A Proposed Verification Check when Developing Stress Intensity Solutions

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Overview

- Validation vs. Verification
- Previous work
- Benchmark case
- J vs. K
- Material sensitivity
- Radius of integration
- Mesh dependency
- AFGROW solutions

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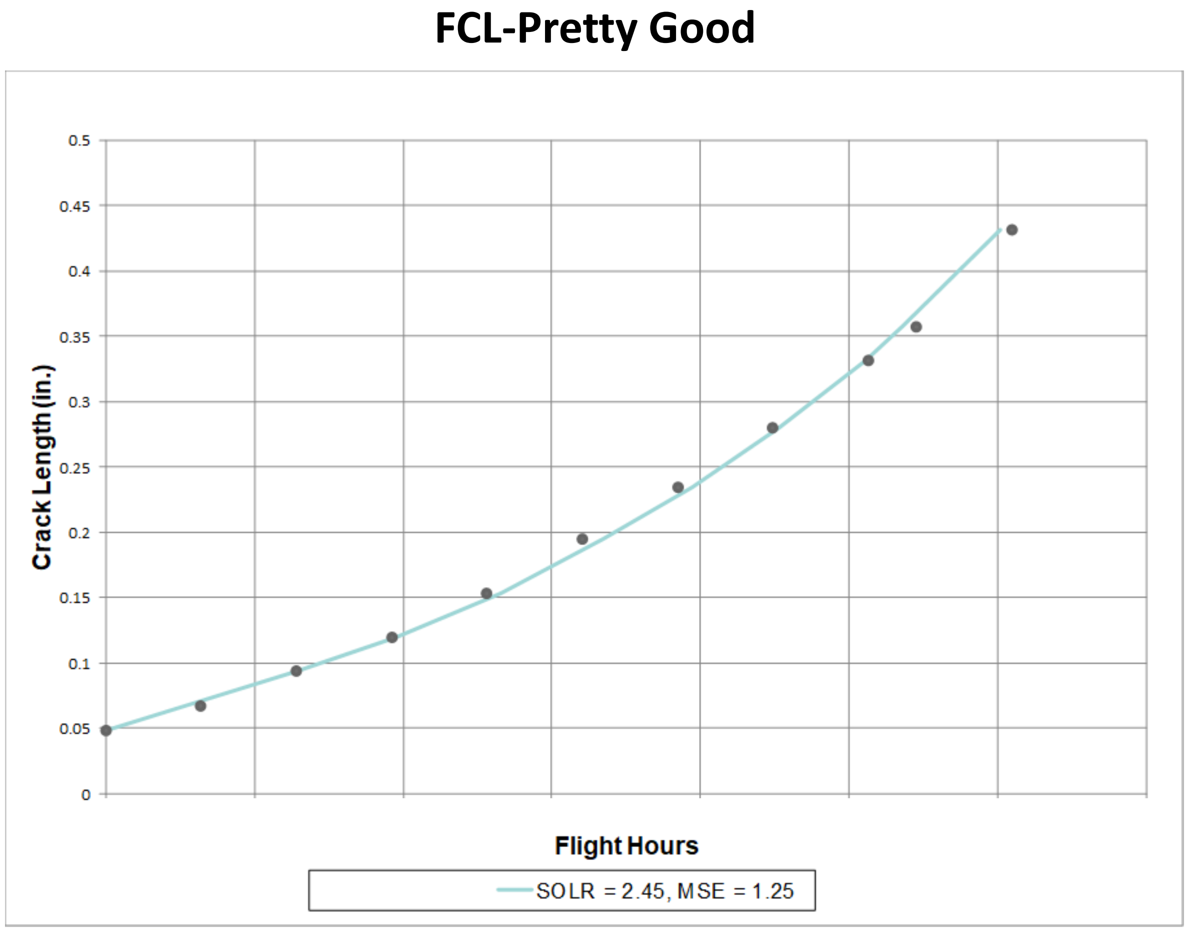
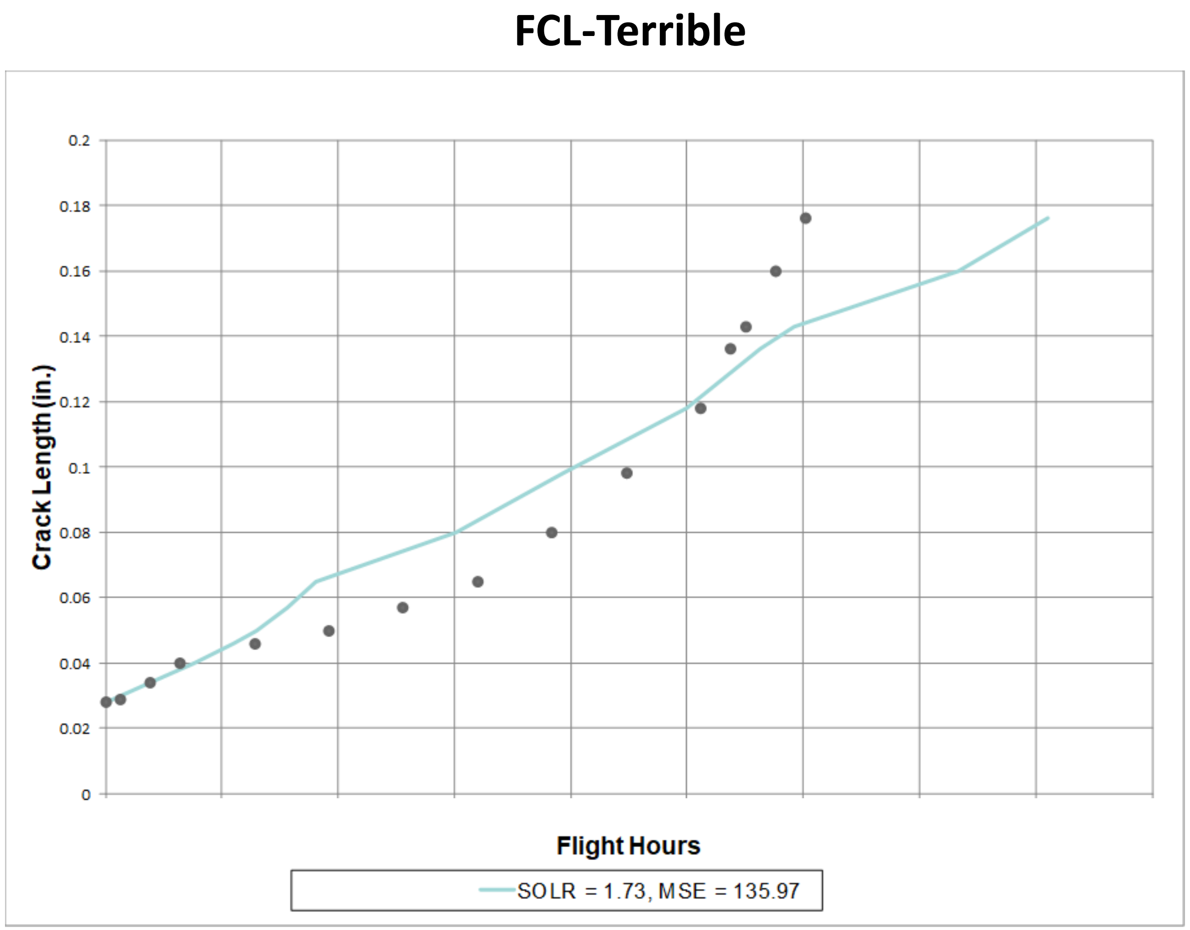
Background

