

# Fracture Analyses of Thin-Ductile Aerospace Materials

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***Layton, Utah***



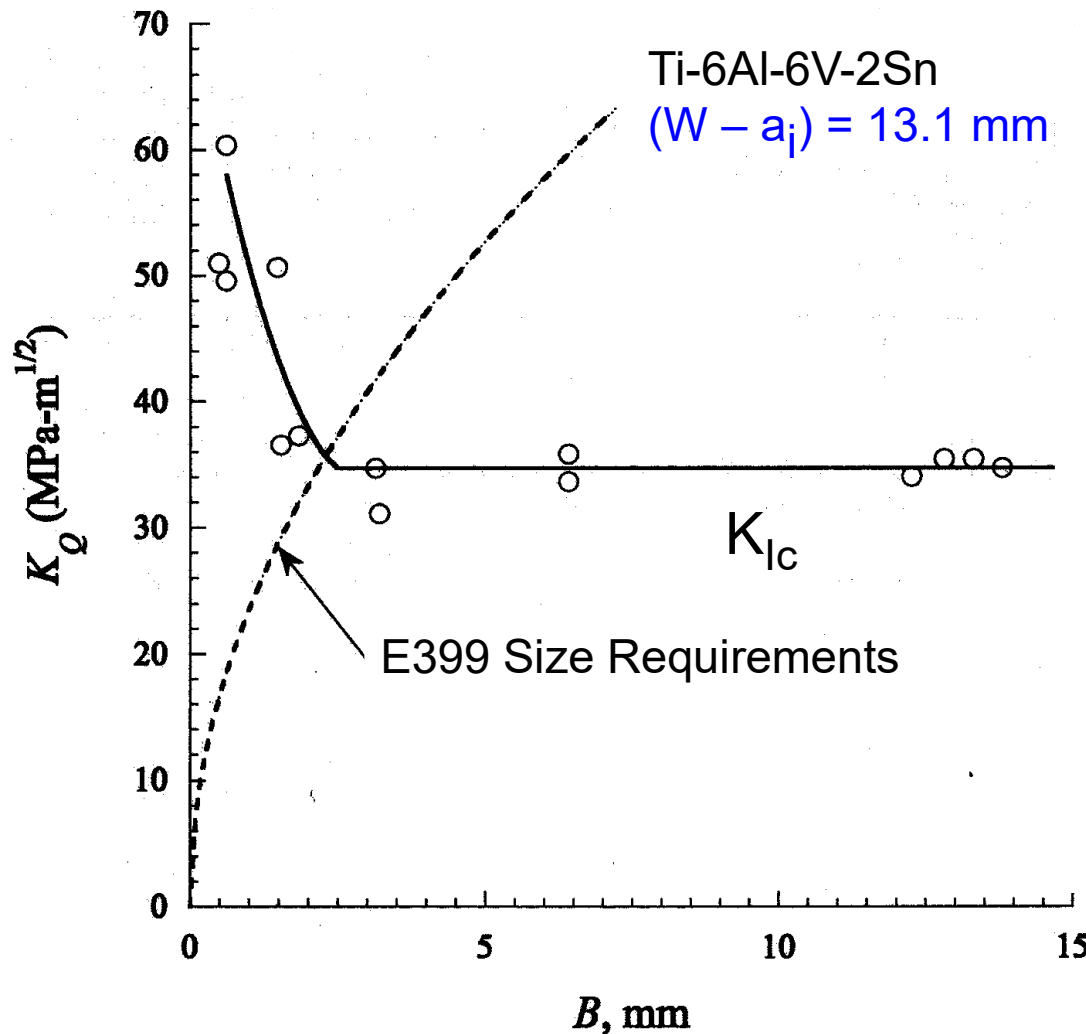
# *Two-Parameter* Fracture Mechanics for Residual Strength

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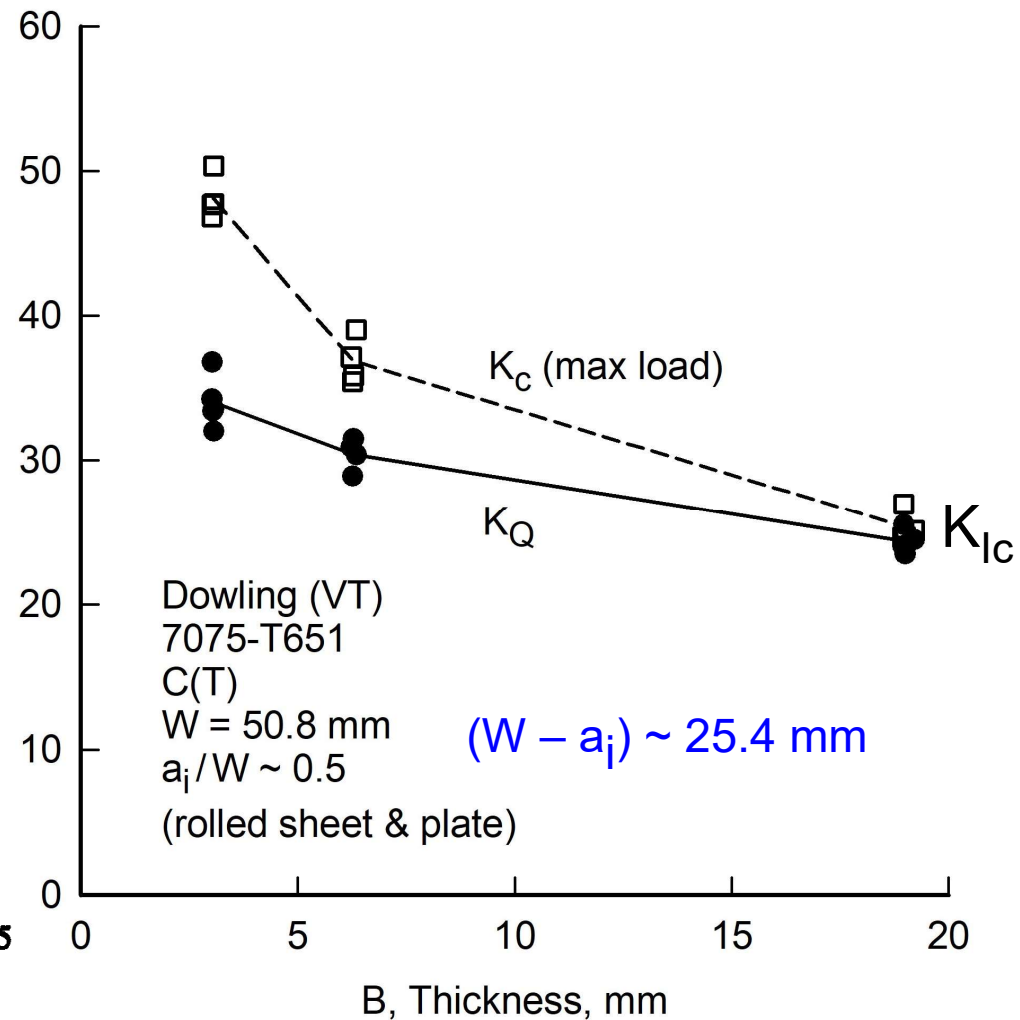
- $K_R$  - Curve (Irwin and Kies, 1954)  $K_R \sim C(\Delta a)^n$
- **$K_C$  - Plane-stress fracture toughness (Irwin, 1957) – not constant**
- $K_{IC}$ , B - Plane-strain fracture toughness (Brown-Srawley, 1964) →

# $K_{Ic}$ Test Results on Two Materials

Jones & Brown, NASA LeRC, 1970



Dowling, VT, ~2000



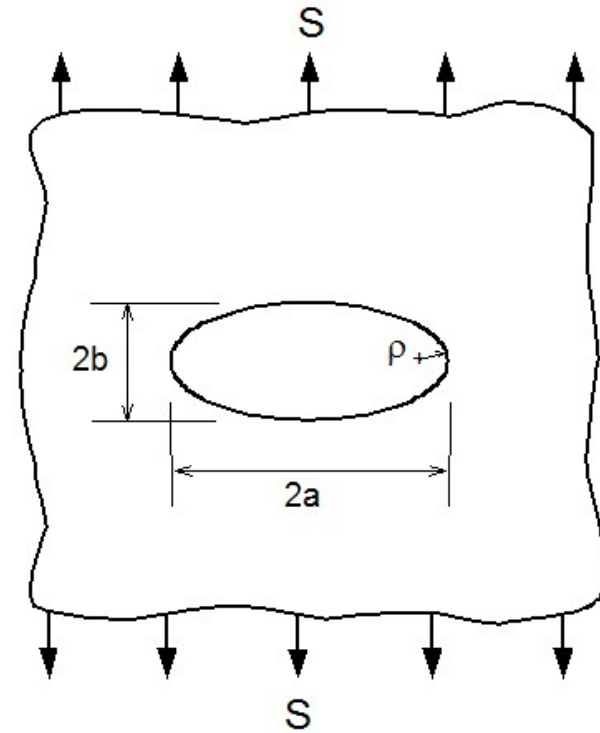
$B$  is the governing parameter for  $K_{Ic}$  and “not” un-cracked ligament ( $W - a_i$ )!

# **Two-Parameter Fracture Mechanics for Residual Strength**

- $K_R$  - Curve (Irwin and Kies, 1954)  $K_R \sim C(\Delta a)^n$
- $K_C$  - *Plane-stress fracture toughness (Irwin, 1957) – not constant*
- $K_{IC}$ , B - *Plane-strain fracture toughness (Brown-Srawley, 1964)*
- $K_u$ ,  $\rho'$  - Notch-Strength Analysis (NSA; Kuhn and Figge, 1964)
- K and T (Larrson & Carlsson, 1973; Hancock et.al., 1991)
- $K_F$ , m - Two-parameter fracture criterion (Newman, 1973)
- Two-criteria (K & S) approach (Dowling and Townley, 1975)
- $J_R$  - Curve (Albrecht et.al., 1982)  $J_R \sim C(\Delta a)^n$
- J and Q (Shih & O'Dowd, 1990)
- CTOA ( $\psi_c$ ) and PSC (plane-strain core) (Newman et.al., 1993)
- J and Stress tri-axiality parameter (Brocks et.al., 1995)

# Modern Fracture Mechanics

Inglis (1913)  
Griffith (1920)



$$K_T = 1 + 2\sqrt{a/\rho}$$

$$\sigma_e = S + 2S\sqrt{a/\rho}$$

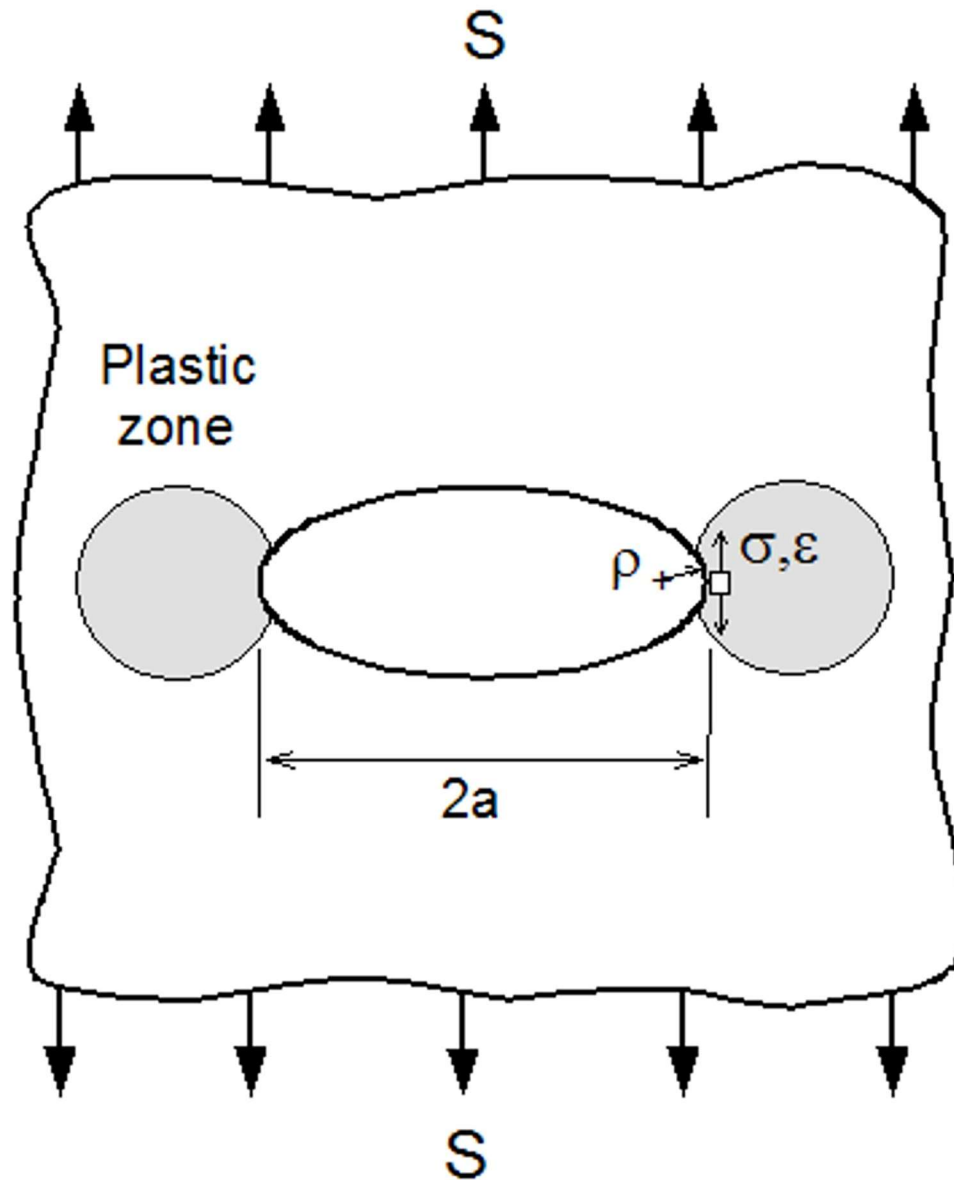
Paul Kuhn

George Irwin

Notch Strength Analysis

Fracture Mechanics

# Elastic-Plastic Stress- and Strain-Concentration Factors using Neuber's Equation



Neuber (1961):

$$K_{\sigma} K_{\epsilon} = K_T^2$$

$$\sigma \epsilon E = \sigma_e^2$$

**Crews** (1974) experimentally validated Neuber's equation for **elliptical hole** in finite plate under remote uniform stress

**Hutchinson** (1968) showed that the stress-strain field for a **crack** in a non-linear elastic material verified Neuber's equation

