEA K_b fatigue crack growth solution



Reference: Adrian Loghin, Fatigue Crack Growth Modeling Validation and Sensitivity Study: Corner Crack Round Robin Challenge, ASIP 2021.

Solution Verification

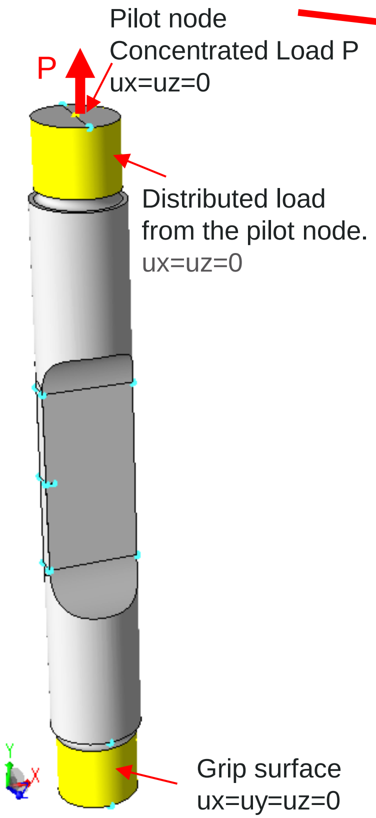


Model verification: the crack growth rate solution matches the input data (the (C, n) pair for Paris-Erdogan relationship).

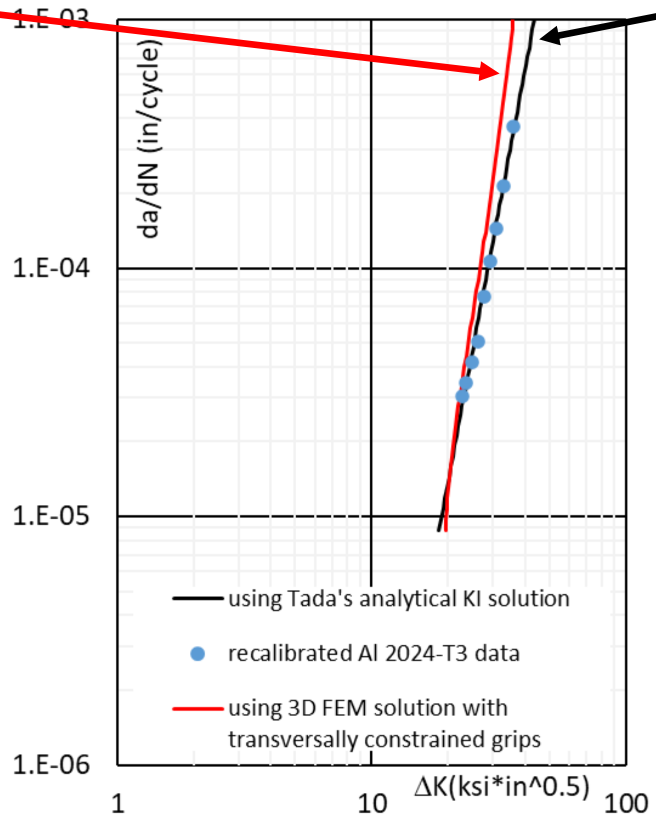
Model verification is reached

Usage of 3D FEA against SEN handbook solutions in FCGR assessment

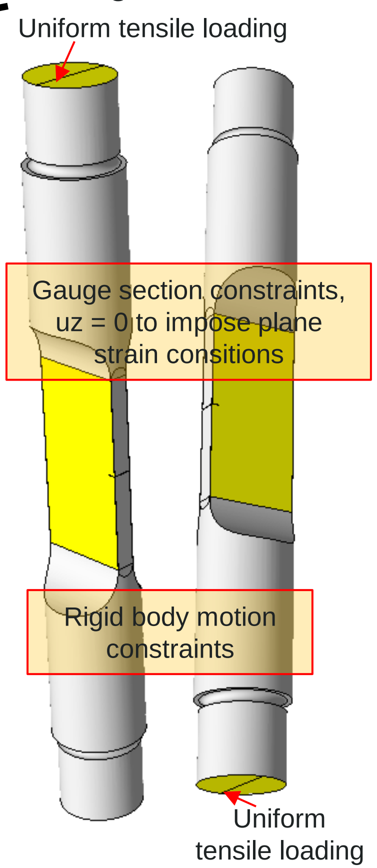
Configuration #1



Unit switch: from (MPa, mm) to (ksi, in)



Configuration #3



If SEN handbook solution is used to assess FCGR using a K_b bar, a modeling error is introduced which leads to a non-conservative FCGR assessment

Conclusions

- ✓ 3D FEA is much more capable than 50 years ago to provide K_I solutions that are more representative of the test conditions. Maybe it is the time to adjust testing procedures using 3D FEA-based solutions.

- ✓ For different specimen geometries, the study needs to be repeated but similar conclusion is expected: the handbook K_I solution contains a modeling error in comparison to a 3D FEA-based solution that is representative of the test conditions (grips, frame stiffness).

- ✓ The numerical error that is carried into the FCGR assessment affects validation efforts that are conducted under ERSI's round robin challenges. For example, in the round-robin challenges where cracks are initiated at the rim of a hole in a flat panel (corner crack type), the latest 3D FEA capabilities are used to provide a crack size vs. cycles solution (or marker band vs. crack front edge at a given accumulated number of cycles). If the FCGR data that is used in the 3D FEA solutions is evaluated using handbook K_I solutions, an error is carried into the validation benchmark.