Validation – "How bad do I suck at Physics?"





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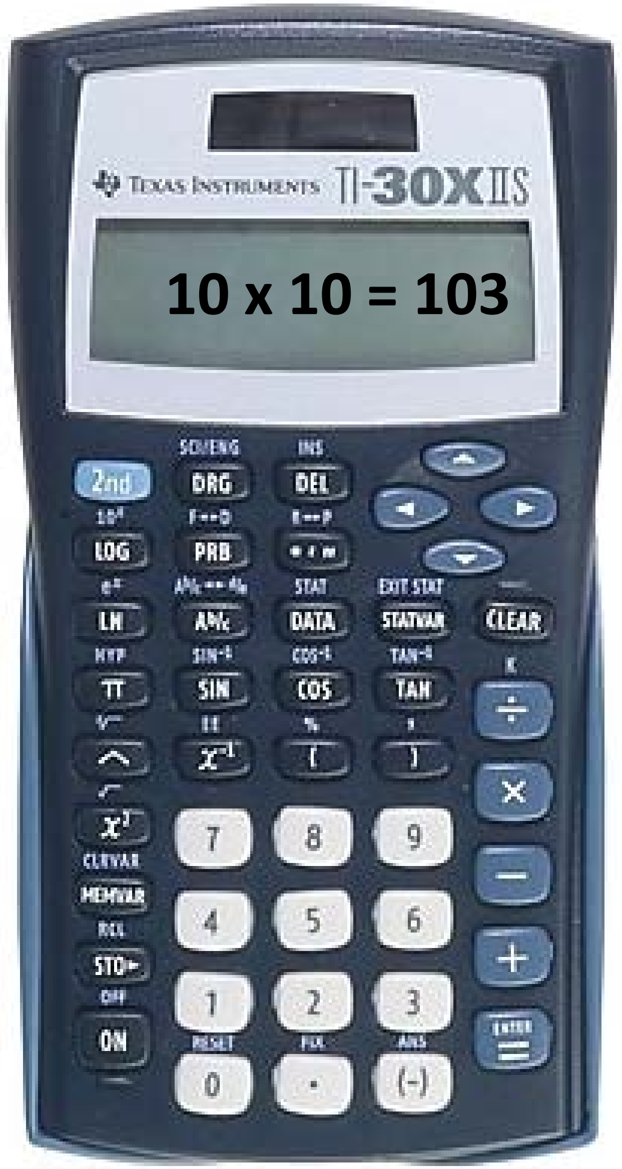
Background

Verification – "How bad do I suck at Math?"

Would you buy this calculator?

Different levels of accuracy required for validation and verification



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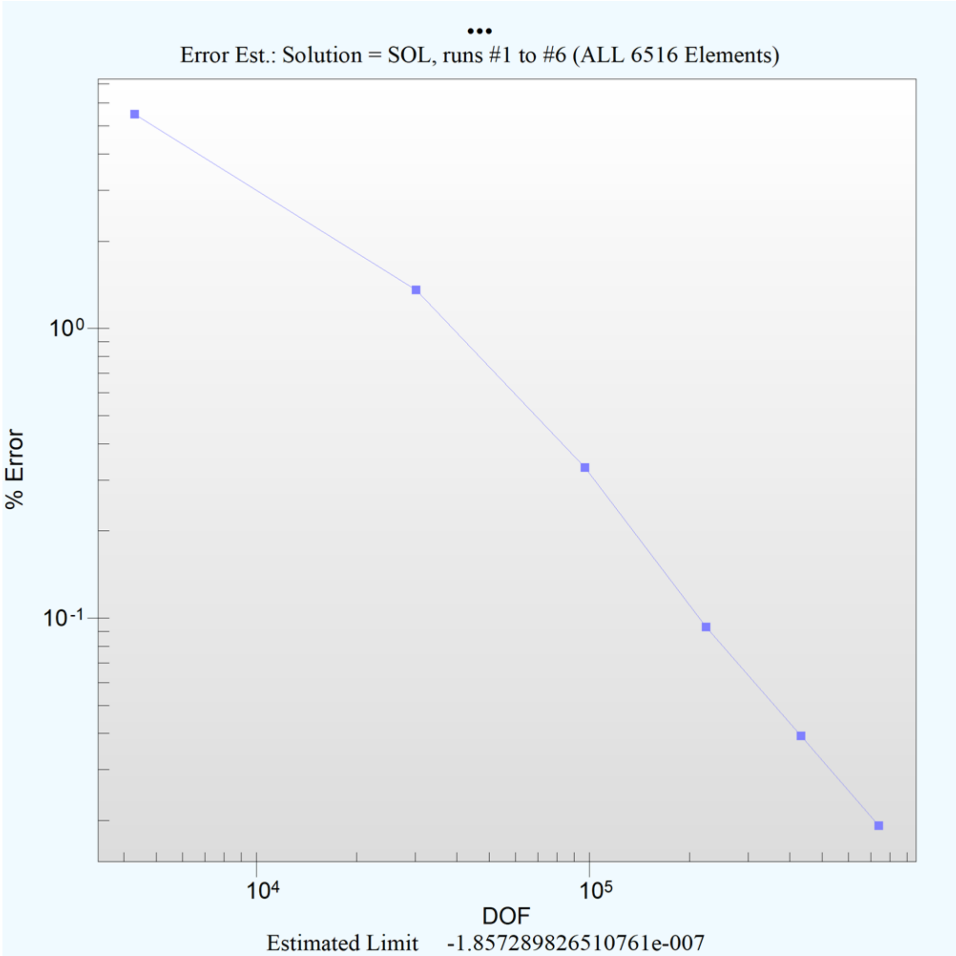
StressCheck Verification

