A Pin-Load Model for Worst Scenario K_I(ϕ)-Analysis

by

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Contents

- Accurate and reliable stress intensity factor *K*-calculation for cracks in *spectrum loaded joints* is practically impossible due to the very complex contact, friction and stick-slip conditions that take place between plates and fastener-plates during the fatigue life. We exemplify this by summarizing some experimental findings.

- An easy-to-use analysis tool is needed as a basis for safe design and maintenance of old aircraft. In the lecture we describe how we determined the *maximum K-value* that can exist at each point along a crack front in a structural joint.

- We also show that the pin-load model USAFA is using to derive hundred of millions of *K*-functions to be used in AFGROW provide *K*-values that are within a few % of the highest *K*-value possible, i.e. a value that will probably never be reached under practical situations.

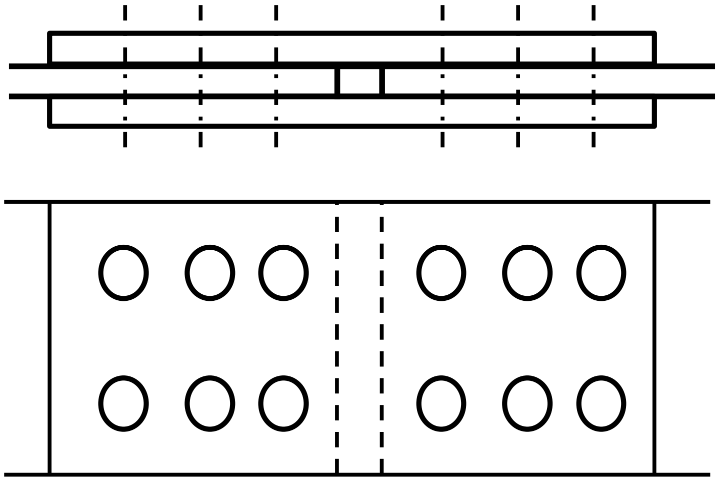
Use of instrumented bolts to monitor complex structural joint behavior

Instrumented bolts for measurement of shear load transfer in single and double shear joints.

B. Palmberg. The Aeronautical Research Institute of Sweden, FFA TN 1991-09, Aug 1991, 87 pp.

Structural Joints:

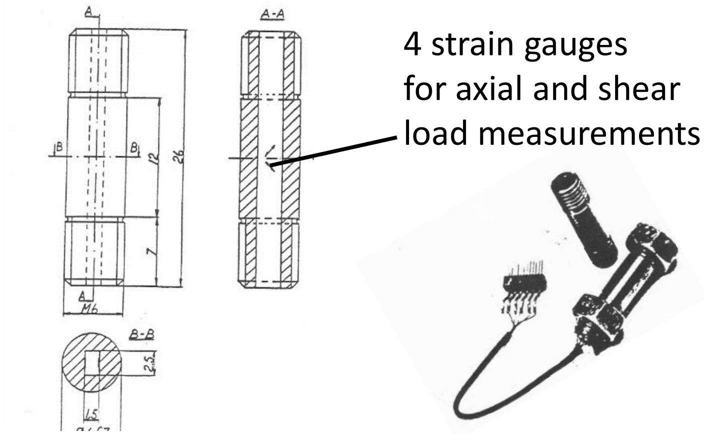
- Instrumented single-lap joints
- Instrumented double-lap joints
- One-, or two bolt rows
- 1-3 bolts/row
- With and without backing plate
- Static loading, constant amplitude and spectrum loading.
- Plate thicknesses 1.5 and 3.0 mm



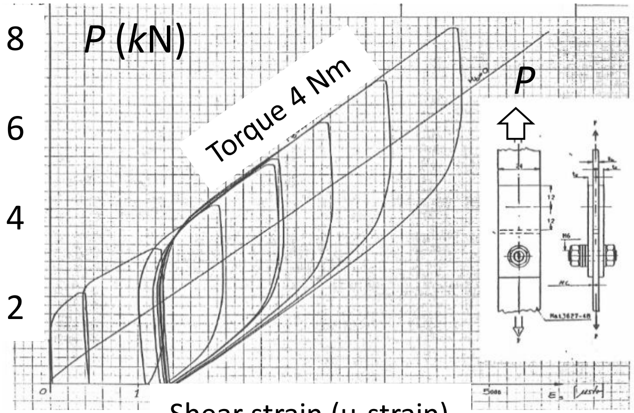
Double shear joint with 3 bolts in 2 rows

Instrumented bolt, 6 mm diam.

Torque 4 Nm, $F_{axial} = 4\ kN$



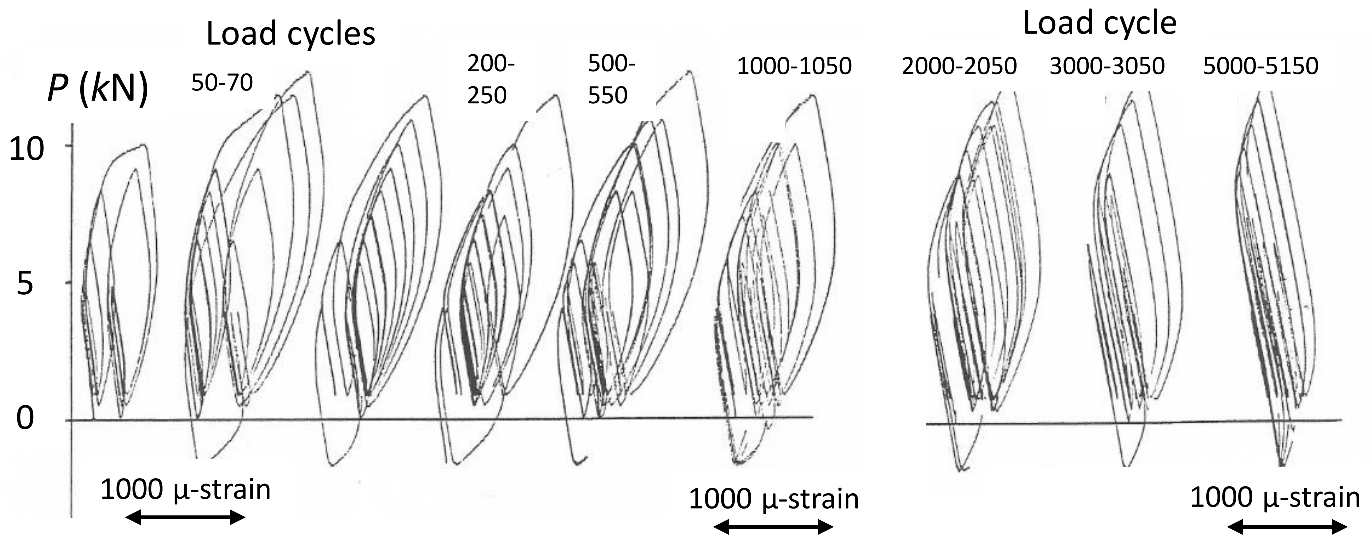
4 strain gauges for axial and shear load measurements



Shear strain (µ-strain)

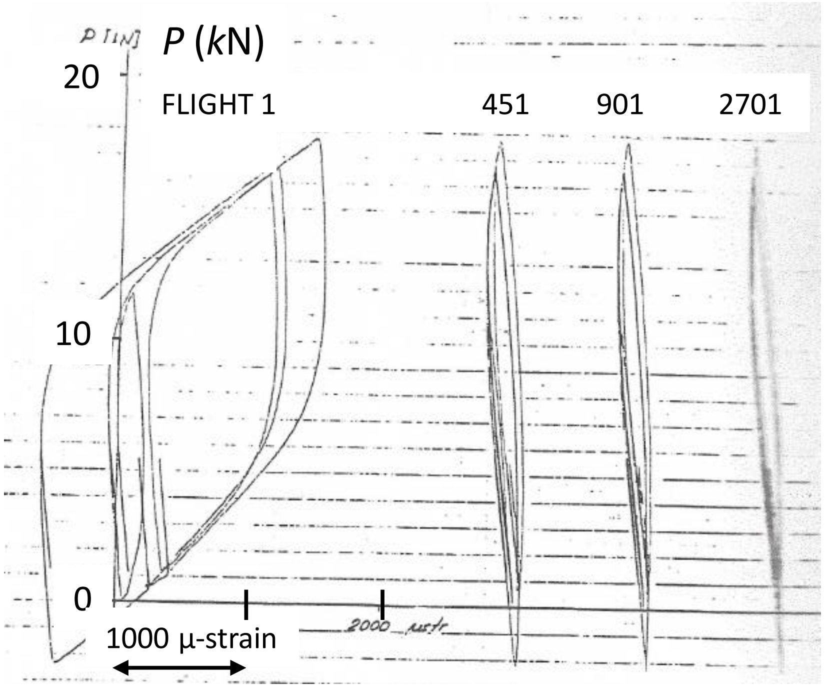
Measured shear strain during cycling with stepwise increase in maximum load.

Example: Evolution of hysteresis loops during spectrum loading. Double-shear, triple row, 1.5,3,1,5mm.