# Two-Parameter Fracture Criterion (Newman, 1973)

- Inglis' stress-concentration equation for elliptical hole:

$$K_T = 1 + 2 \sqrt{c/\rho}$$

- Neuber's equation:  $K_\sigma K_\varepsilon = K_T^2$

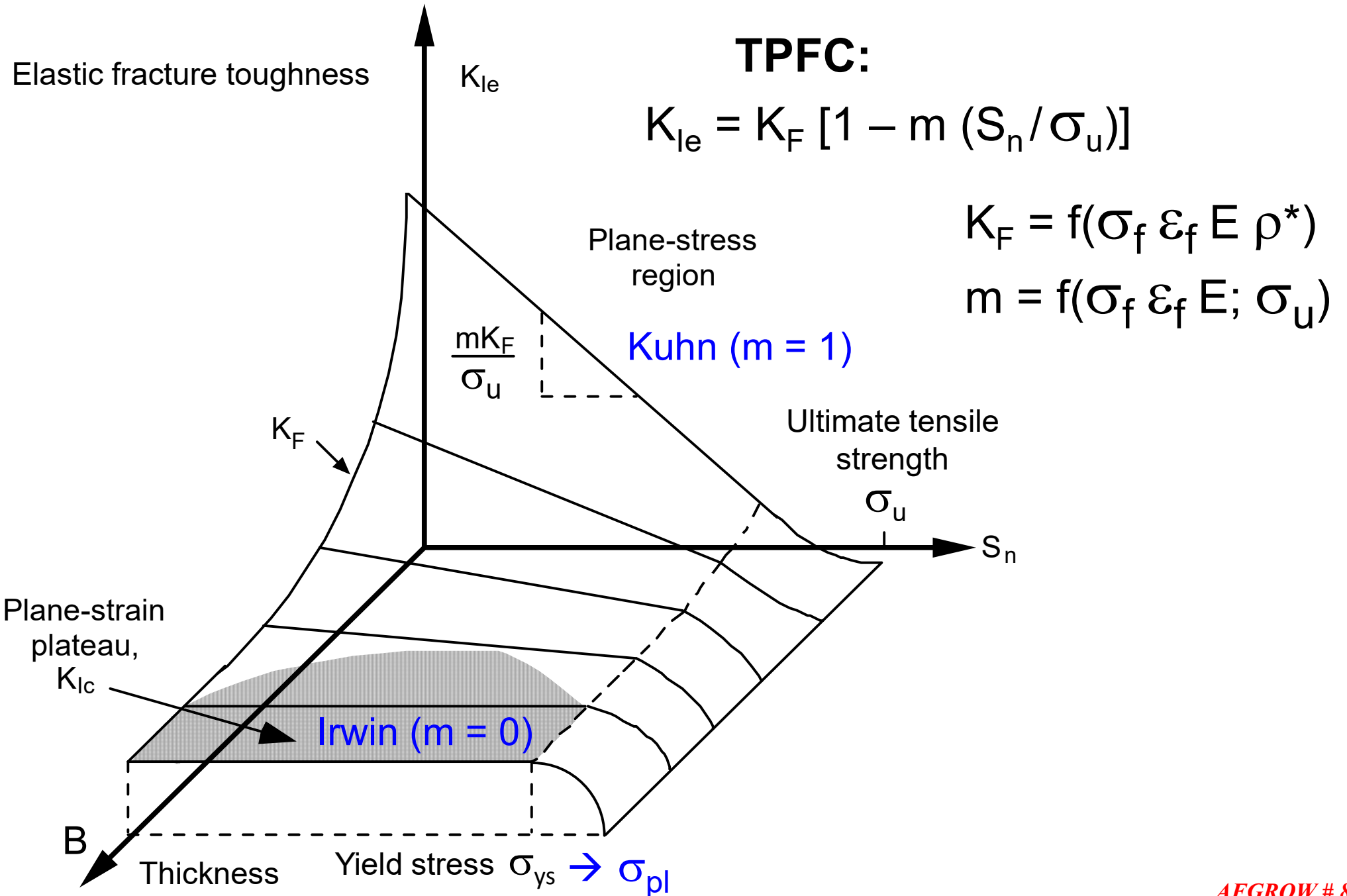
$$K_F = K_{le} / \Phi$$

$$\Phi = 1 - m (S_n / \sigma_u) \quad \text{for } S_n < \sigma_{ys} \quad \rightarrow \sigma_{pl}$$

$$\Phi \approx (\sigma_{ys} / S_n) [1 - m (S_n / \sigma_u)] \quad \text{for } S_n \geq \sigma_{ys}$$

**Simple approximation for “*tension*”—more work needed, especially for “*bending*”.**

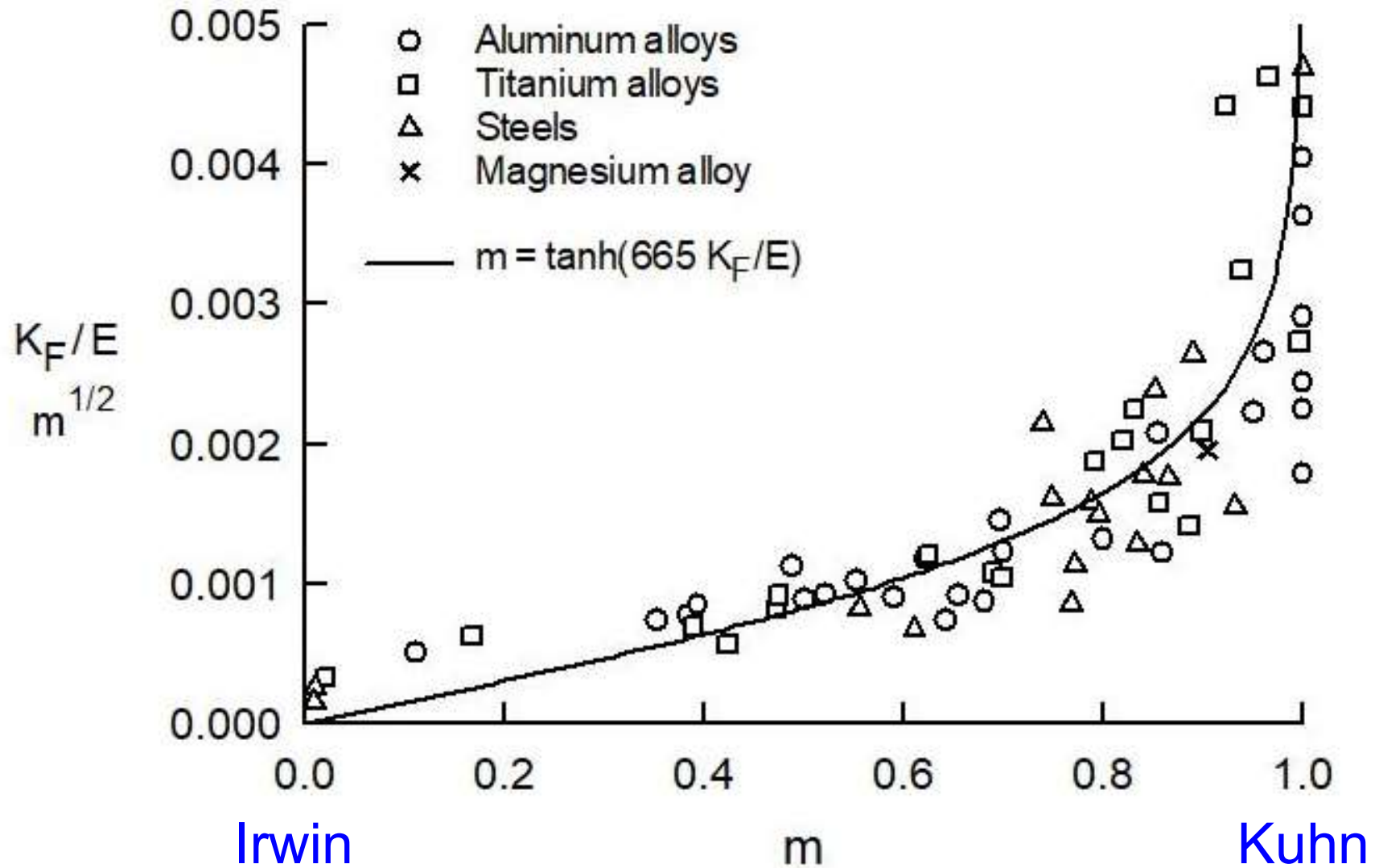
# Fracture Locus for Thin Sheet to Thick Plate Materials





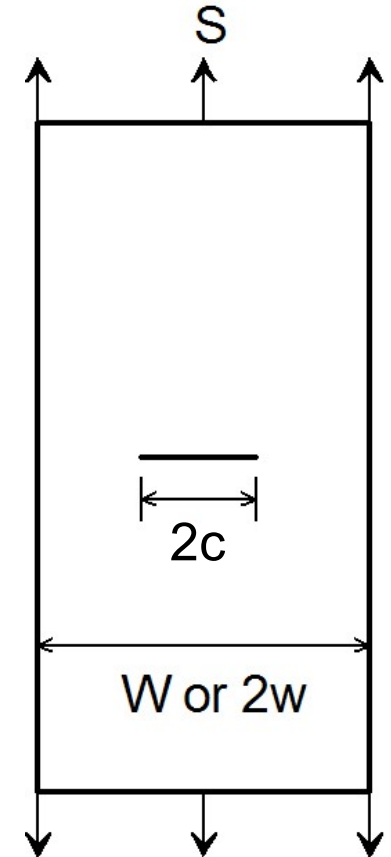
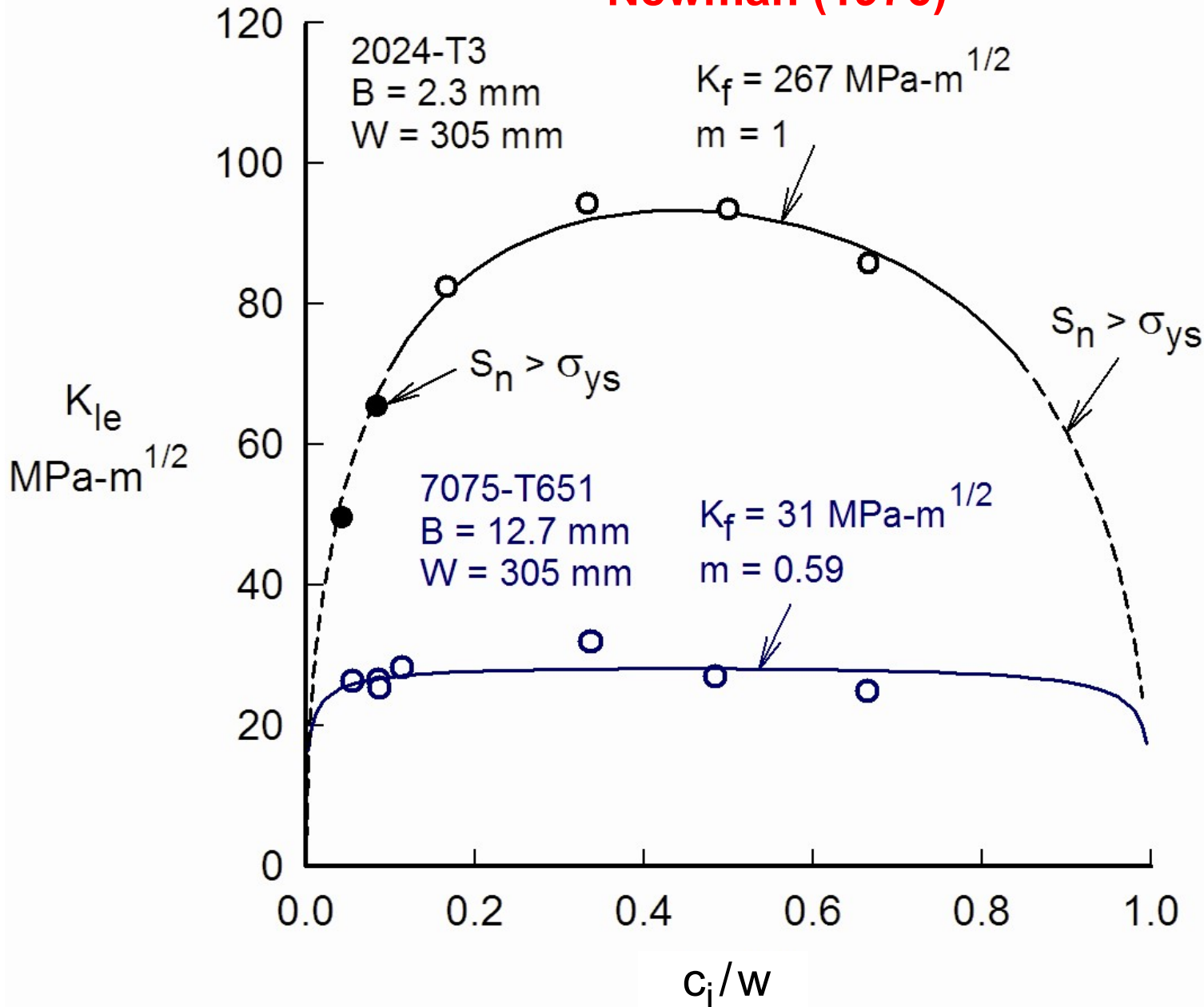
# Relationship between $K_F/E$ and $m$ for Wide Variety of Materials for *Through- and Surface-Crack* Configurations

Newman (1973)

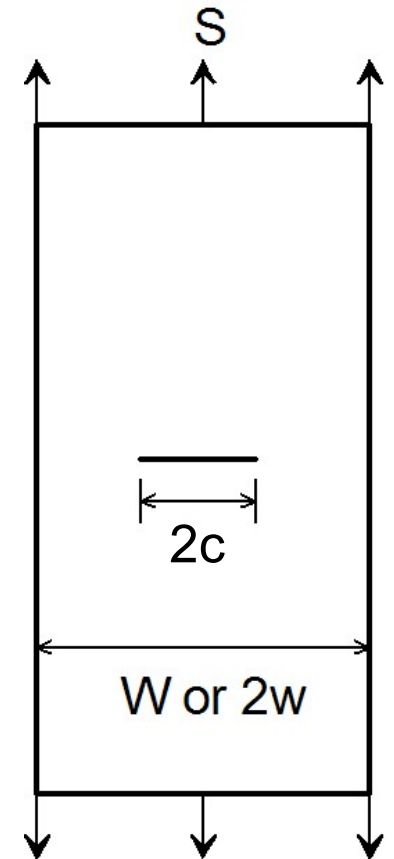
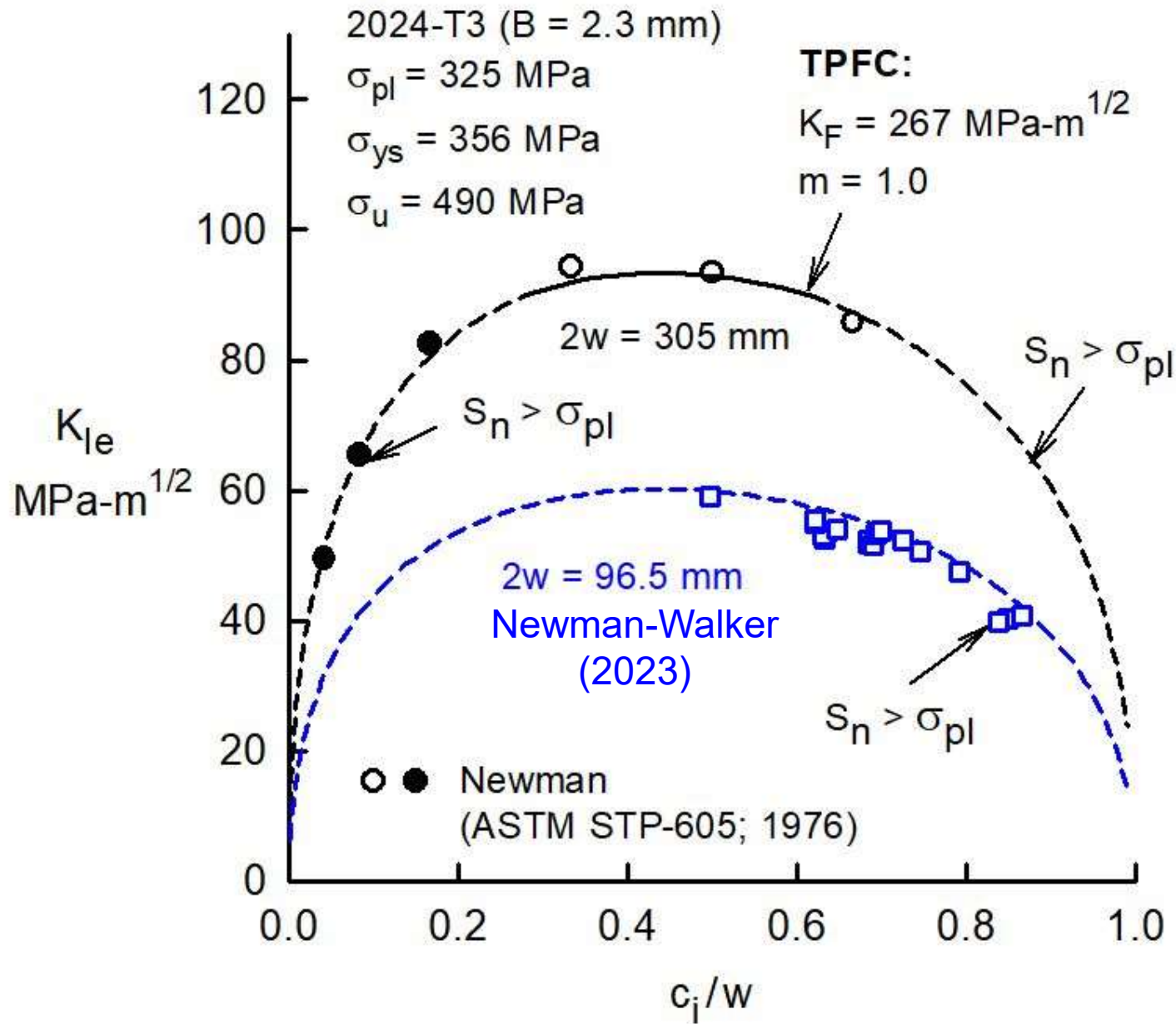