- Increasing polynomial order to converge energy

p-level	DOF	Total Potential Energy	Convergence Rate	% Error
1	4323	-1.85E-07	0	5.48
2	30189	-1.86E-07	0.72	1.36
3	97142	-1.86E-07	1.21	0.33
4	224730	-1.86E-07	1.51	0.09
5	432501	-1.86E-07	1.33	0.04
6	740003	-1.86E-07	1.33	0.02



- K (CIM) is super convergent
- Necessary, but not sufficient

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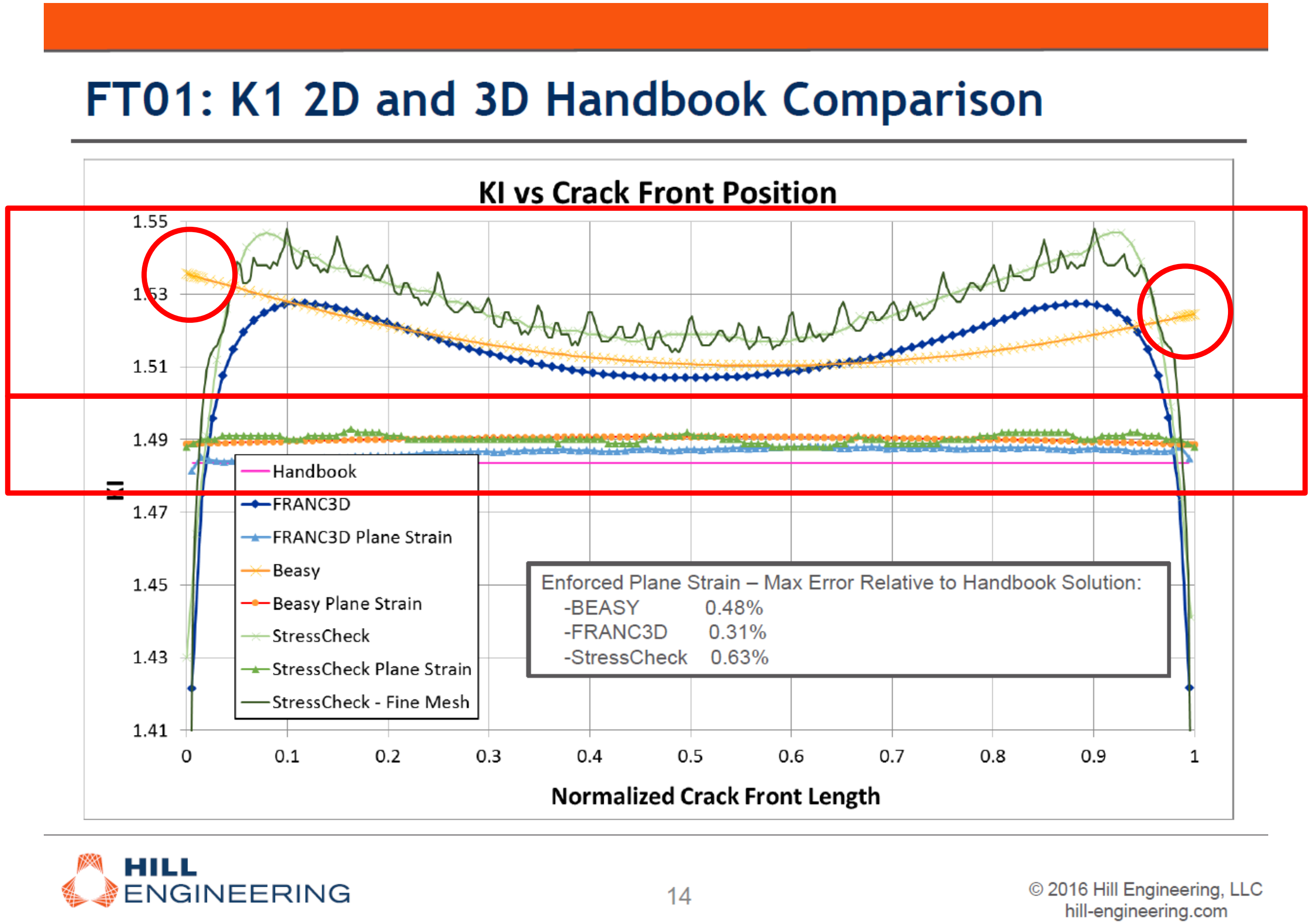
Previous Work