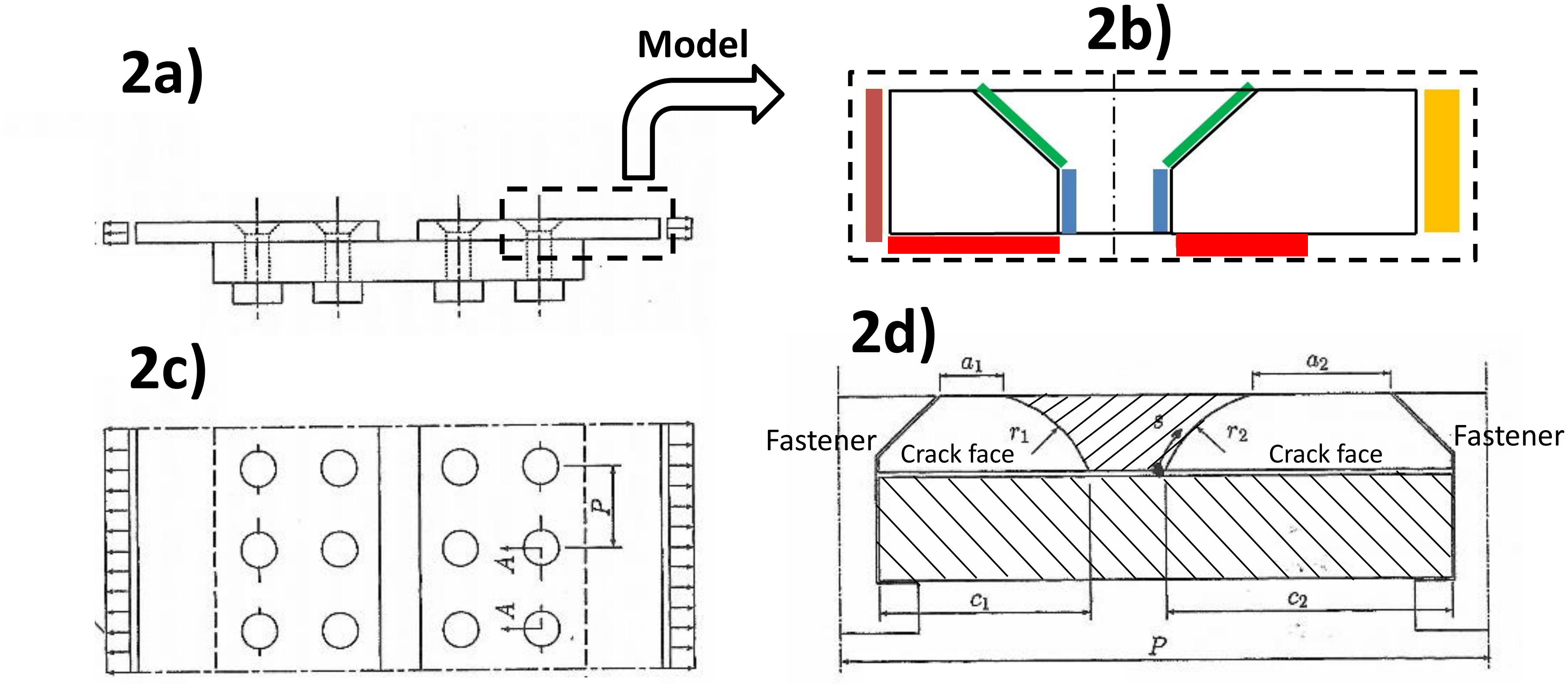
Example: Hysteresis loops during spectrum fatigue loading (wing root bending, 5000 max load cycles corresponds to 300 flights). Single shear, double row, double column with a T-shaped splice plate.

- After 4501 flights, the bolt load is out of phase with the applied load and the load transfer variation is less than 10% of the specimen load variation.
- The base and splice plates perform almost as one integrated material.
- Crack growth predictions which do not consider these effects tend to be overly conservative?



The Pin-Load Model

- Figure 2b shows the 3D model domain we will use for *K*(ϕ)-analysis. Red, blue … rectangular areas mark where surface loads are applied in the model. The model in Figure 2b will *provide exact K*(ϕ)-data provided that *the surface tractions are exactly known.*



Section A-A

Figure Model for calculation of conservative *K*-data

Basic load systems used for *K*-calculation in the optimization process

Figure 3 shows a cross section of a part of the mathematical model used to derive conservative *K*(ϕ)-data. Contact, friction, pre-strain in the fastener/bolt and external loads are represented exactly in the load systems $q_i(r,z,\psi)$, $i=1\ldots9$ and $q_{11}(r,z,\psi)$, $q_{12}(r,z,\psi)$,

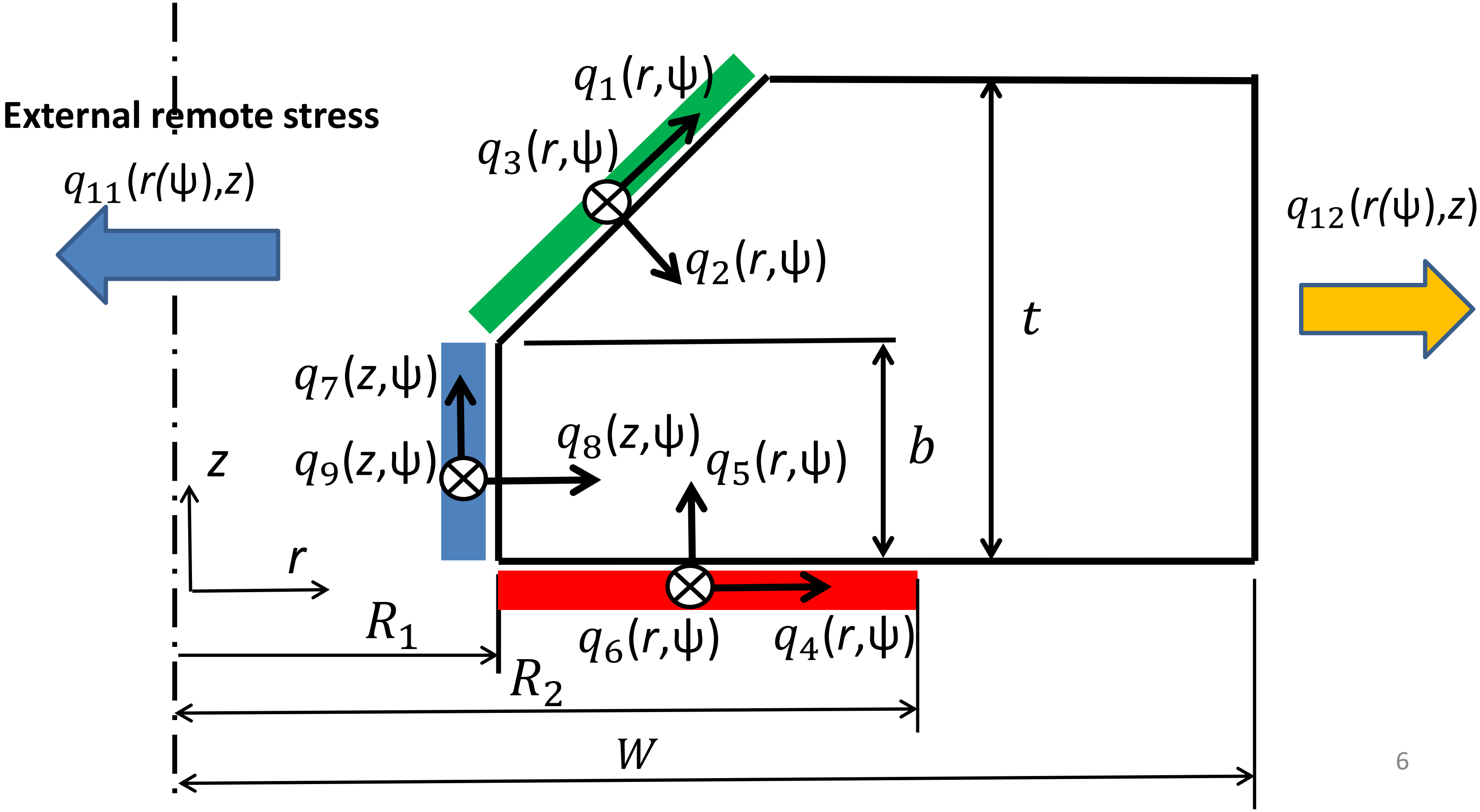
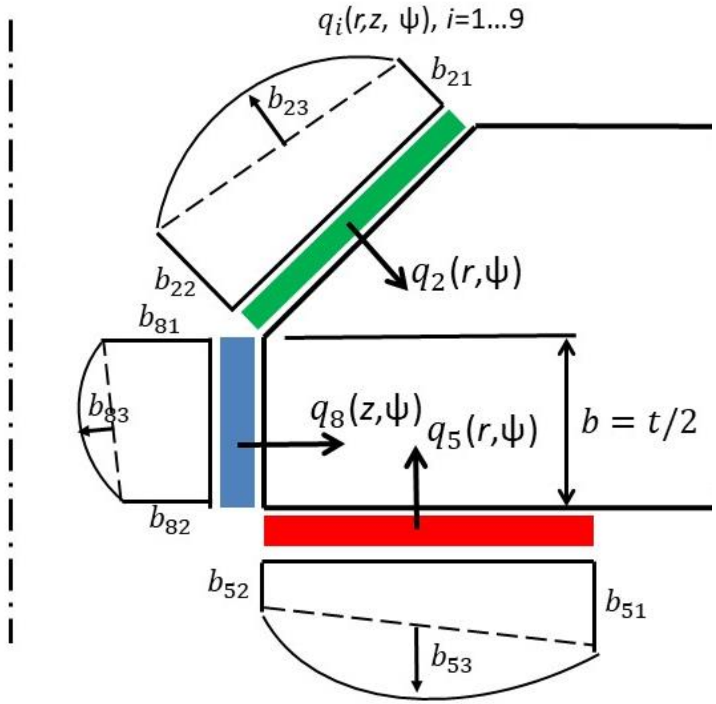


Figure 3 Definition of load functions q_1 to q_9

Approximations ! Functions q_1 *to* q_9 used in numerical analysis.