# Fracture of 2024-T3 Thin Sheet and 7075-T651 Thick Plate M(T) Specimens for Large Width

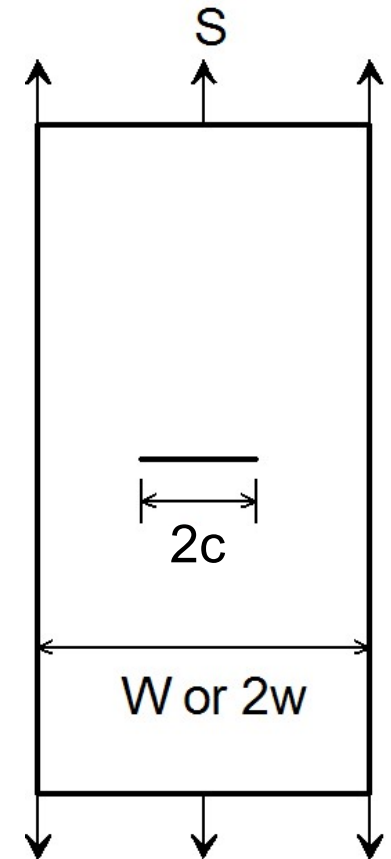
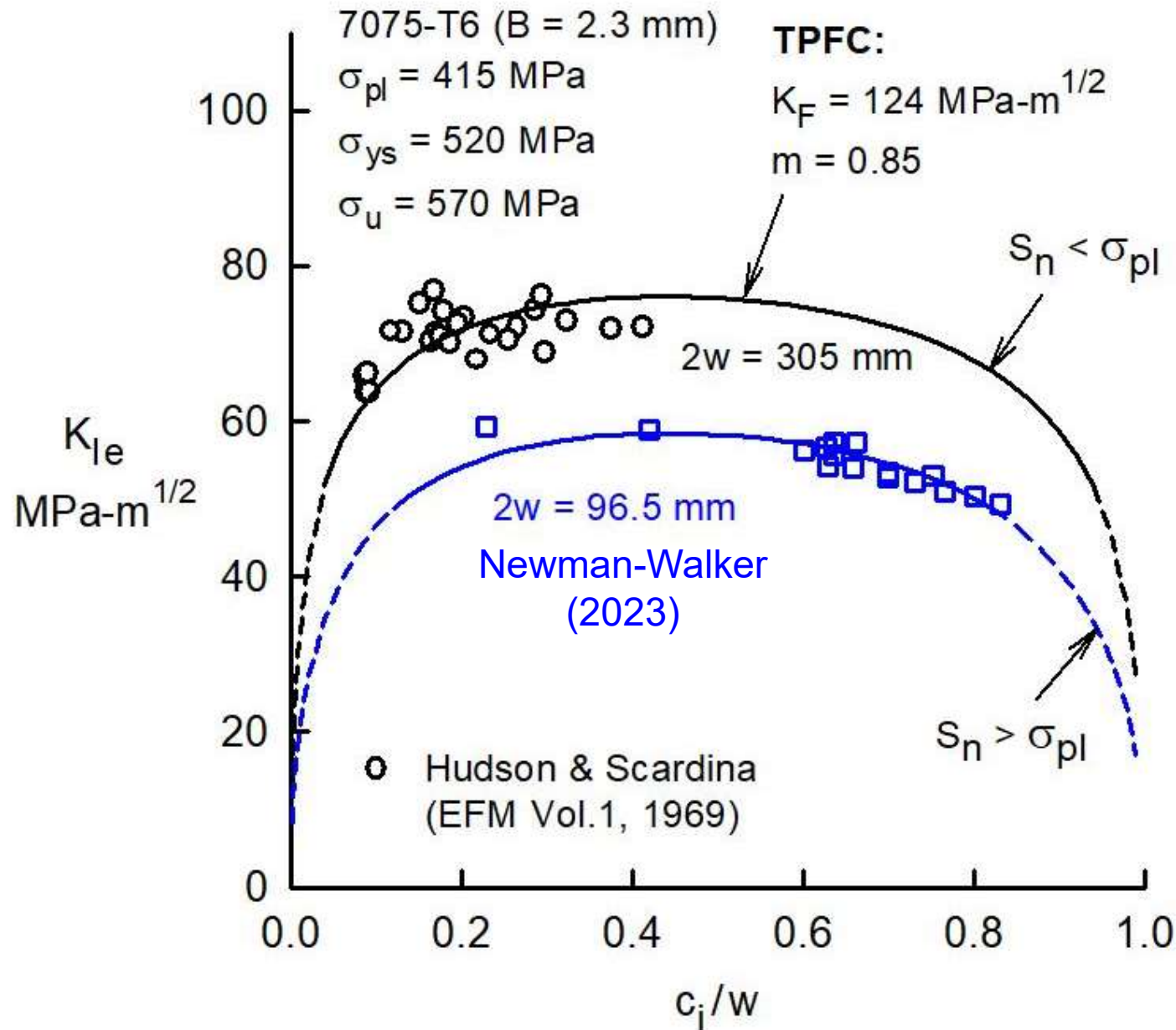
**Newman (1976)**



# Fracture of 2024-T3 Thin Sheet M(T) Specimens for Two Widths

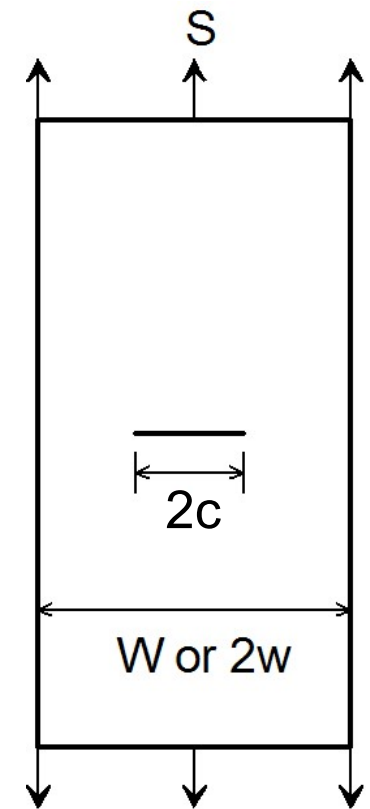
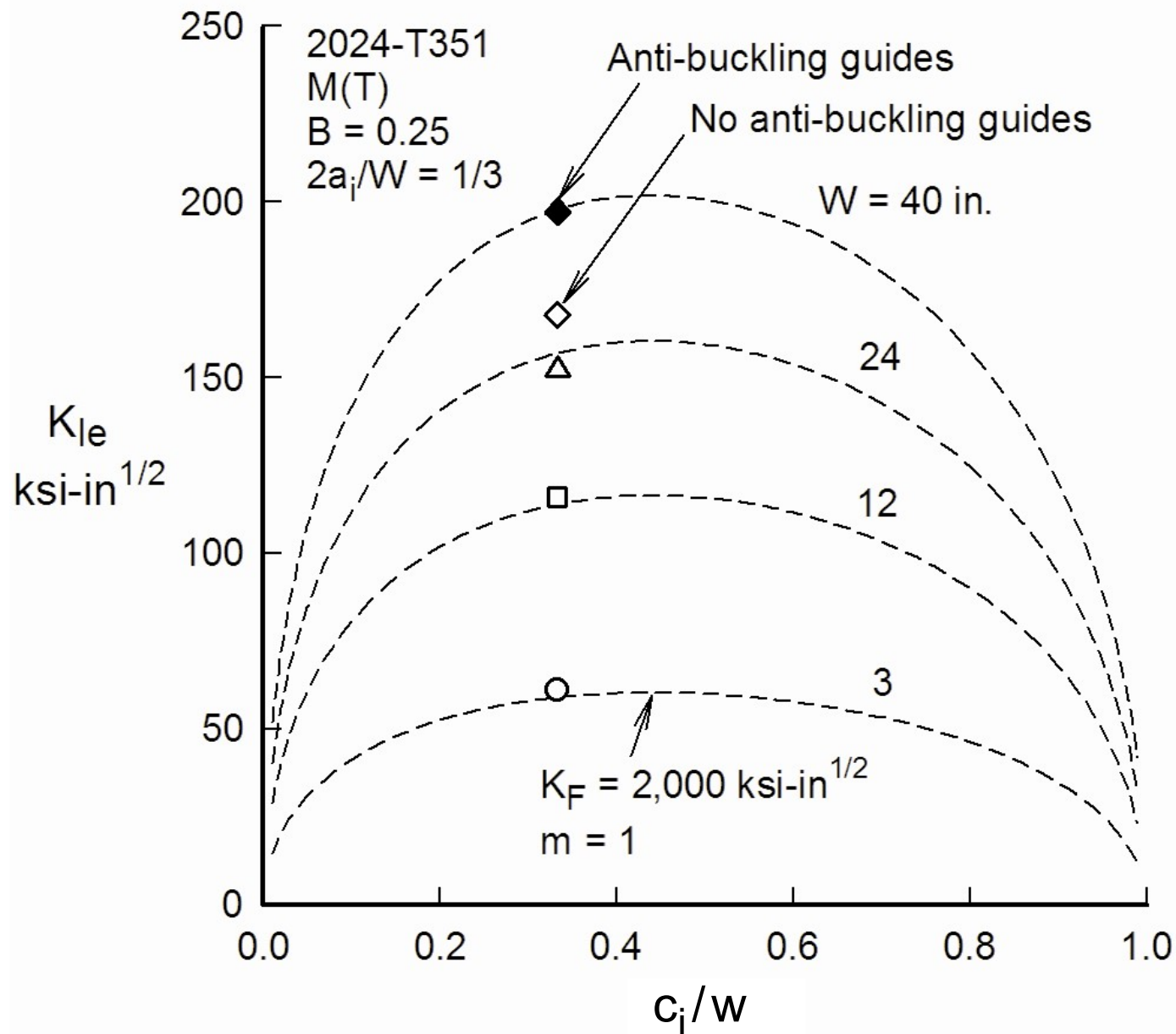


# Fracture of 7075-T6 Thin Sheet M(T) Specimens for Two Widths



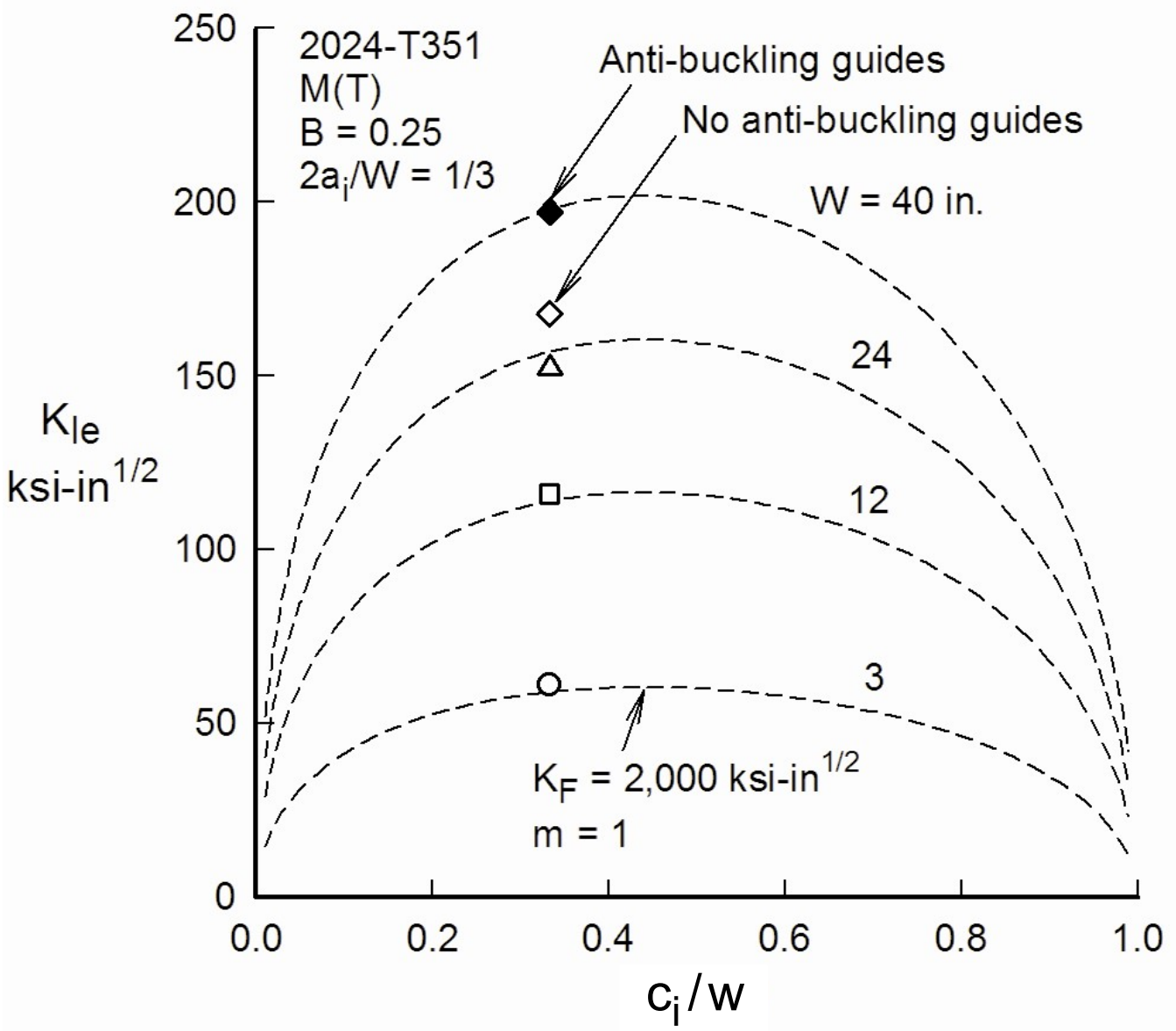
# Fracture of 2024-T351 Small- and Large-M(T) Specimens

(Note: All tests and analyses have  $S_n > \sigma_{ys}$ )



# Fracture of 2024-T351 Small- and Large-M(T) Specimens (2)

(Note: All tests and analyses have  $S_n > \sigma_{ys}$ )



**TPFC:**

Tension  $S_n > \sigma_{ys}$

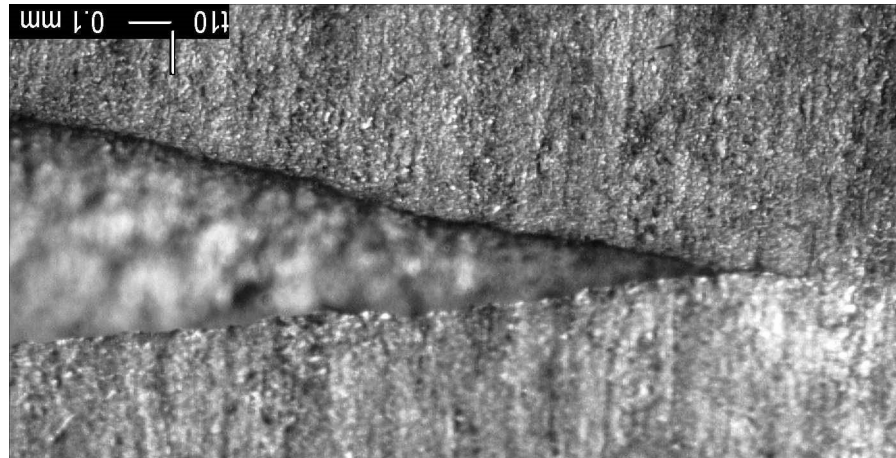
$$S_n = \frac{\sigma_u}{1 + K_{Ic}/K_F}$$

$$K_F \rightarrow \infty$$

$$S_n \rightarrow \sigma_u$$

Stress failure criterion!

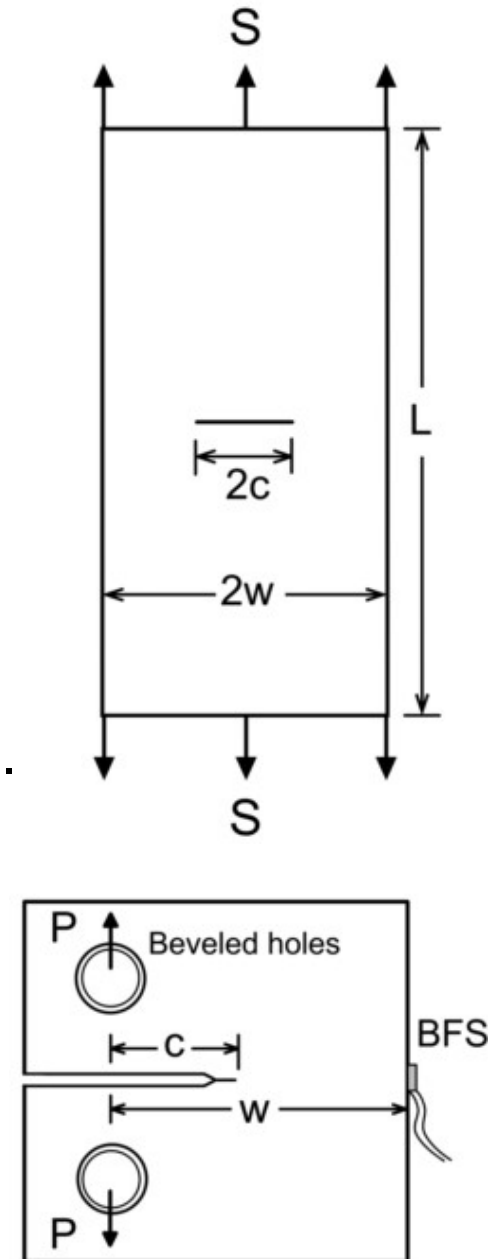
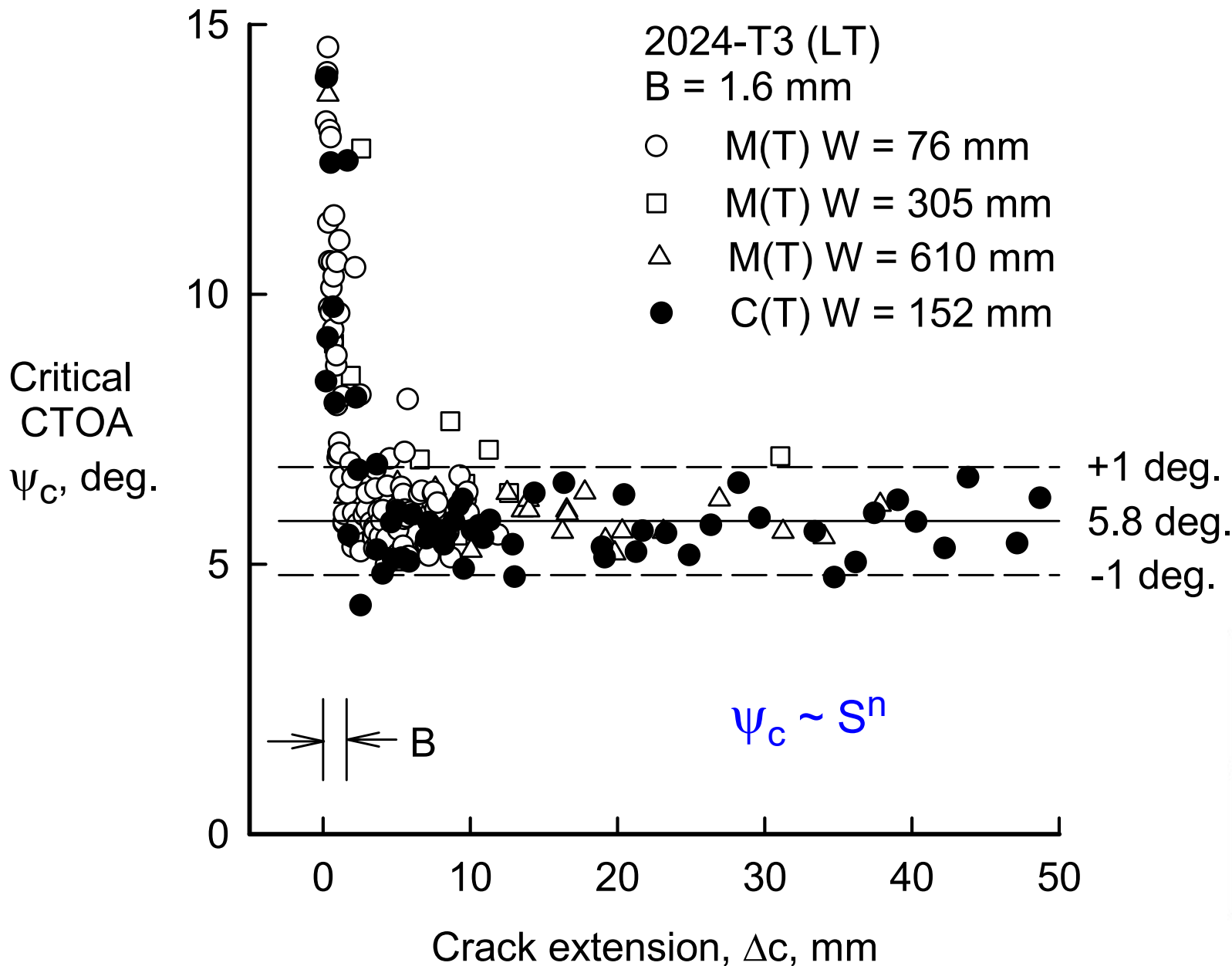
# Fracture Simulations using Elastic-Plastic Finite-Element Analyses and Critical CTOA



Prof. C.T. Sun (1995)  
Purdue University  
Mild steel

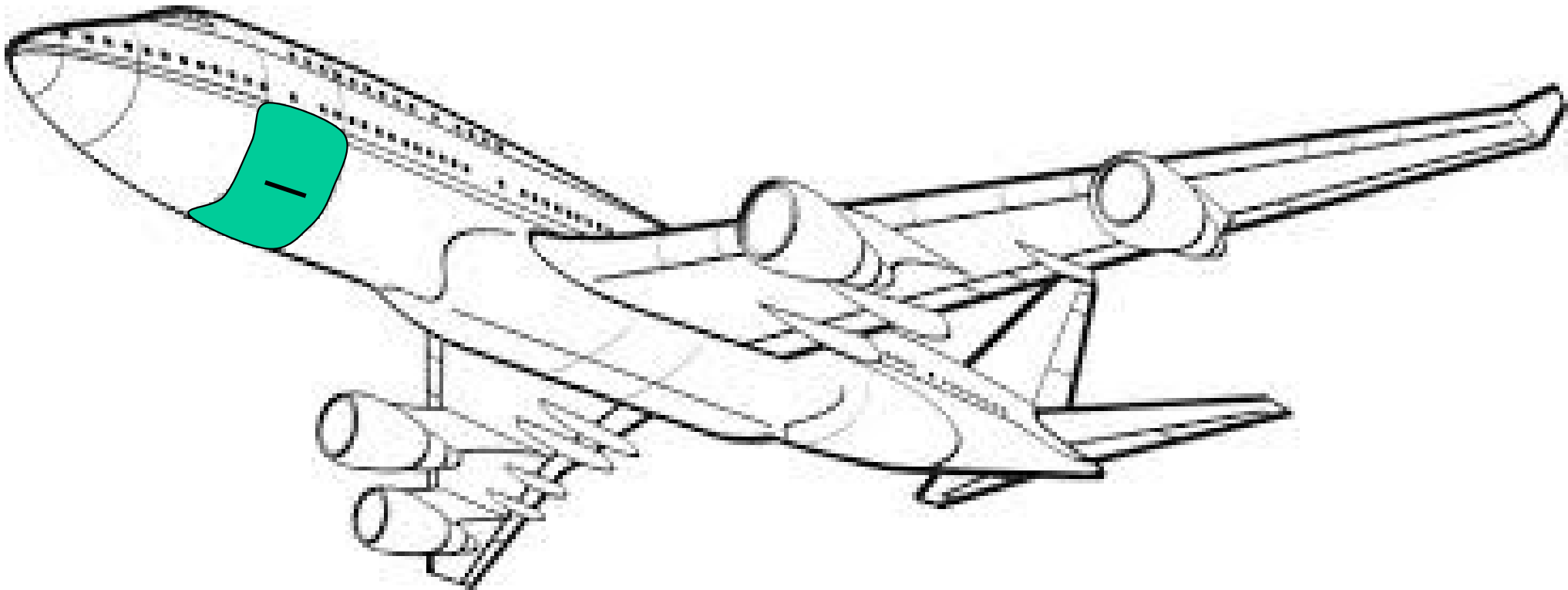
# Measured Critical Crack-Tip-Opening Angles (CTOA)

## Newman and Dawicke (1993)





# Why are “large” cracked panels important?



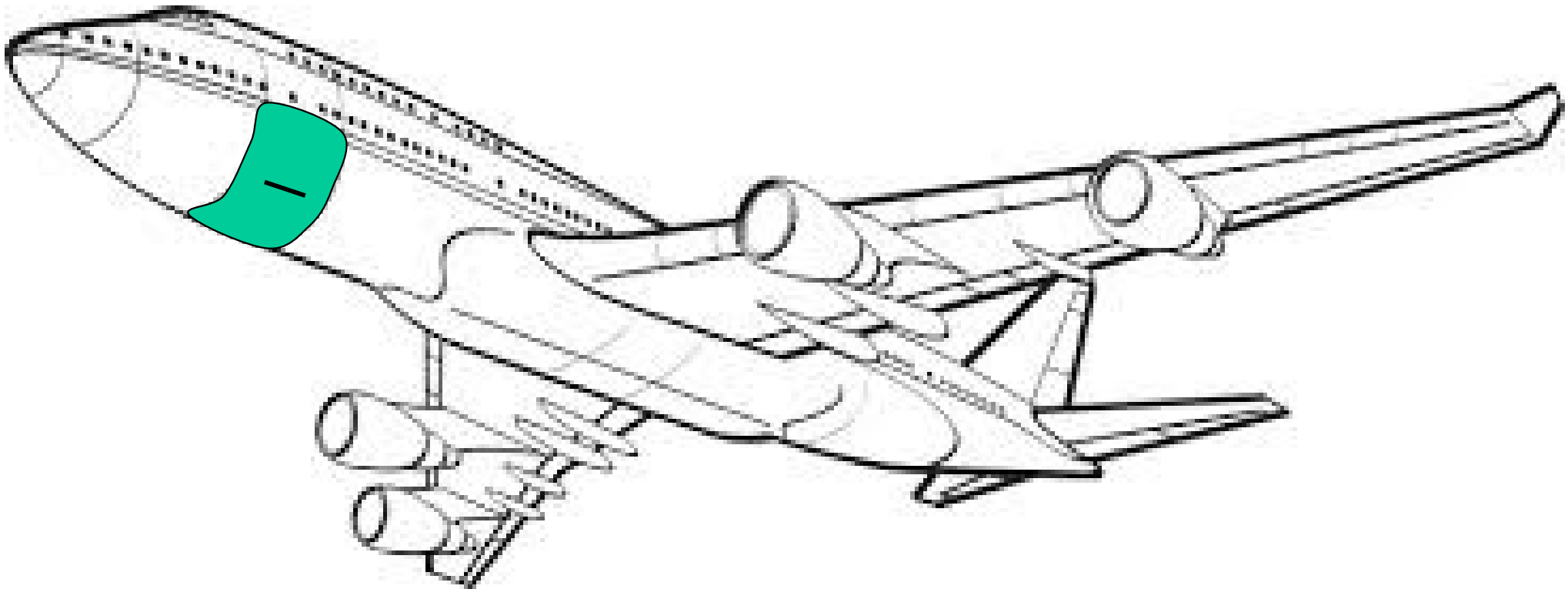
STAGS and



Hsu et al., “Residual Strength Analysis using CTOA Criteria for Fuselage Structures containing Multiple-Site Damage, Engineering Fracture Mechanics, Vol. 70, 2003.