- In the numerical analysis we approximate load q_1 to q_9 with simple polynomials.
- Hence we define 9x3 basic load systems for *K*-calculation.
- Each of these 27 load systems we construct in such a way that each of them are in force and moment equilibrium. The figure shows the basic functional behavior of the load functions q_1 to q_9 used in the numerical analysis. The coefficients α_{ij} are the unknowns to be determined in the optimization process.



$$\boldsymbol{q_i(r,z,\psi)=\sum_{j=1}^{J}\left(\alpha_{ij}\cdot b_{ij}\,(r,z)\cdot f_{ij}(\psi)\right)}$$

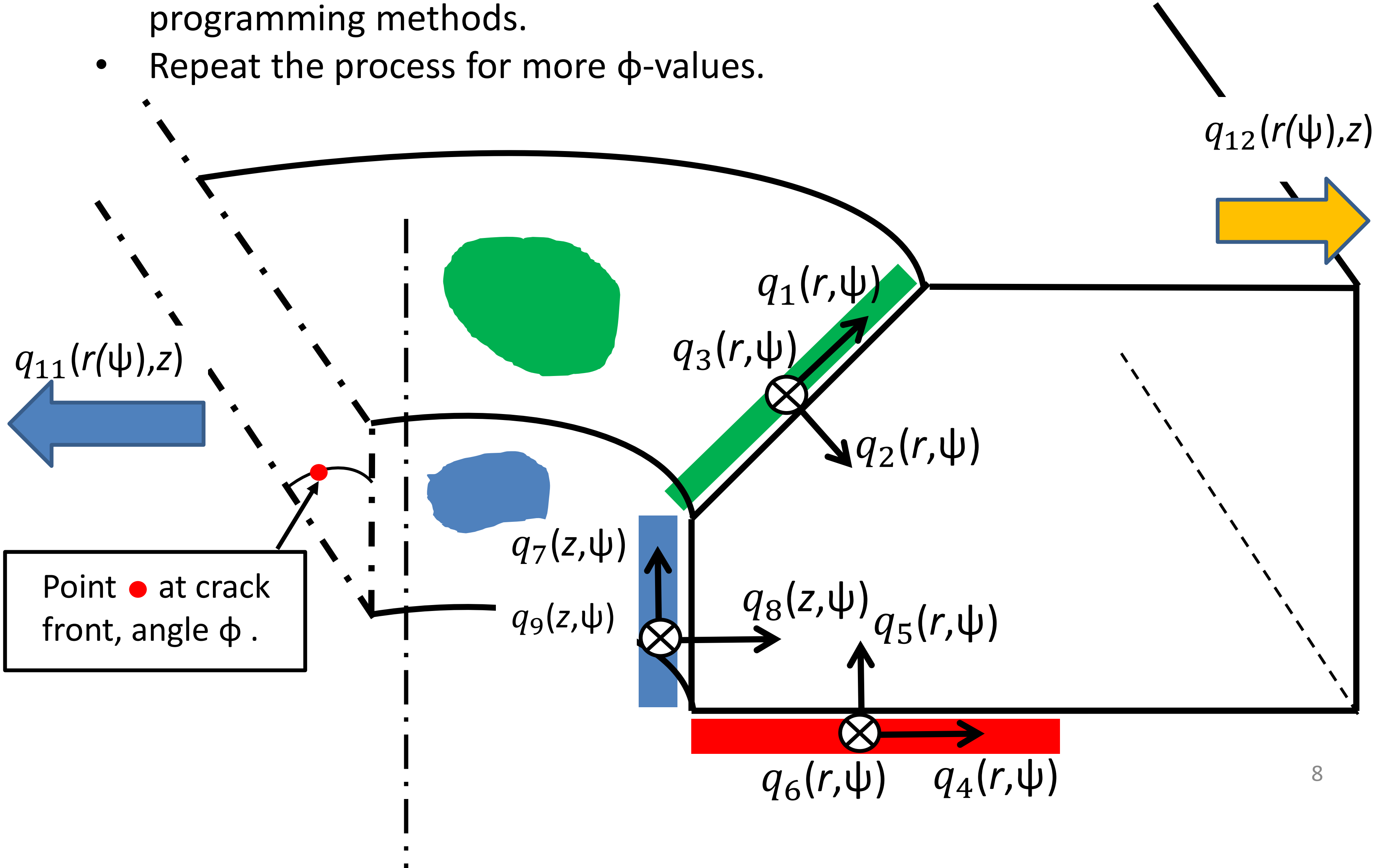
$$\boldsymbol{b_{11}=\frac{z-b}{t-b}}$$

$$\boldsymbol{b_{12}=\left(1-\frac{z-b}{t-b}\right)}$$

$$\boldsymbol{b_{13}=4\cdot\frac{z-b}{t-b}\cdot\left(1-\frac{z-b}{t-b}\right)}$$

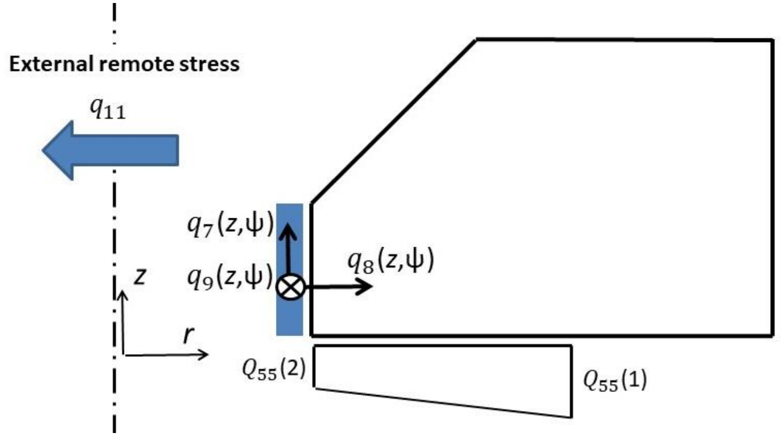
The optimization process

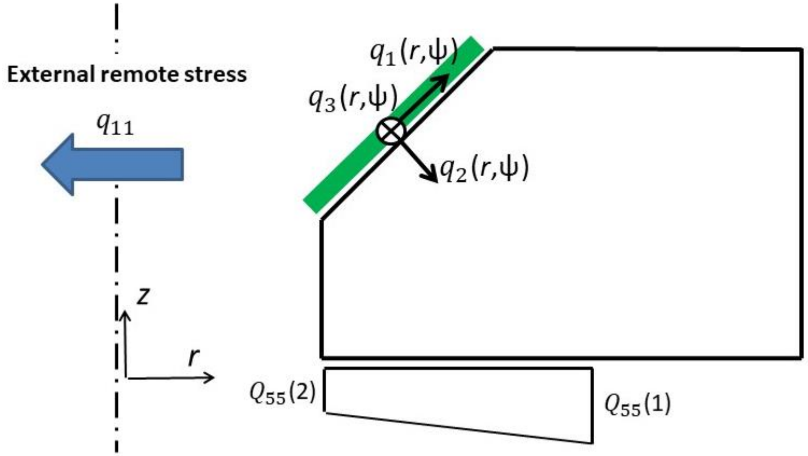
- Select ϕ where maximum $\boldsymbol{K_I(q_1(r,z,\psi), \ldots q_9(r,z,\psi))}$ is sought.
- Determine $q_1(r,z,\psi) \ldots q_9(r,z,\psi)$ by using mathematical programming methods.
- Repeat the process for more ϕ-values.

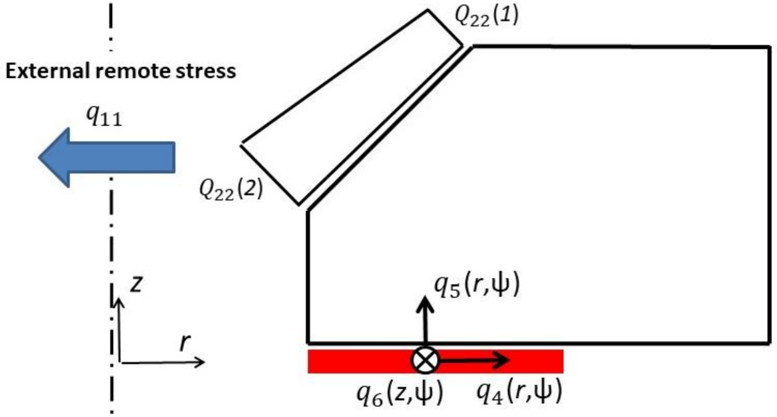


Round-off errors: How to get rid of them practically.

- It is possible to apply loads $q_i(r, z, \psi)$ function by function and satisfy equilibrium by allowing large reaction forces in nodes with prescribed rigid body displacements. This may lead to large round-off errors, so we avoid doing so here.
- The principle we use to satisfy equilibrium is to add one or several of three auxiliary load systems labelled Q_{11}(horizontal), Q_{22}(normal to cone surface) and Q_{55}(vertical), respectively.







3. The test case

We provide numerical data for the countersunk geometry shown in the Figure. (The choice of q_{11} = 1 and the large c-value only needs some care when defining the clamping force in the optimisation studies.)

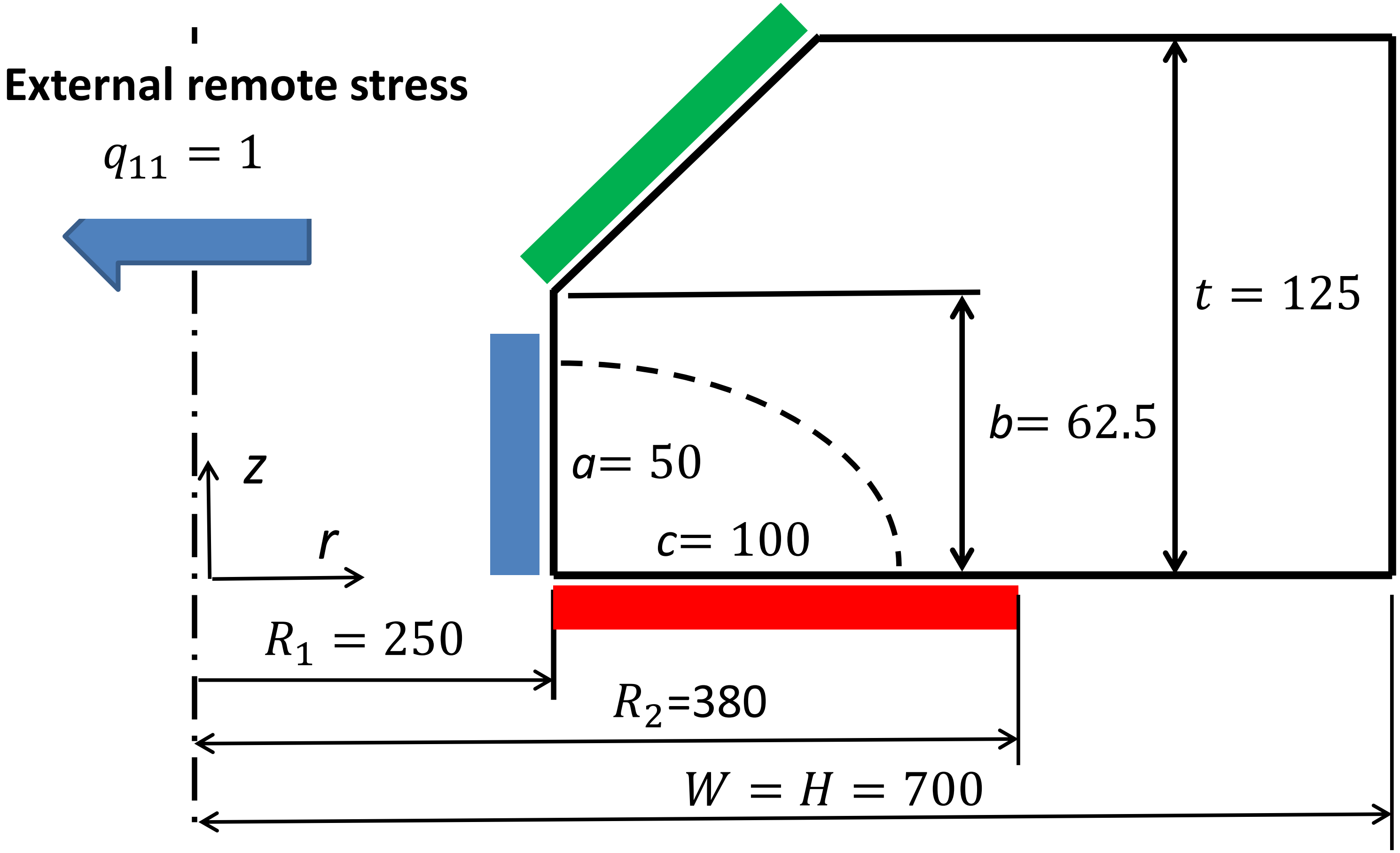
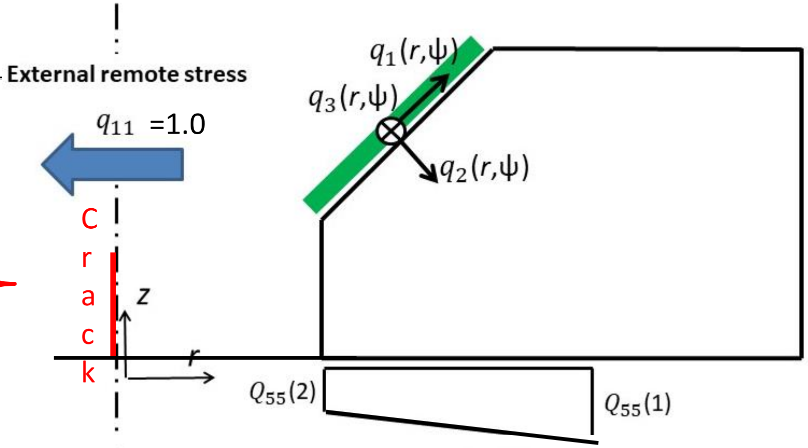


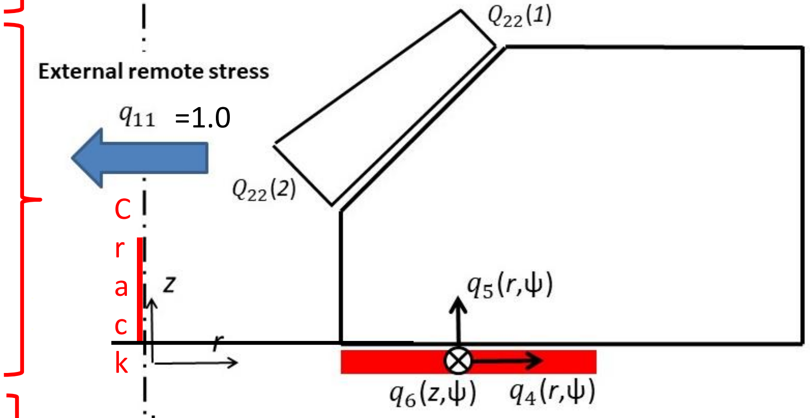
Figure Test Geometry, *R/t=2, b/t=0.5, a/t=0.4, c/a=2*.

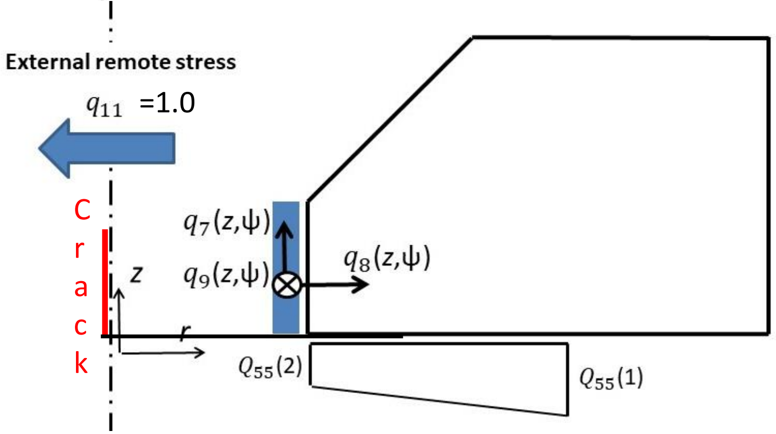
- Maximum K_I for tension 1 unit.
- K_{max} is ranging from 57-72 units, that is rather uniform.

The ψ-variation is the simple $\cos^2(\psi)$-function.