Predicted internal pressure to fail damaged DC-9 fuselage at WPAFB was within 4% of the test pressure at failure

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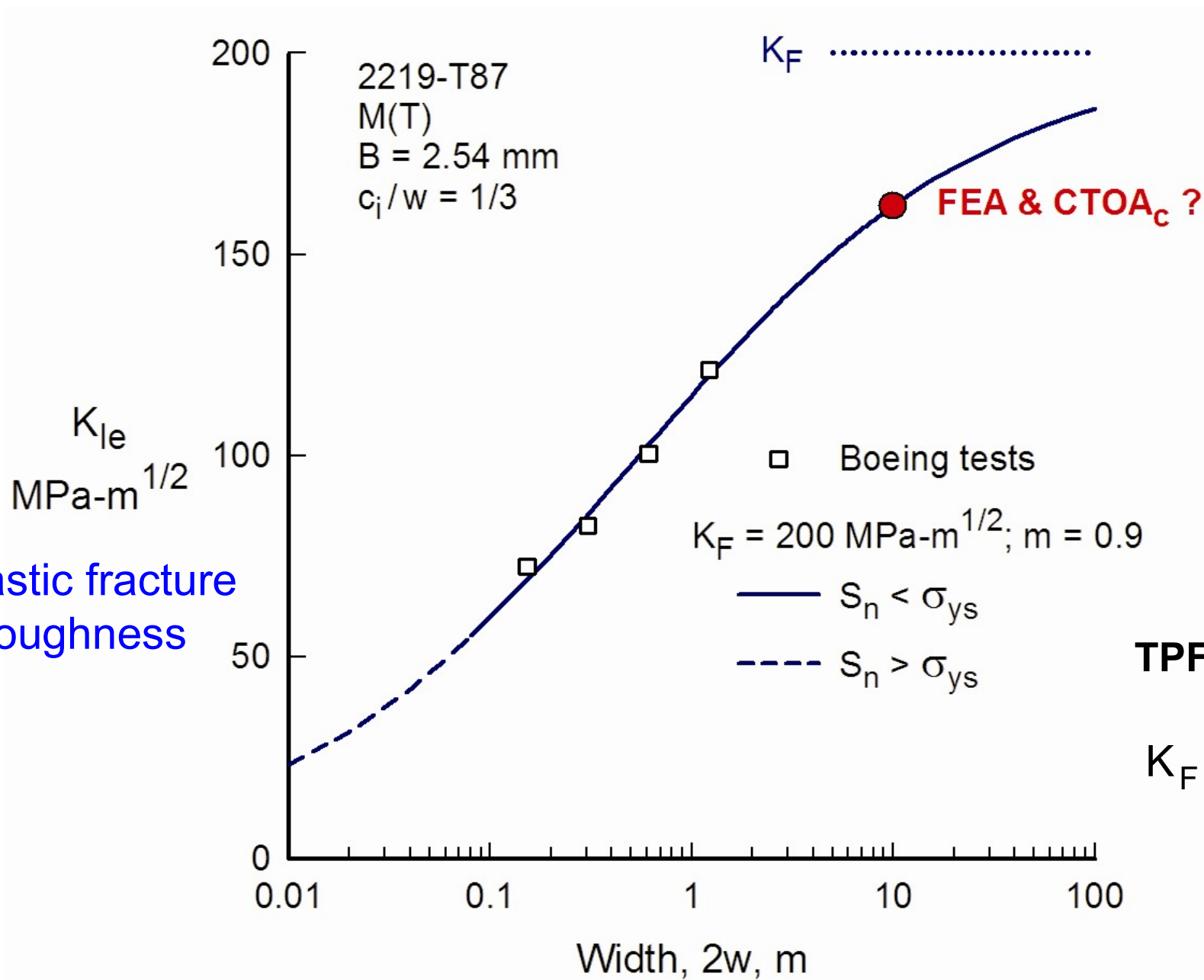
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Hsu et al., “Residual Strength Analysis using CTOA Criteria for Fuselage Structures containing Multiple-Site Damage, Engineering Fracture Mechanics, Vol. 70, 2003.

Predicted internal pressure to fail damaged DC-9 fuselage at WPAFB was within 4% of the test pressure at failure, **before the test was ever conducted !!!**

# Experimental and Analytical Relationship between $K_{Ie}$ and Specimen Width at Constant $c_i/w$ Ratio

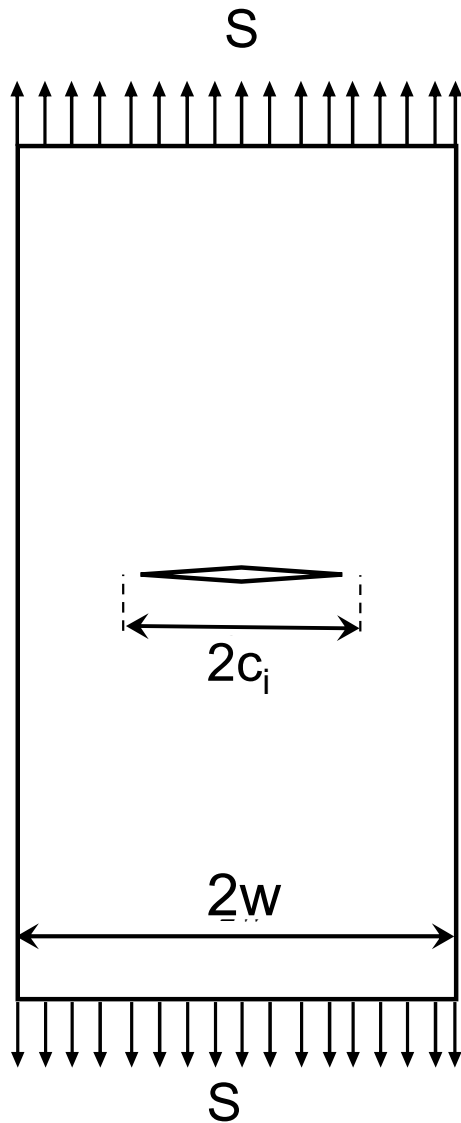


**TPFC (1973):**

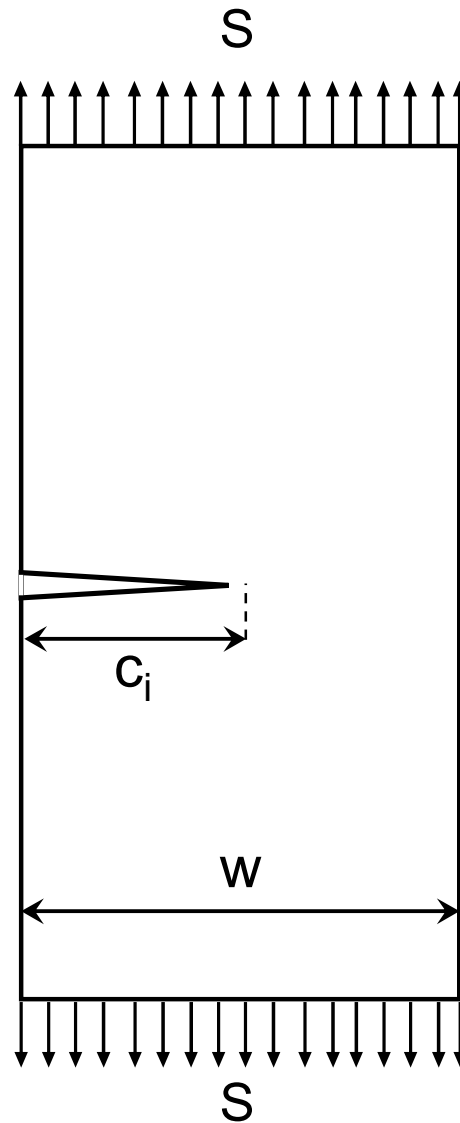
$$K_F = \frac{K_{Ie}}{1 - m (S_n / \sigma_u)}$$

$$S_n < \sigma_{ys}$$

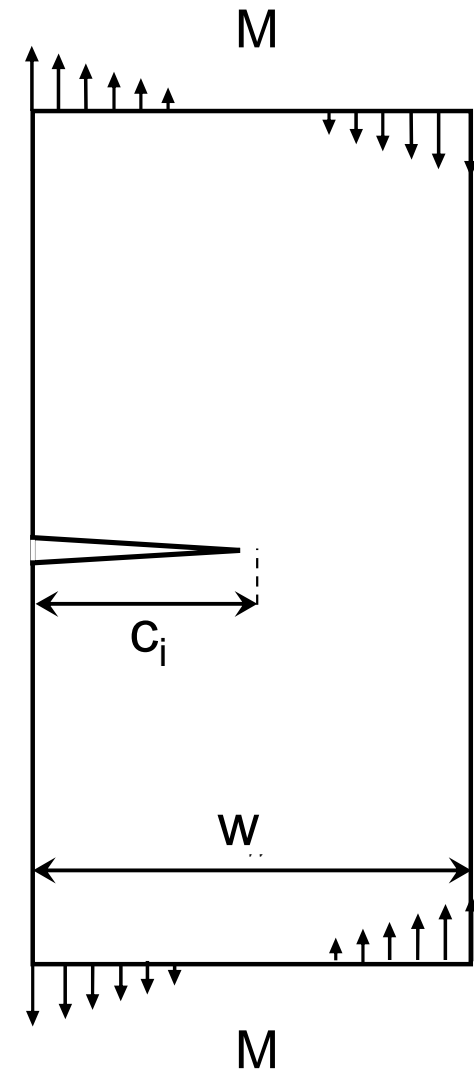
# Crack Configurations Analyzed



(a) Middle tension

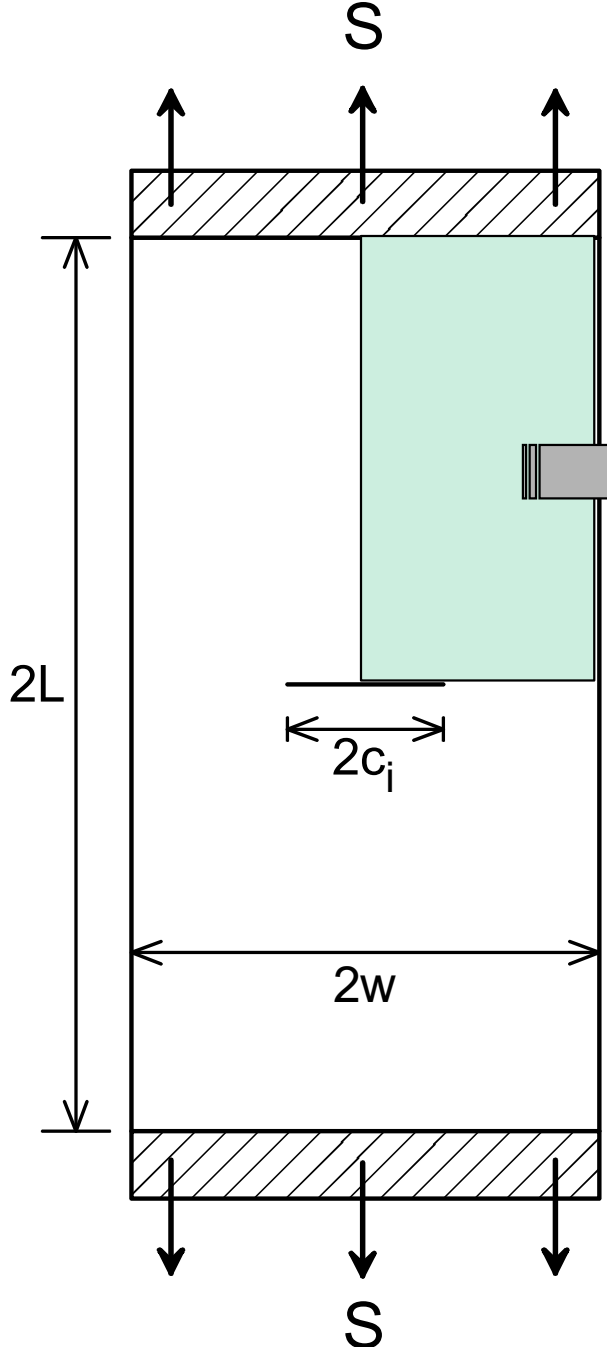


(b) Single-edge tension

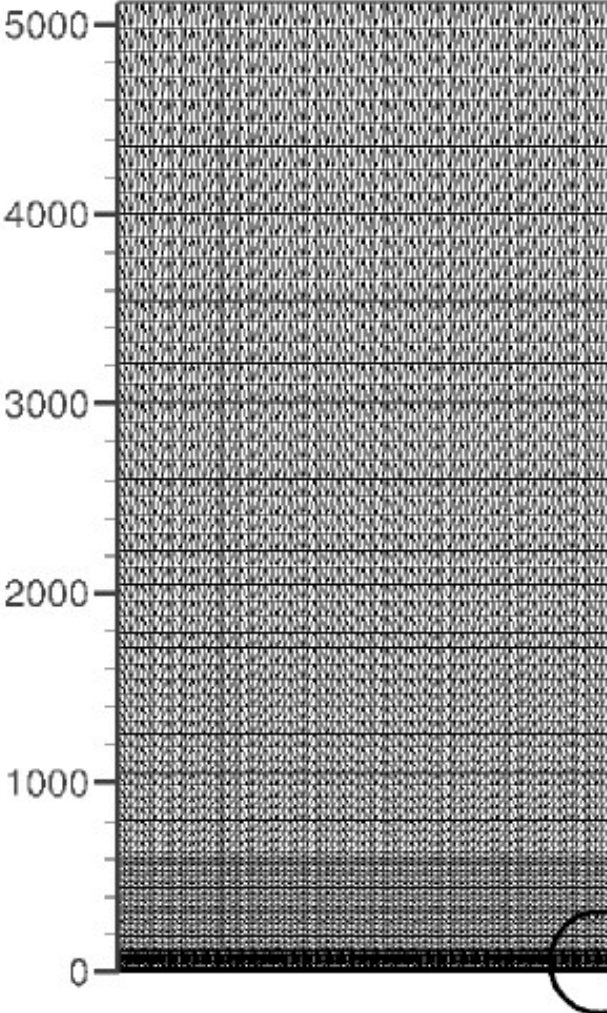


(c) Single-edge bending

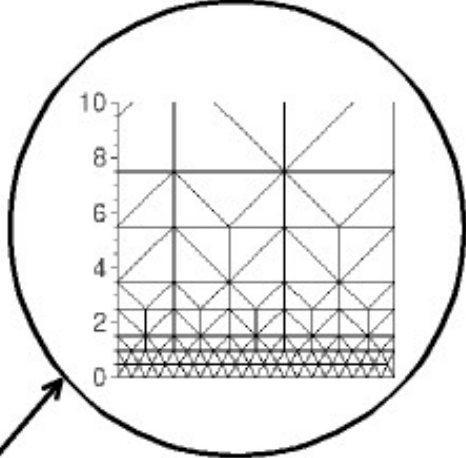
# Largest M(T) Specimen Model and Normal Size Man



Width =  $2w = 60$  to  $4,880$  mm (**2.4 in. to 16 ft**)  
 $c_i/w = 0.05$  to  $0.95$



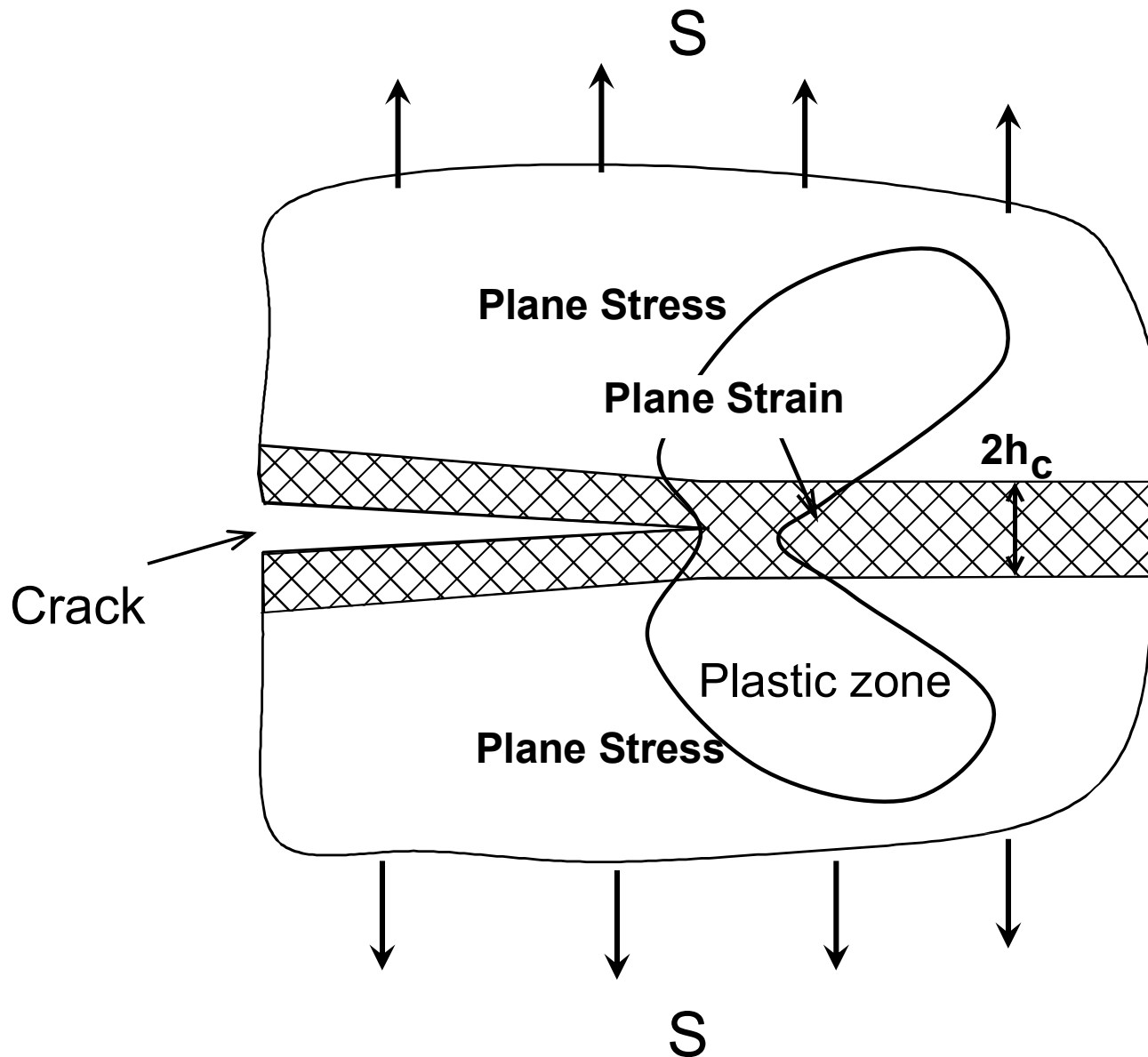
ZIP2D:  
 ~ 57000 elements  
 ~ 31000 nodes



(a) One-quarter model

(b) Local mesh pattern

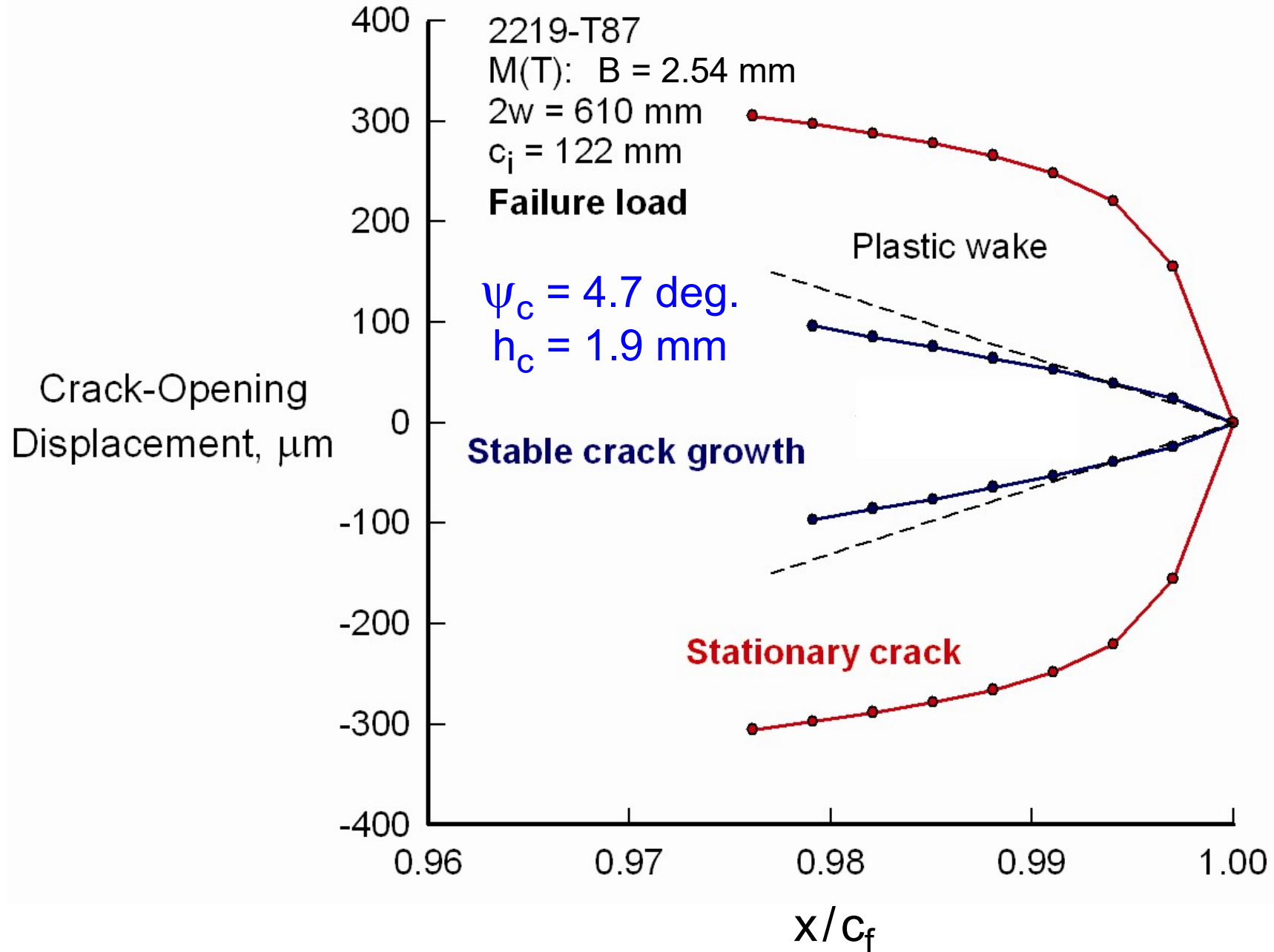
# Plane-Strain Core in Plane-Stress Finite-Element Models



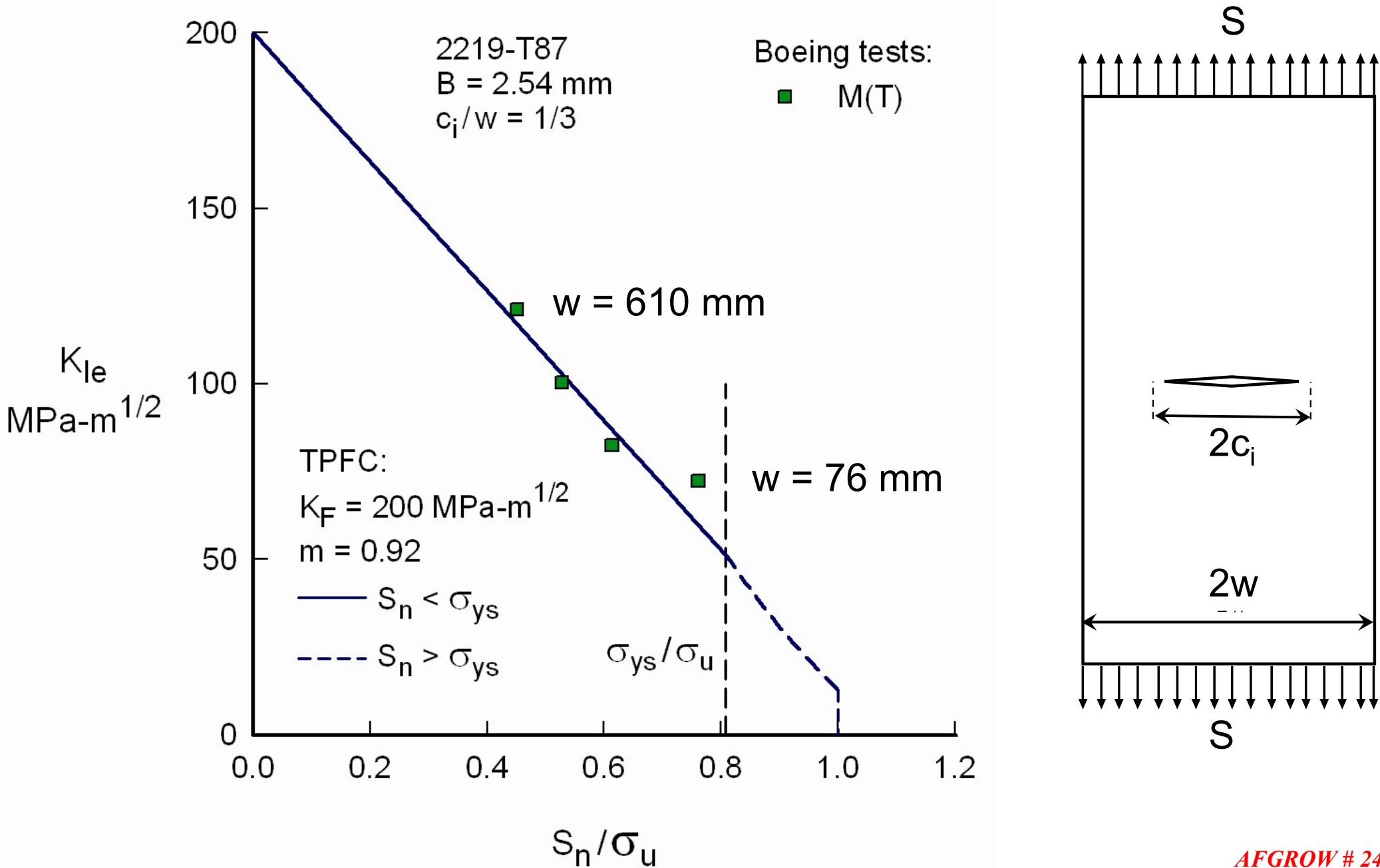
## Finite-element codes:

- ZIP2D
- STAGS
- FEA – C.T. Sun (Purdue Univ.)

# Crack-Opening Displacements for Stationary and Stably Tearing Crack



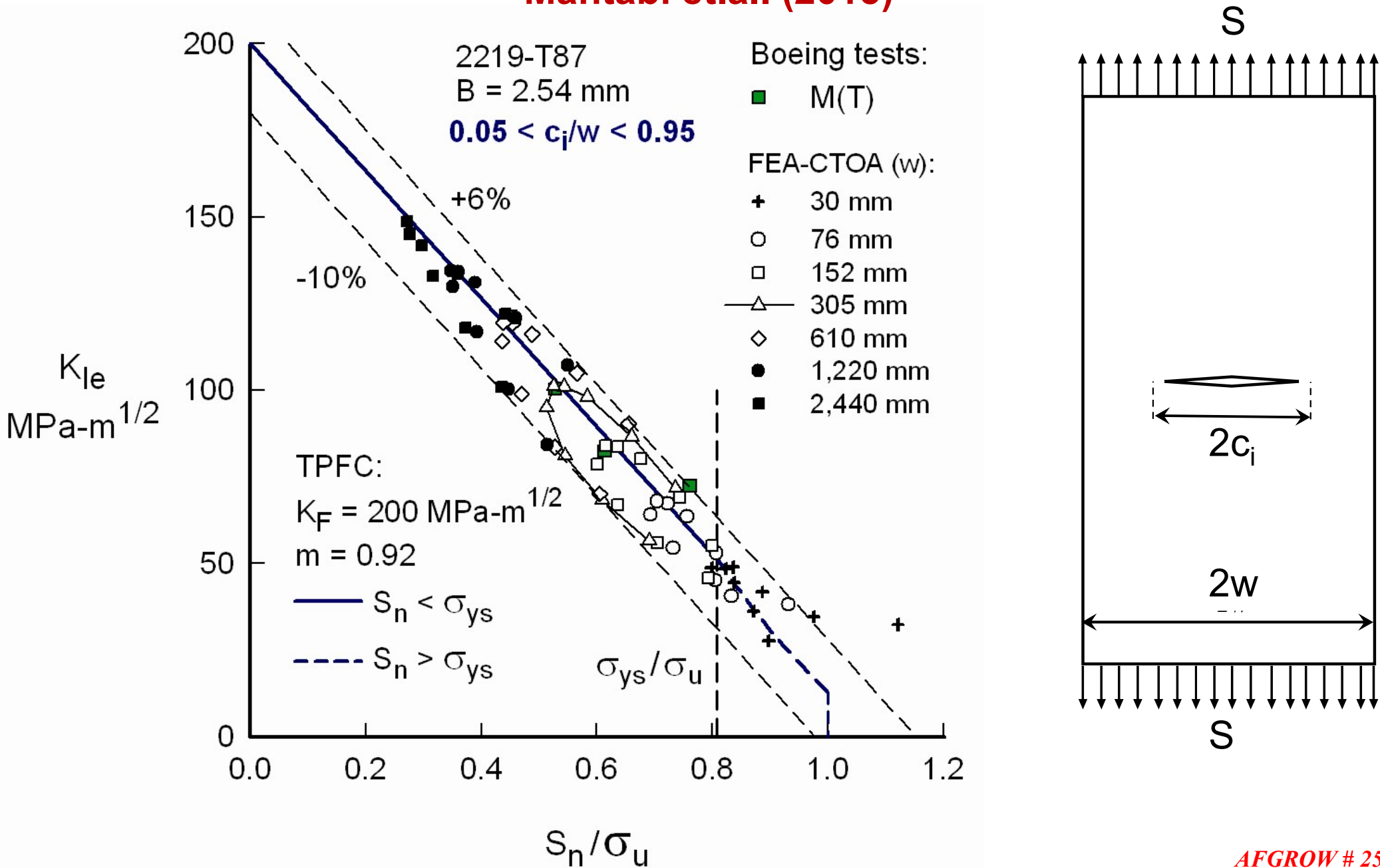
# Two-Parameter Fracture Criterion (TPFC) Analysis on 2219-T87 Aluminum Alloy M(T) Specimens