	Load	K_{max}
q1	Q1,1	71.76
	Q1,2	71.14
	Q1,3	71.47
q2	Q2,1	65.08
	Q2,2	64.97
	Q2,3	64.93
q3	Q3,1	57.26
	Q3,2	58.71
	Q3,3	57.84
q4	Q4,1	67.38
	Q4,2	68.81
	Q4,3	68.30
q5	Q5,1	65.08
	Q5,2	64.98
	Q5,3	65.20
q6	Q6,1	60.28
	Q6,2	63.69
	Q6,3	62.19
q7	Q7,1	
	Q7,2	
	Q7,3	
q8	Q8,1	68.25
	Q8,2	68.45
	Q8,3	68.34
q9	Q9,1	62.31
	Q9,2	63.61
	Q9,3	63.01



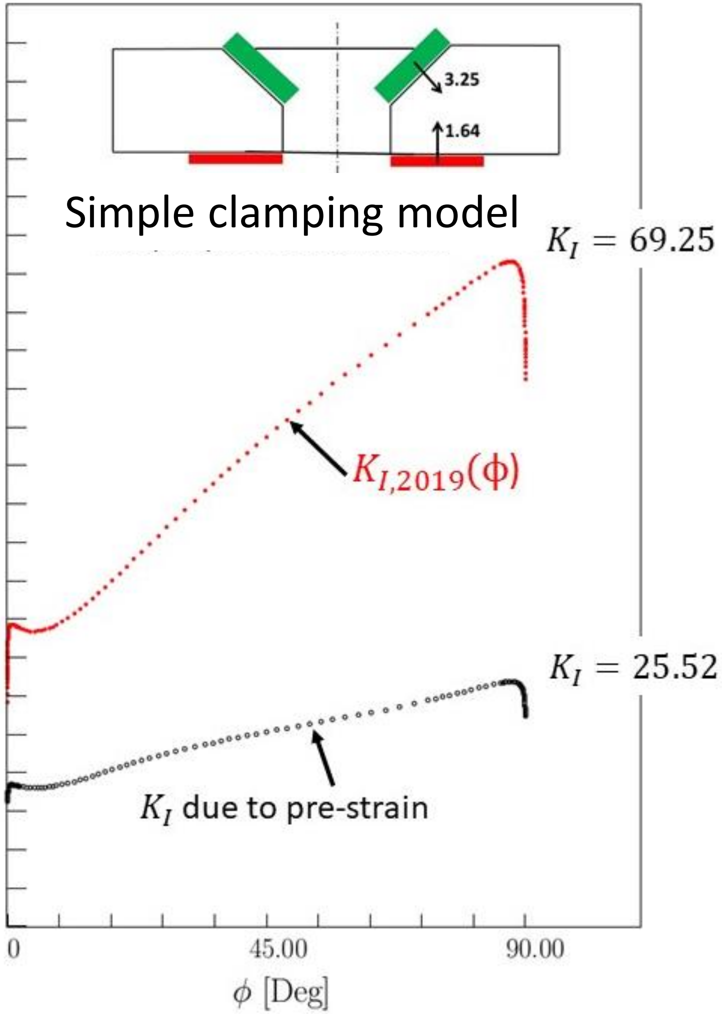




K_I due to clamping force

The following bolt pre-strain data is used in the optimization studies: A steel fastener with diameter 6.0 mm, torque 4 Nm leading to an axial force 6.3 kN or an axial average stress of 223 MPa. The remote stress is 1Mpa.

- The figure shows calculated K_I due to the clamping only. The model assumes friction free conditions and uniform contact pressures.
- For comparison K_I available from 2019 years data base for pin-loading is shown for comparison.
- Frictional forces between plate-plate and plate fastener will decrease K_I due to clamping.



Friction and contact assumptions in the model

- All contact pressures on the three contact surfaces are positive (i.e. no tension)
- All frictional forces are proportional to the contact pressure.
- Friction coefficients μ might be different on the three contact surfaces (due to different stick-slip phenomena, fretting etc.).
- The sliding direction is optimized, for maximum $K_I(\phi)$ for all ϕ, by selecting the sign ± of the friction coefficients μ.

The following equations then hold:

- $q_1 = \mu_1(q_2 + Q_{22}), q_3 = \mu_3(q_2 + Q_{22}), q_4 = \mu_4(q_5 + Q_{55}), q_6 = \mu_6(q_5 + Q_{55}),\ q_7 = \mu_7 q_8,\ q_9 = \mu_9 q_8$. Note that Q_{22} and Q_{55} depends on $q_1 - q_9$.
- The six friction coefficients are assumed to be in the range $-0.75 \leq \mu \leq 0.75$.
- Of physical reasons we assume that μ_1 is of same magnitude as $\mu_3$but the sign might be different (to model sliding in opposite directions). The same holds for μ_4 - μ_6 and μ_7 - μ_9, respectively.

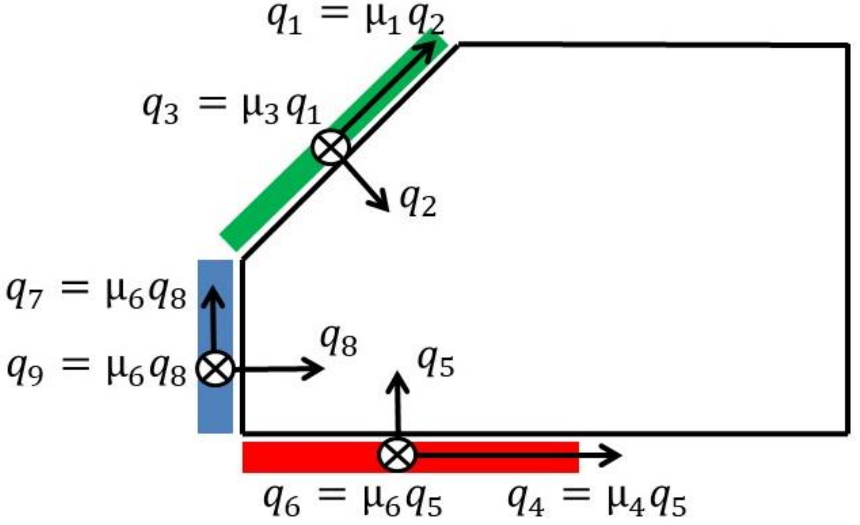
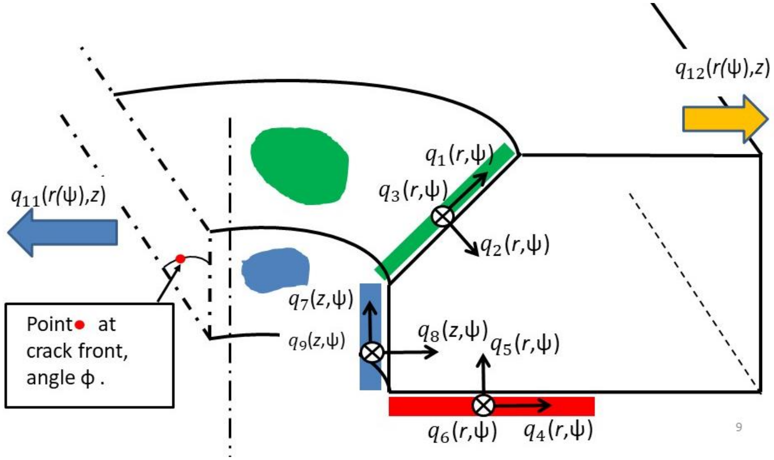


Figure (For clearness of the figure the terms Q_{22} and Q_{55} have been left out).

The optimization problem

- We will determine the highest possible value of K_I at one point ϕ^* on the crack front, and then repeat this analysis for each point on the crack front (i.e. 16 domains and 11 points/domain).



- The assumptions on friction make the $K(\phi)$-value dependent on q_2, q_5 and q_8 and the 6 friction coefficients $\mu_1, \mu_3, \mu_4, \mu_6, \mu_7$ and μ_9, respectively.
- The $K(\phi)$-value can, for fixed friction coefficients, be written,

$$K(\phi) = \sum_{k=2,5,8} \sum_{m=1}^{m=3} \alpha_{km} q_{k,m} \qquad \text{Eq. 1}$$

Where $\{\alpha_{k,m}\}$ are the nine stress intensity factors corresponding to the three polynomial terms defining the functions q_2, q_5 and q_8 (the contributions from q_1, q_3, q_4, q_6, q_7 and q_9 via friction have of course also been considered in equation Eq. 1).

Constraints on the optimization problem

- The constraints that the remote stress q_{11} shall be one unit leads to a constraint of the following type:

$$\sum_{k=2,5,8} \sum_{m=1}^{m=3} \beta_{km} q_{k,m} + C = 1 \qquad \text{Eq. 2}$$

- The conditions that only positive contact pressure (or zero) are allowed between surfaces leads to four constraints of this type,

$$q_{2,1} + Q_{22}(1) + 3.25 \geq 0 \qquad \text{Eq. 3a}$$

.....

$$q_{5,2} + Q_{55}(2) + 1.64 \geq 0 \qquad \text{Eq. 3d}$$

- Where $Q_{22}(1)$, depends on all *q*-values and the constant 3.25 is due to the pre-strain (if considered) etc.
- We solve the linear optimization problem, i.e. find maximum $K(\phi)$ subject to Eq.2 and Eq. 3a – 3d, with the standard *simplex method*.
- Note that the solution obtained in this way is for the preselected set of (six) friction coefficients.

7. Numerical scheme

- For each point ϕ*, of 176 points, Do:
  Set K(ϕ) = - 999*

176 loops

- For $\mu_7 = -0.75, -0.60, -0.50 \ldots 0.50, 0.60, 0.75$ Do:
- For $\mu_9 = +\mu_7$ and $\mu_9 = -\mu_7$ Do:
- For $\mu_1 = -0.75, -0.60, -0.50 \ldots 0.50, 0.60, 0.75$ Do:
- For $\mu_3 = +\mu_1$ and $\mu_3 = -\mu_1$ Do:
- For $\mu_4 = -0.75, -0.60, -0.50 \ldots 0.50, 0.60, 0.75$ Do:
- For $\mu_6 = +\mu_4$ and $\mu_6 = -\mu_4$ Do:

$(2 * 15)^3 = 27000$ loops.

- Create coefficients in equations Eq.1, Eq. 2 and Eq. 3.a – 3.d.
- Use the Simplex method to find the maximum *K (ϕ*)* -value, *if it exists*[1]. Denote this value S.

  If S exists, and $S > K(\phi^*)$, then put $K(\phi*) = S$.

- End of six μ-loops
  Store $K(\phi^*)$
- End loop over 176 points

With[1] general sliding directions and friction coefficients a solution may not exist that satisfy all contact constraints 3a – 3d.

The 2019 years pin-load model used by USAFA

The model shown in Figure was 2019 used to derive 36 million *K*-functions (See AFGROW lecture 2019, Andersson and Greer). This model is a special case of the general model presented in this lecture, i.e. the only nonzero components (are q_1, q_2 $(= q_1 \tan(50^o))$ and q_8.

$$q_{120} = q_8 \cdot (R \cdot b^2/2)/(R \cdot (t-b)^2/2 + (t-b)^3 \cdot \tan(\beta)/3)$$
$$F = 4 \cdot q_8 \cdot b \cdot R/3 + 4 \cdot q_{120} \cdot ((t-b) \cdot R + (t-b)^2 \cdot \tan(\beta)/2)/3$$

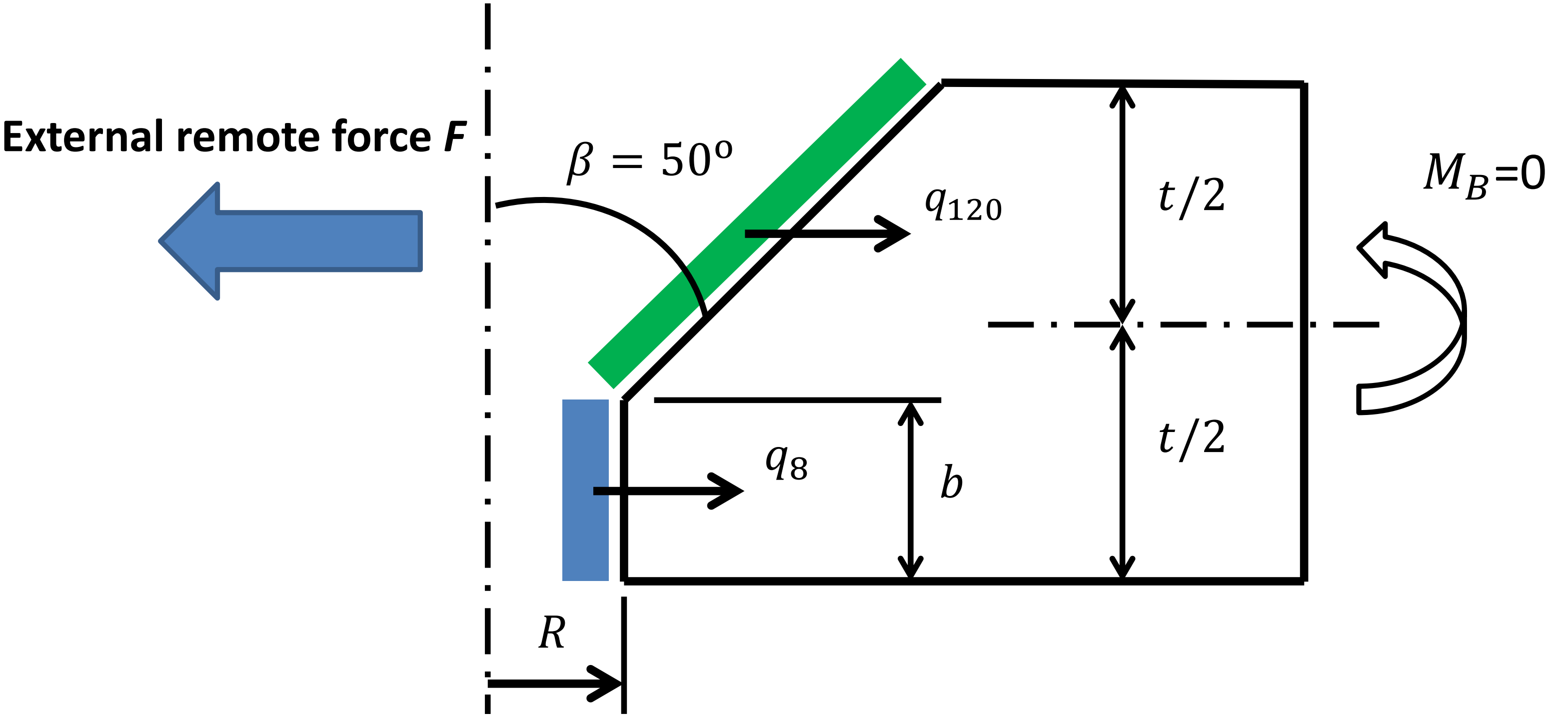
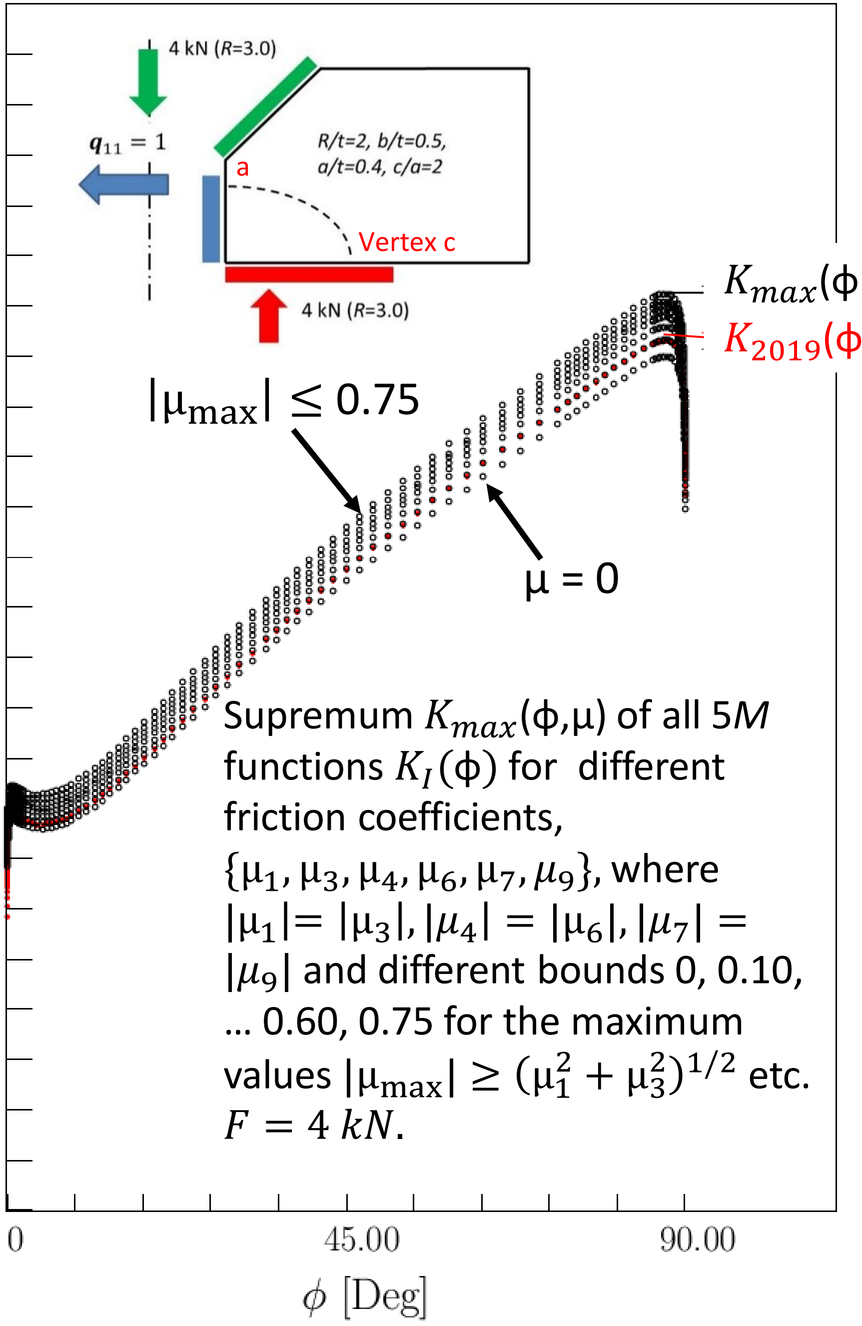
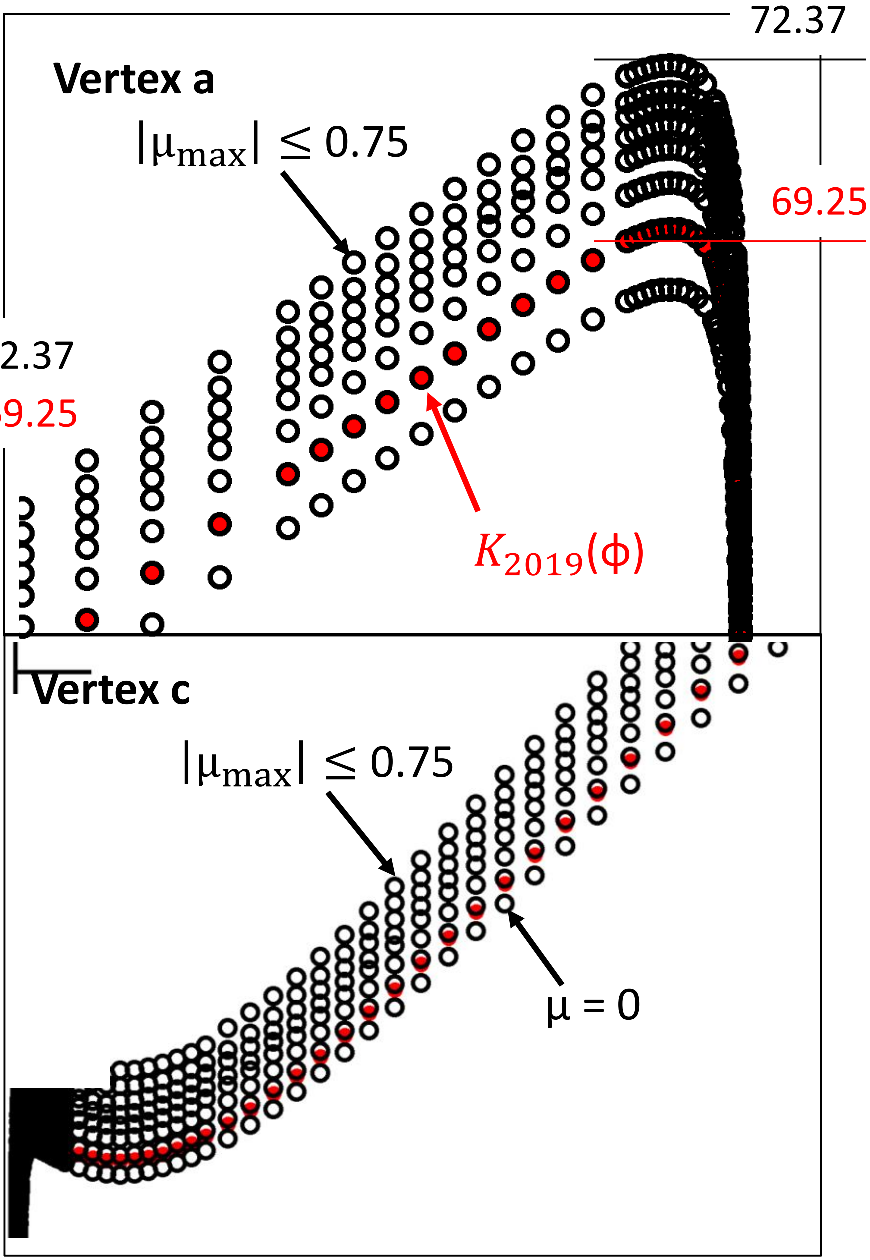