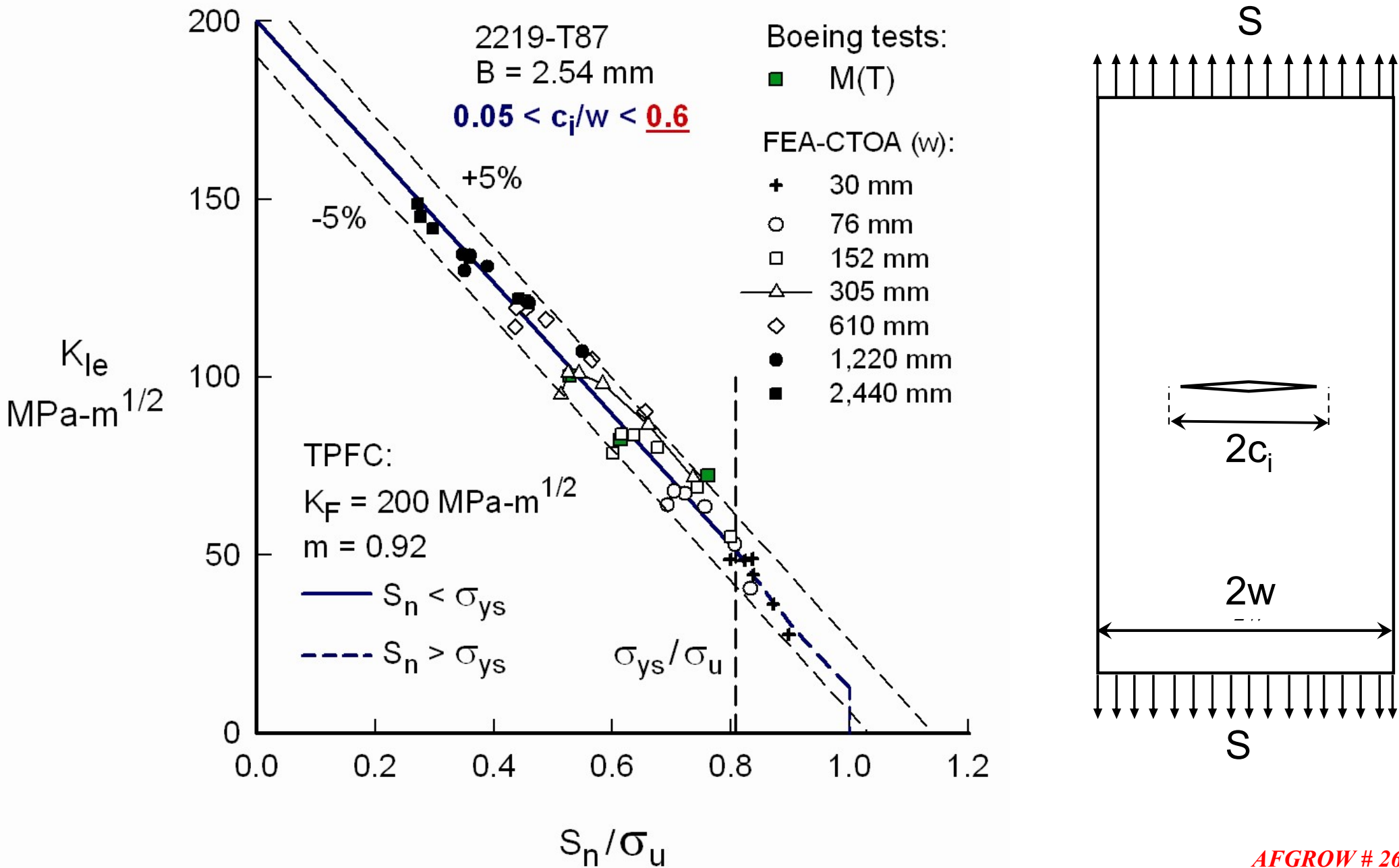
# Elastic Stress-Intensity Factor at Failure for Wide Range of *Middle-Crack Tension* Specimens

Mahtabi et.al. (2015)



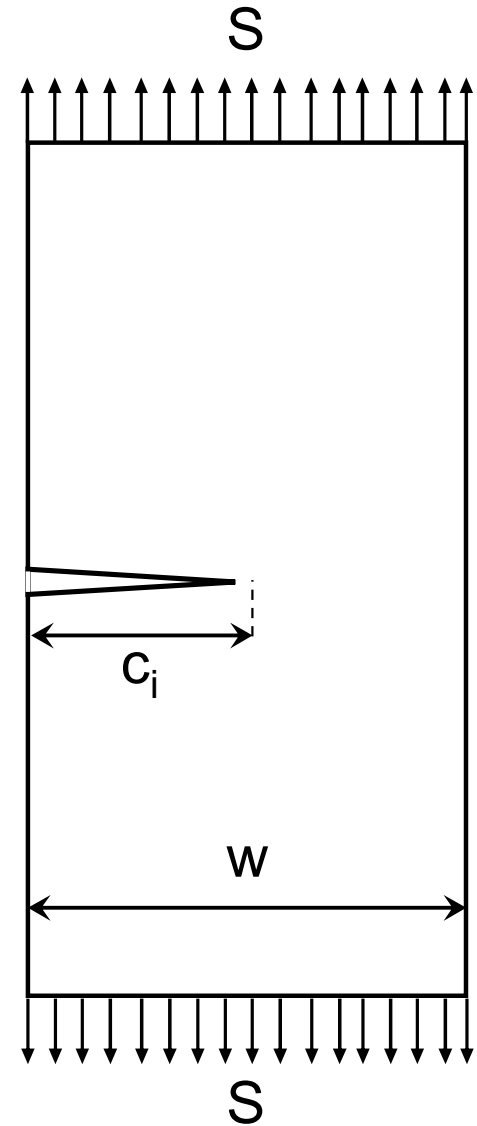
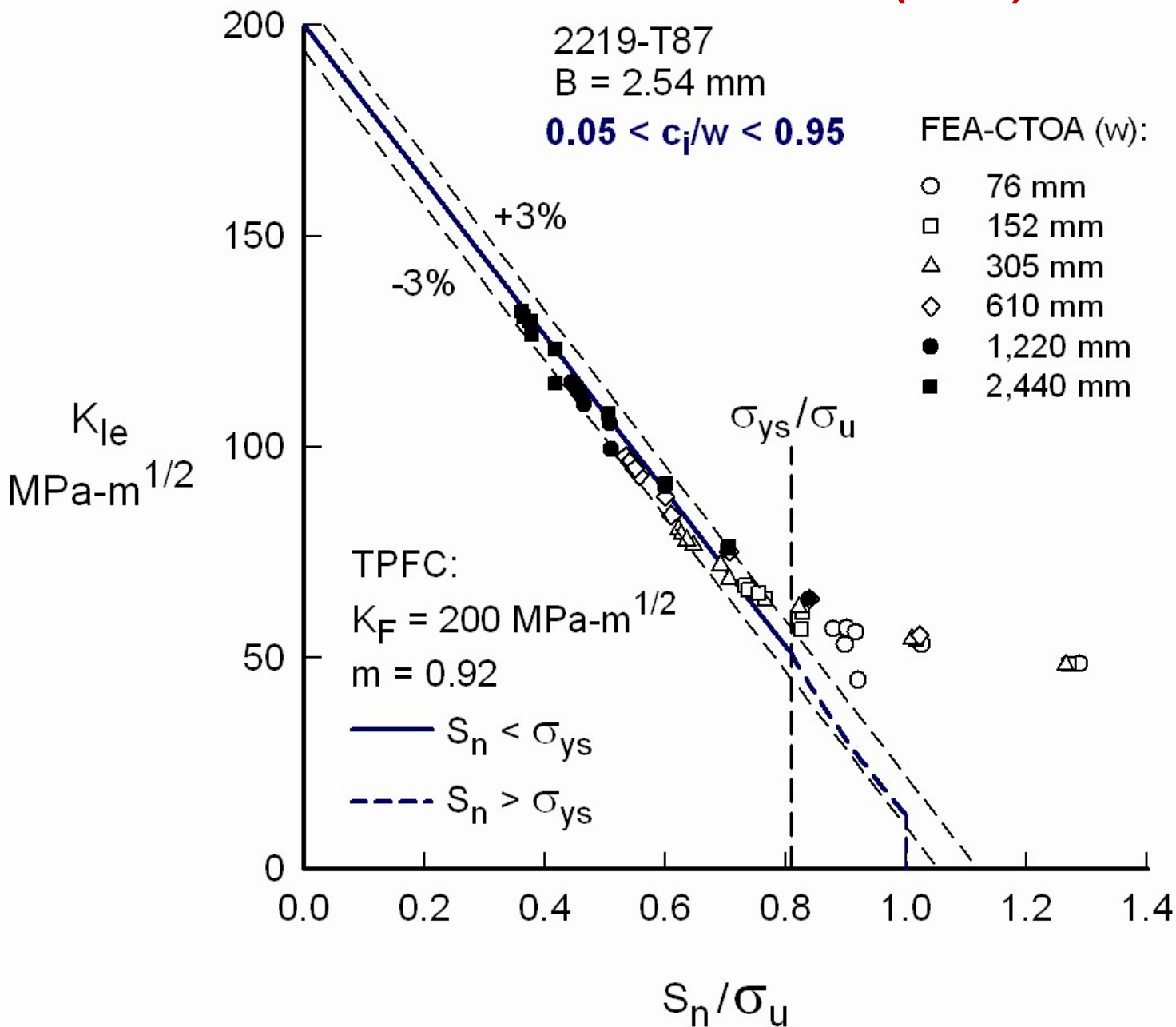
# Elastic Stress-Intensity Factor at Failure for Limited Range of *Middle-Crack Tension Specimens* (2)

Mahtabi et.al. (2015)



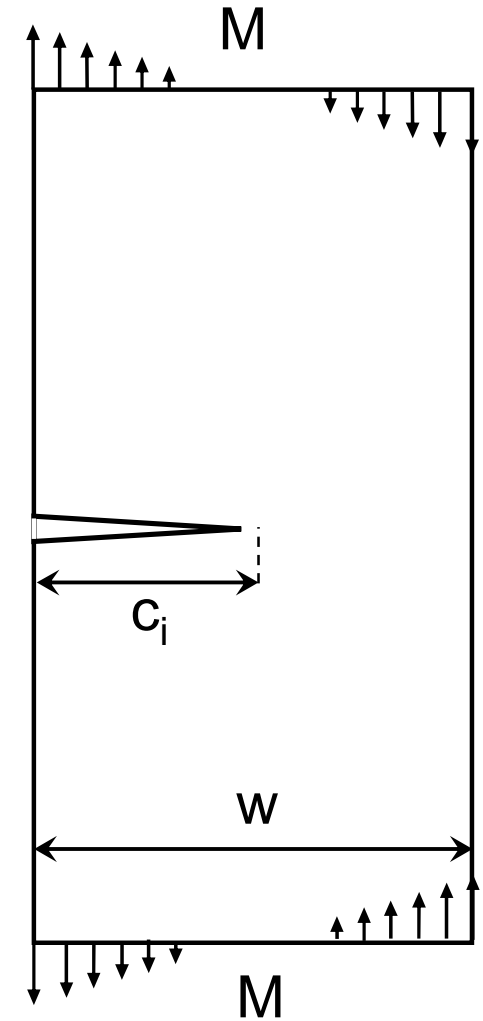
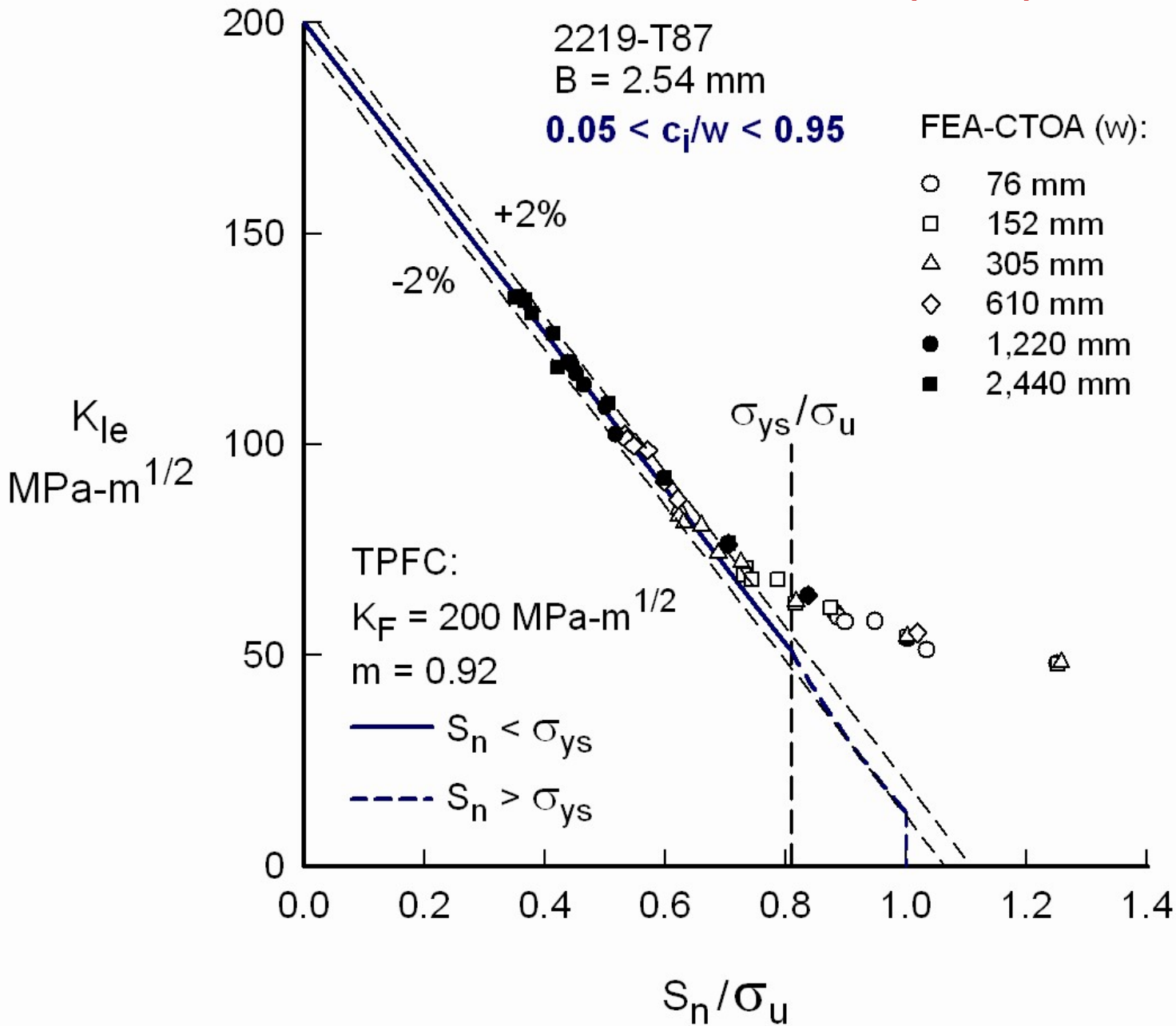
# Elastic Stress-Intensity Factor at Failure for Wide Range of *Single-Edge-Crack Tension* Specimens

Mahtabi et.al. (2015)