Figure Model for pin loading (Andersson, Greer 2019)

Major Result: Supremum $K_{max}(\phi,\mu)$



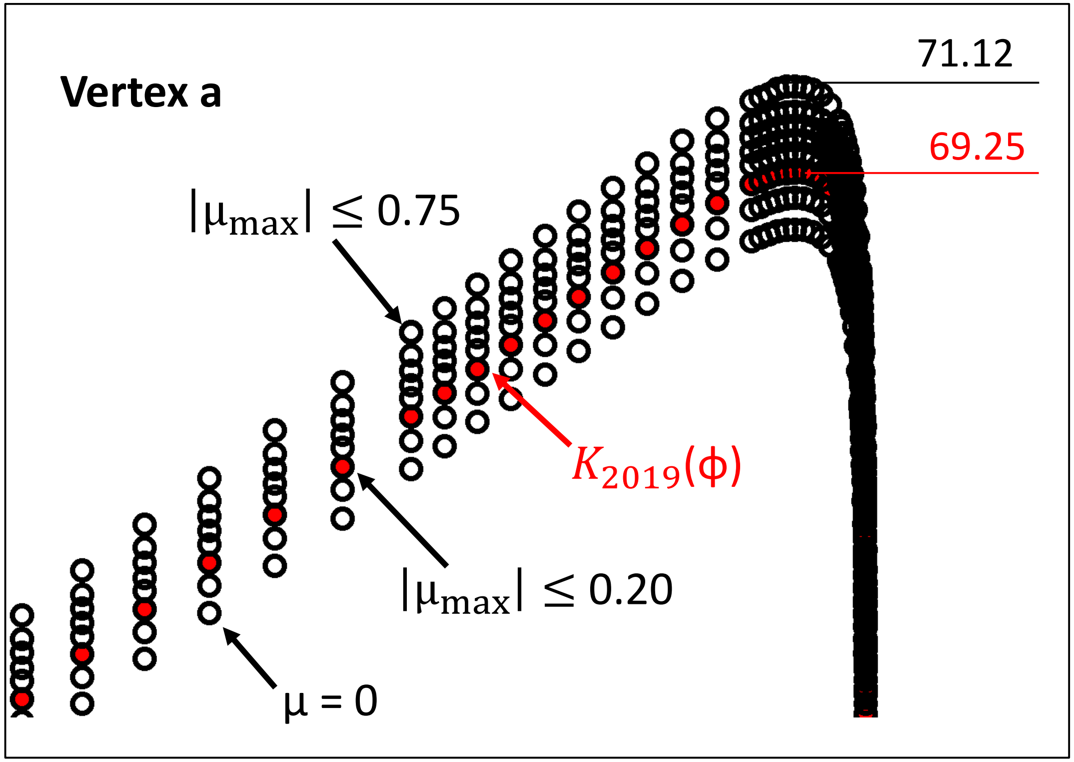
Supremum $K_{max}(\phi,\mu)$ of all 5M functions $K_I(\phi)$ for different friction coefficients, $\{\mu_1,\mu_3,\mu_4,\mu_6,\mu_7,\mu_9\}$, where $|\mu_1| = |\mu_3|, |\mu_4| = |\mu_6|, |\mu_7| = |\mu_9|$ and different bounds 0, 0.10, … 0.60, 0.75 for the maximum values $|\mu_{max}| \geq (\mu_1^2 + \mu_3^2)^{1/2}$ etc. $F = 4\ kN$.



Supremum $K_{I,max}(\phi, \mu)$ -functions joint with no clamping

- The figure shows $K_{I,max}(\phi, \mu)$ *near vertex a* for the case when the clamping force is zero.
- The maximum value of $K_{max}(\phi)$, for all possible sliding directions and friction coefficients is somewhat lower, i.e. 71.12, compared to 72.37 for the case with a nonzero clamping force.
- The 2019 years pin-load model underestimates this value by less than 3%.

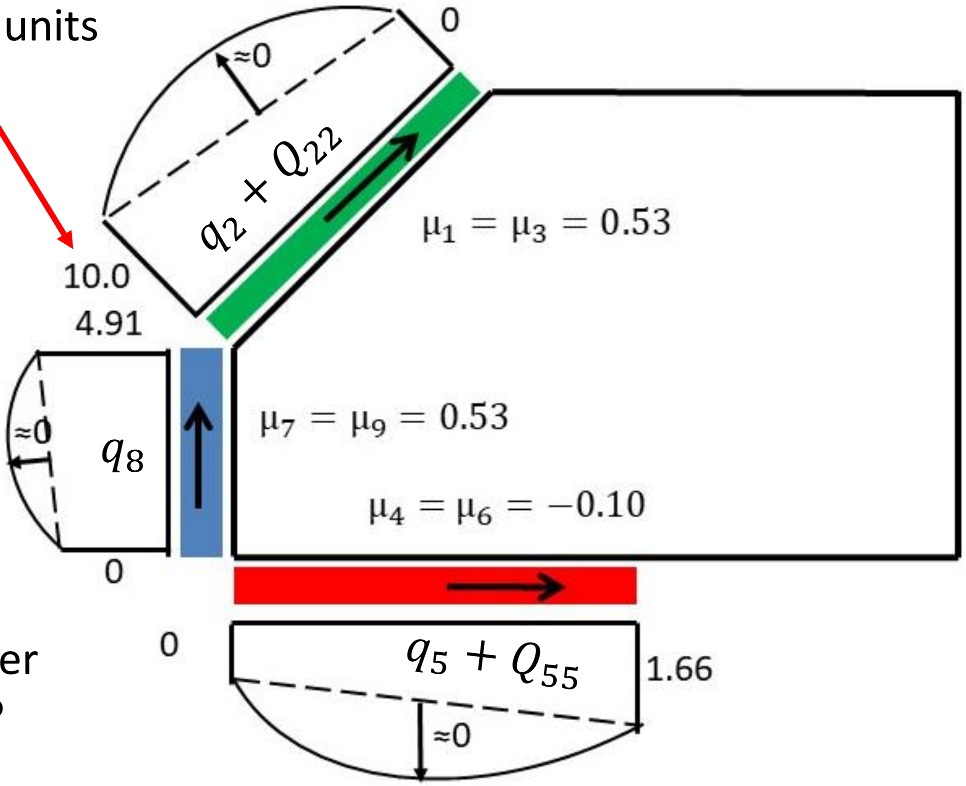
Extremum $K_{I,max}(\phi, \mu)$-functions near vertex *a* for a joint with zero clamping force and $f_{ij} = \cos^2(\psi)$.



Which load system leads to the highest possible K_I-value?

- The figure shows which load system $q_2 + Q_{22}$ and $q_5 + Q_{55}$ that leads to the maximum value K_{max}(ϕ)= 71.12 at vertex a for zero clamping force.
- K_{max}(ϕ)= 71.12 is obtained for the maximum friction coefficient 0.75, except at the bottom surface.
- The load system shown is much more complex (and the result of solving 5M optimization problems) than for 2019 years model but the highest values differ only by less than 3%!

NOTE the compressive stress 10.0 units at the conical surface.
Scaled to the "engineering case", that is bolt radius 3 mm and external stress $q_{11} = 100$ MPa, the contact stress becomes 1000 MPa (yield stress for aluminum 7075-T6 is about 500 MPa).
A constraint on the maximum contact stress would further decrease K_{max}(ϕ) to become closer To results from 2019 years model?



Contact stresses of type $\alpha \cdot cos^m(\psi) + \beta \cdot sin^n(\psi)$

- The results in previous sections are all valid for$f_{ij}(\psi) = \cos^2(\psi)$. However, in practical cases is the contact between plate and fastener very different in the ψ-direction.
- We can distinguish two different axial regions, for the selected geometry, i.e. near the faying surface (i.e. $z \leq 0.4 \cdot t$) where the plate presses like two knife edges into the bolt surface, see Figure. If case of linear elasticity, this contact condition will lead to infinite contact stresses near the 'knife' edge. The functional behavior is exactly as for the crack problem, that is an $r^{-1/2}$-stress singularity.
- For $z > 0.5 \cdot t$, say, the contact distribution is smooth with no singularity.

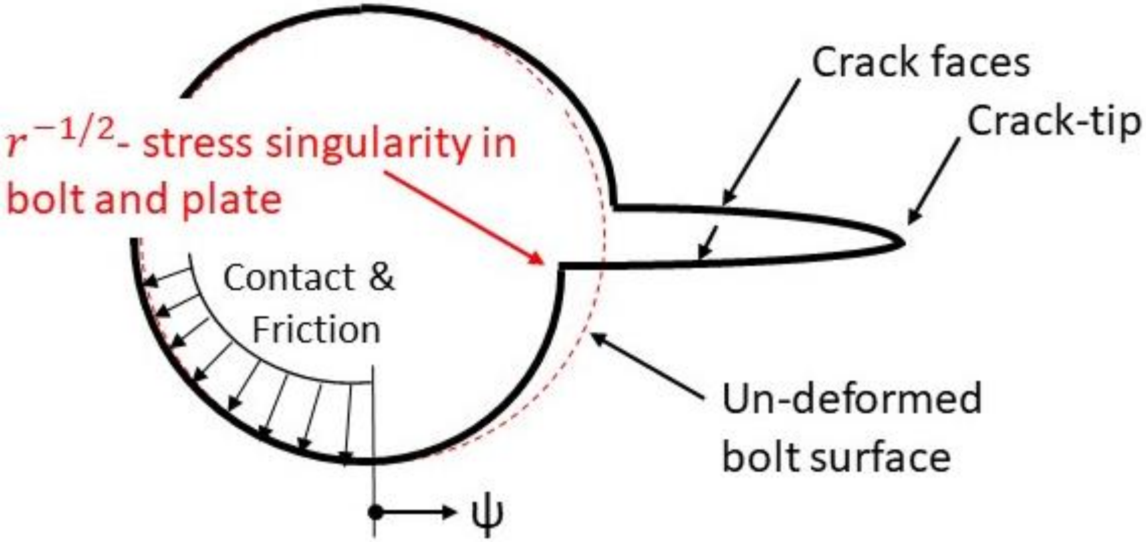


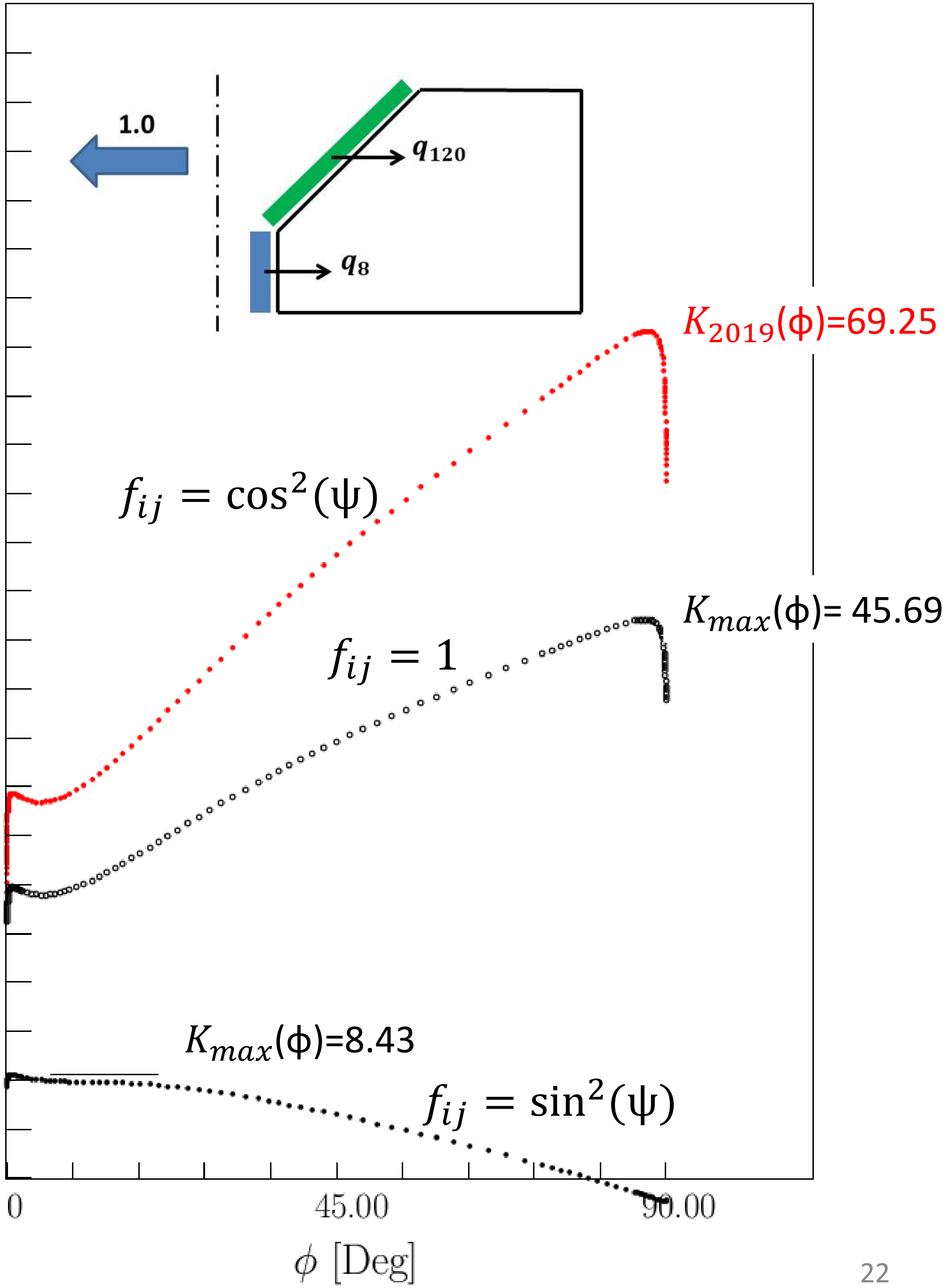
Figure Principal plate deformations for $z \leq 0.4 \cdot t$.

Contact stresses of type $\boldsymbol{\alpha \cdot cos^m(\psi) + \beta \cdot sin^n(\psi)}$

The figure shows the stress intensity functions $K_I(\phi)$ obtained with 2019 years pin-load model that has been modified for various $f_{ij}(\psi)$.

The conclusion is that K_{max} depends strongly on the parameter ***m***, i.e. an increasing ***m*** leads to a larger hole ovalisation and higher K_{max} .

Use of $f_{ij} = \cos^2(\psi)$ in the above analyses leads most likely to very conservative values of K_{max}



Conclusions

The objective of this investigation was to:

- develop a model for calculation of the supremum of all K_I- functions for cracks in structural joints with countersink ‘pin-loaded’ fasteners.
- Determine the degree of conservatism in the pin-load model used by USAFA 2019.
- If there is a need, to suggest a better pin-load model for analysis of countersink hole geometries.

A general mathematical model was developed which in principle considers the general behavior of such joints, i.e. contact behavior, stick-slip, fretting, bolt torque.

- We used the model to analyze a typical structural joint. Over 5 million optimization problems were solved in order to the determine the supremum function K_{max}(ϕ).
- It was found that 2019 year’s pin-load model used by USAFA provided a *K* peak value that was only 4% lower than the theoretical maximum value, never achieved in real-life situations.

Note that the lowest $maximum\ K_I$-values obtained during optimization studies were in many cases negative, despite the applied tensile load on the structural joint.

The main conclusion from this study is that the pin-load model used in 2019 provided conservative K_I-values and we recommend its use in future K-database generation.