# Elastic Stress-Intensity Factor at Failure for Wide Range of *Single-Edge-Crack Bend* Specimens

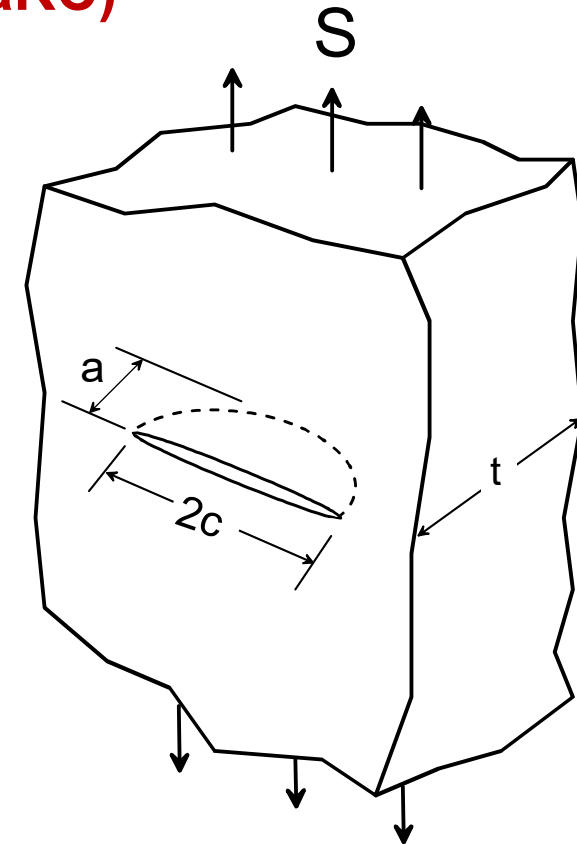
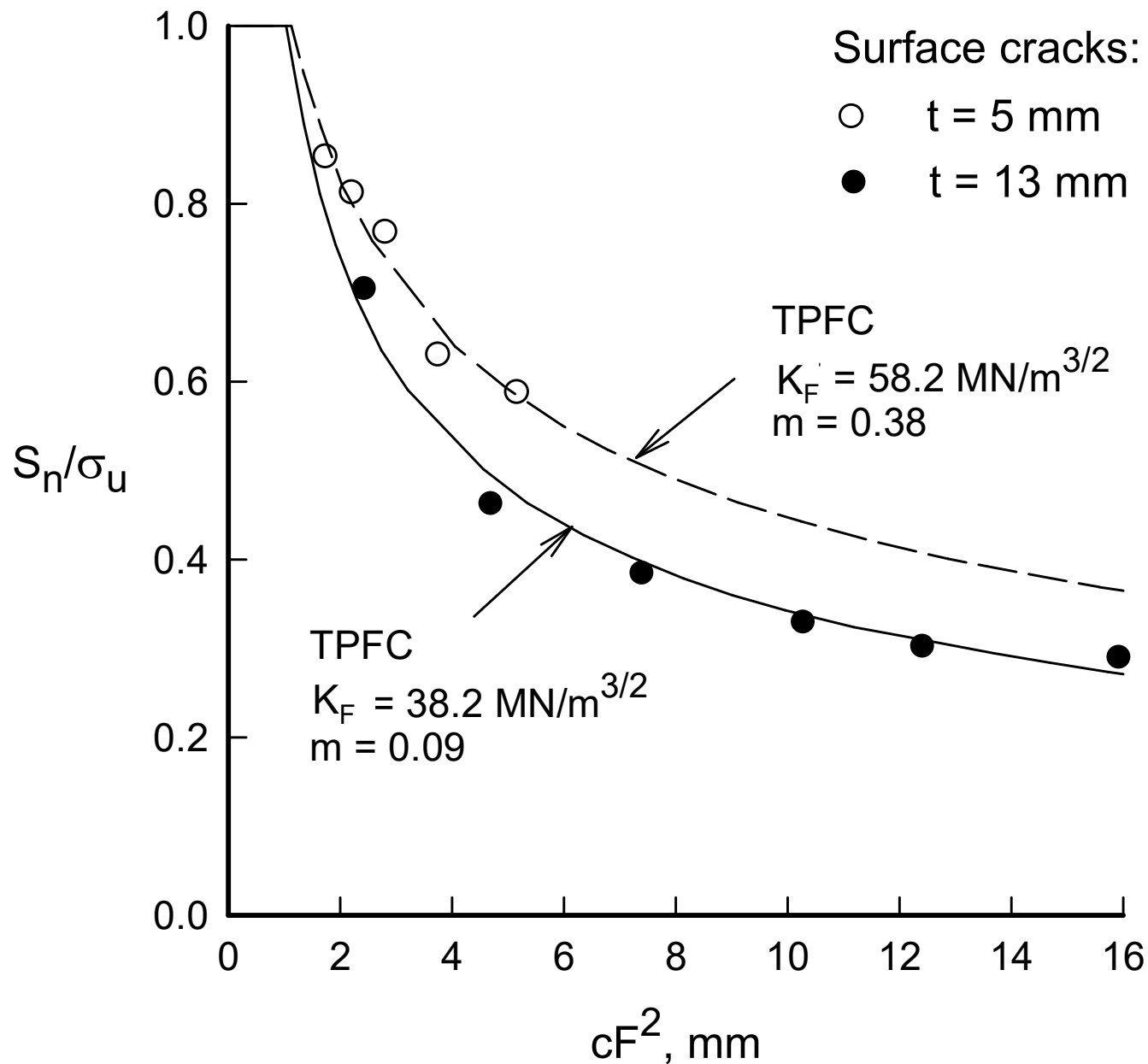
Mahtabi et.al. (2015)



# Surface-Crack Tests and TPFC Analyses

# Residual Strength Analyses of Surface Cracks

## 7075-T651 Aluminum Alloy (NASA LaRC)

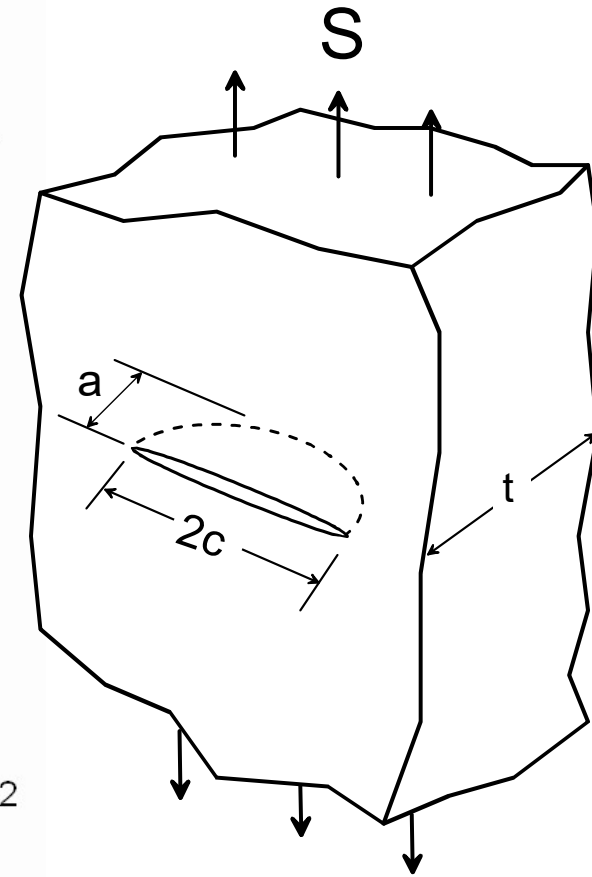
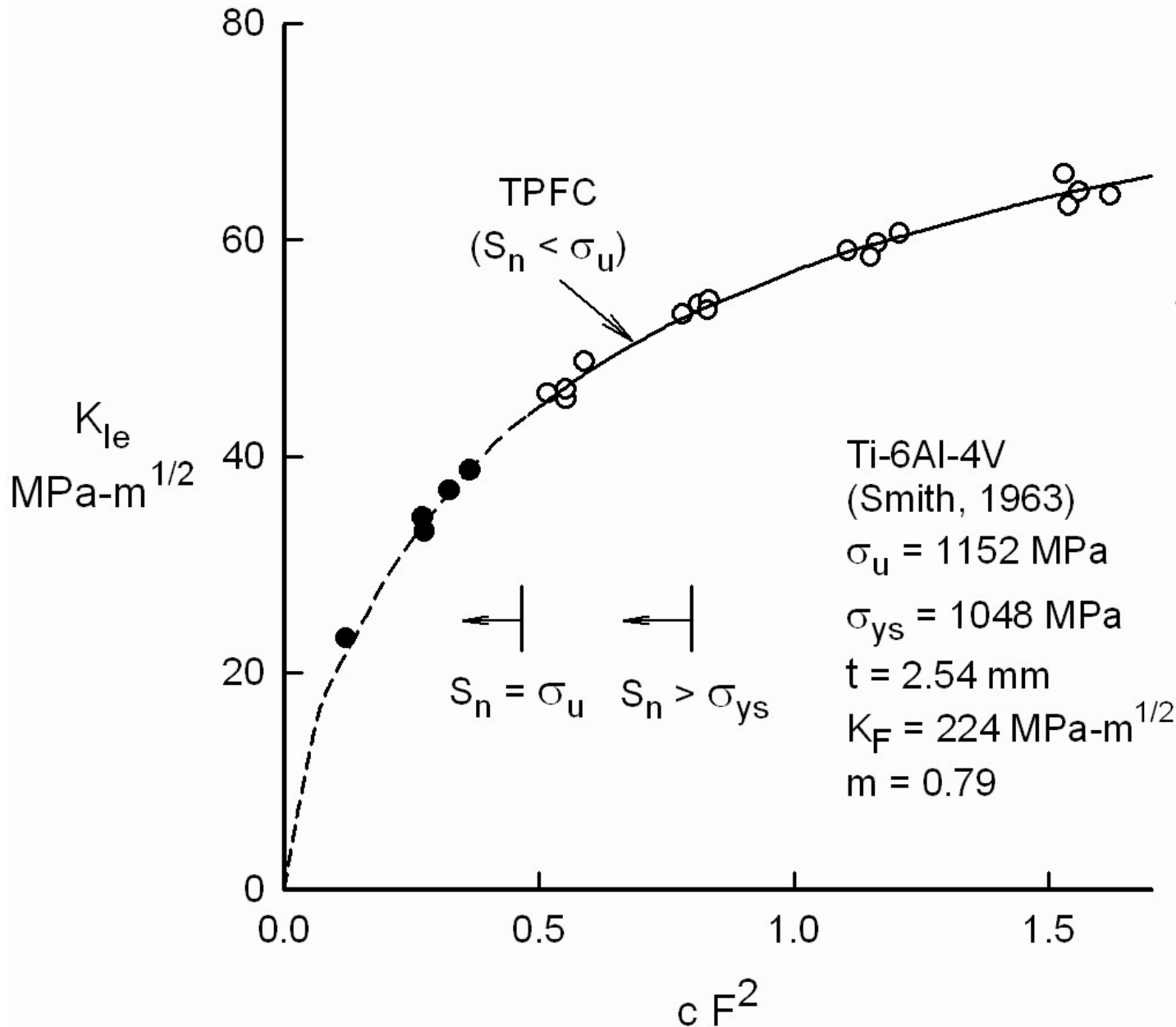


$$K_{Ie} = S \sqrt{(\pi c)} F(a, c, w, t)$$

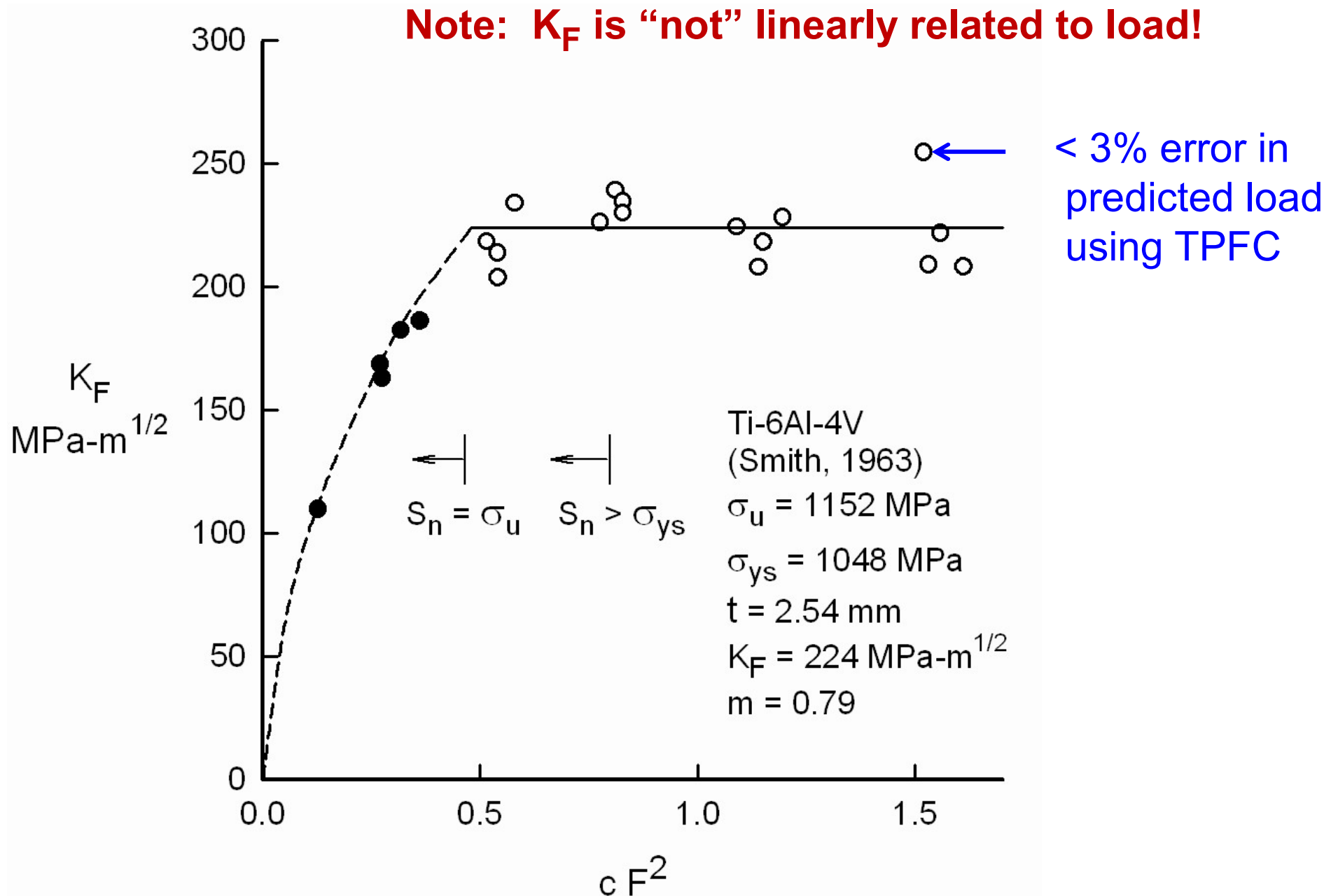
$$\phi_c = 30 \text{ deg}$$

# Elastic $K_{Ie}$ at Failure is Uniquely Related to $cF^2$

Wide range in  $a/c$  and  $a/t$  values!



# Fracture Toughness, $K_F$ , is Constant for $S_n < \sigma_{ys}$





# Concluding Remarks

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- Relationship between  $K_{Ie}$  (elastic stress-intensity factor at failure) and  $S_n$  (net-section stress) is “linear”.

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- EPFEA-CTOA fracture simulations were **consistent** with  **$K_{Ie}$ - $S_n$  linear** behavior in **TPFC** for M(T), SEC(T) and SEC(B) specimens for net-section stresses less than yield stress ( $S_n < \sigma_{ys}$ ).

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- TPFC failure analyses on “tension and bend” specimens were very good for large widths, but not for small widths ( $S_n > \sigma_{ys}$ ).
- **More work** required on TPFC for **bend-type loading** ( $S_n > \sigma_{ys}$ ).

# Future Recommendations

---

1. **ASTM E-740 – Surface-Crack Standard – Does *NOT* have a fracture toughness assessment, but something like the **TPFC** needs to be added. Method is **simple and very accurate** for **residual-strength analyses of surface cracks**. (ASTM E-2899 is only for “initiation” toughness and *NOT* residual strength.)**

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2. ASTM E-08 **needs** a simple **fracture toughness standard** for ***through cracks*** in **low-constraint, high toughness materials** that works for both ***tension and bending*** crack configurations.



*Thank You, Thank You  
Very Much!*

*Questions?*