"Benchmarking Problems in Fatigue Crack Growth Analyses"

Pilarczyk et. al. AFGROW Workshop 2016

- Edge/Center Cracked Plate
  - Handbook is not dependent on t
- 2D (Plane Strain) Comparisons
  - Max Error ~0.5%
- Beasy is not symmetric

- 3D Comparison
  - Max Error ~5%
- StressCheck 3D consistently higher than FRANC3D



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Benchmark Case

Quarter circular crack at hole

- Looking at damage tolerance: a = c = 0.05 inch
- Width = 5 inches
- Height = 6 inches
- Diameter = 0.5 inch
- Thickness = 0.125 inch
- Young's Modulus = 10.3e6 psi
- Poisson's Ratio = 0.33
- Tensile Traction = 1 psi

Handbook solution along crack front

Newman & Raju, Forman, AFGROW DTDHandbook

Determining both K and J

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K vs. J

Stress Intensity Factor (K) vs. Strain Energy Release Rate (J)

How do these compare?

$$K_I = \sqrt{J_I E'}$$

Where

$$E' = \begin{cases} E & plane - stress \\ \dfrac{E}{1 - v^2} & plane - strain \end{cases}$$

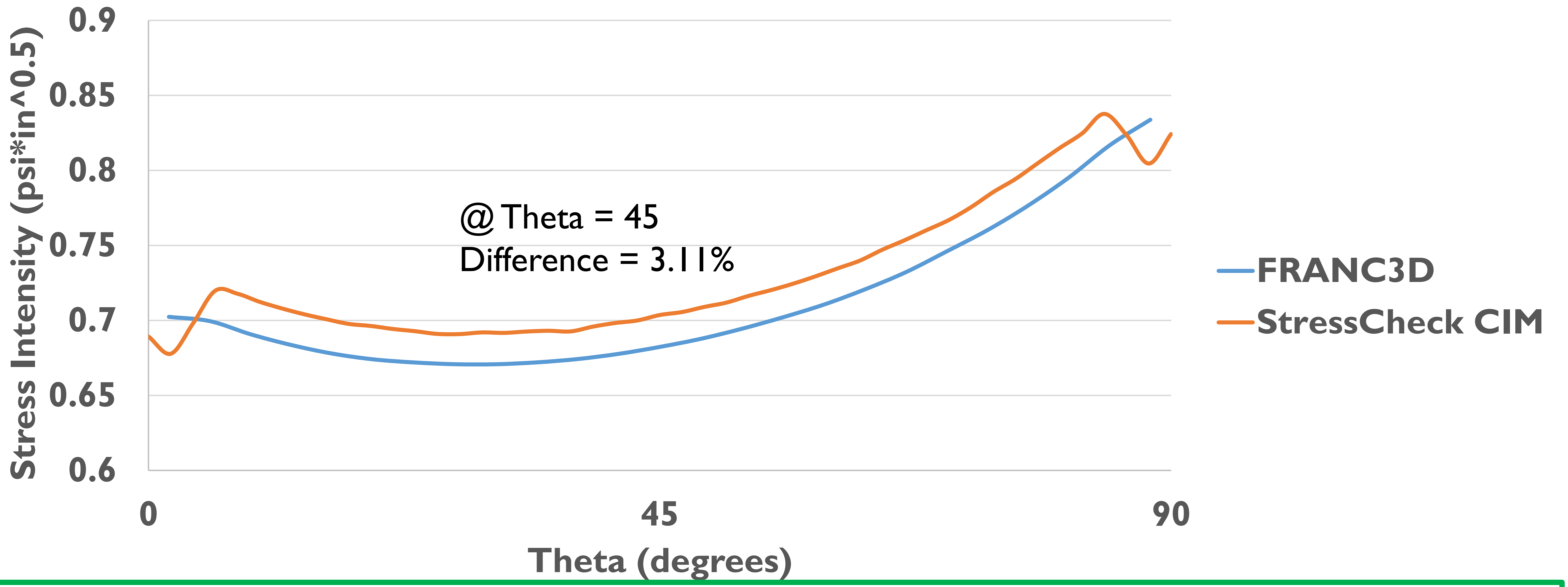
Plane-strain error:

$$error = (1 - v^2)^{-0.5}$$

$=(1 - 0.33^2)^{-0.5}$=5.93% increase

Quarter Circular Crack at Hole

StressCheck vs. FRANC3D



StressCheck is generally higher

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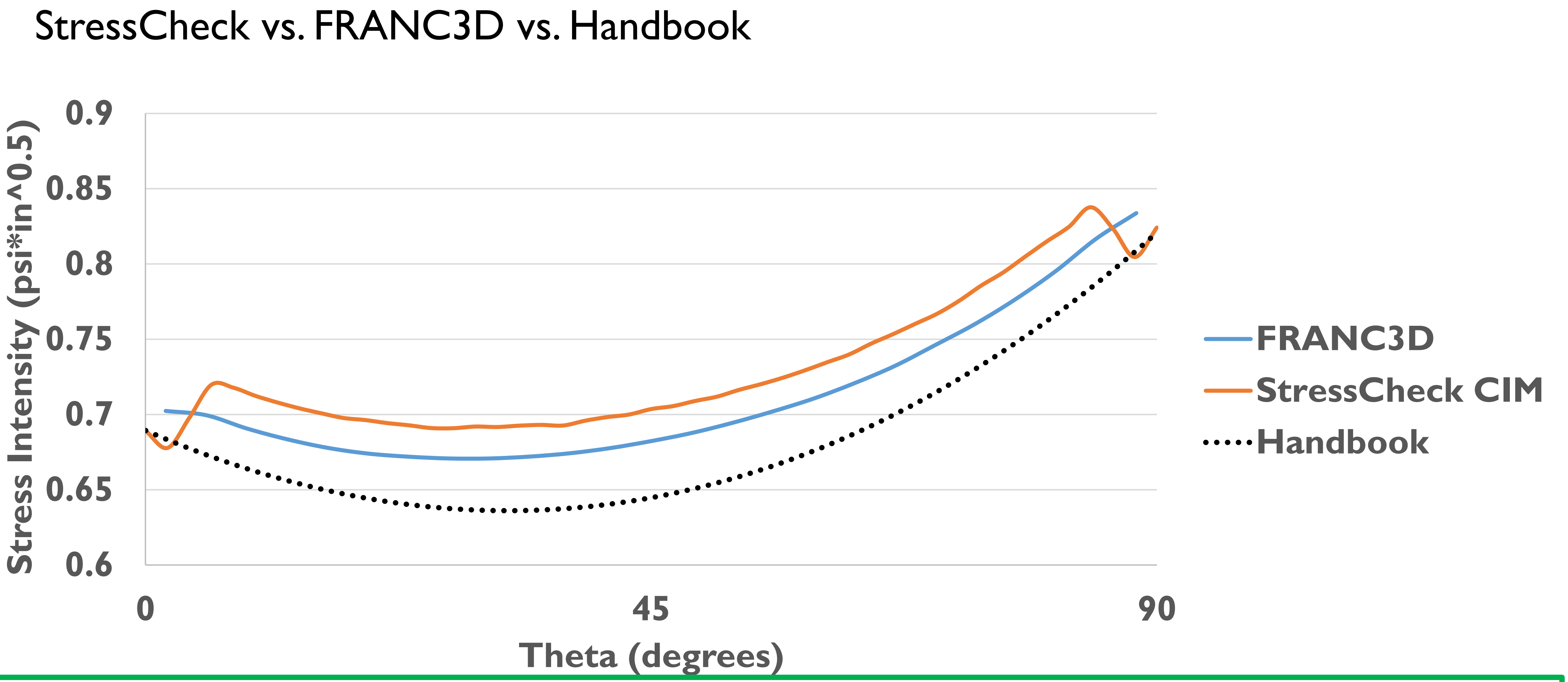
Quarter Circular Crack at Hole



This doesn't help… yet

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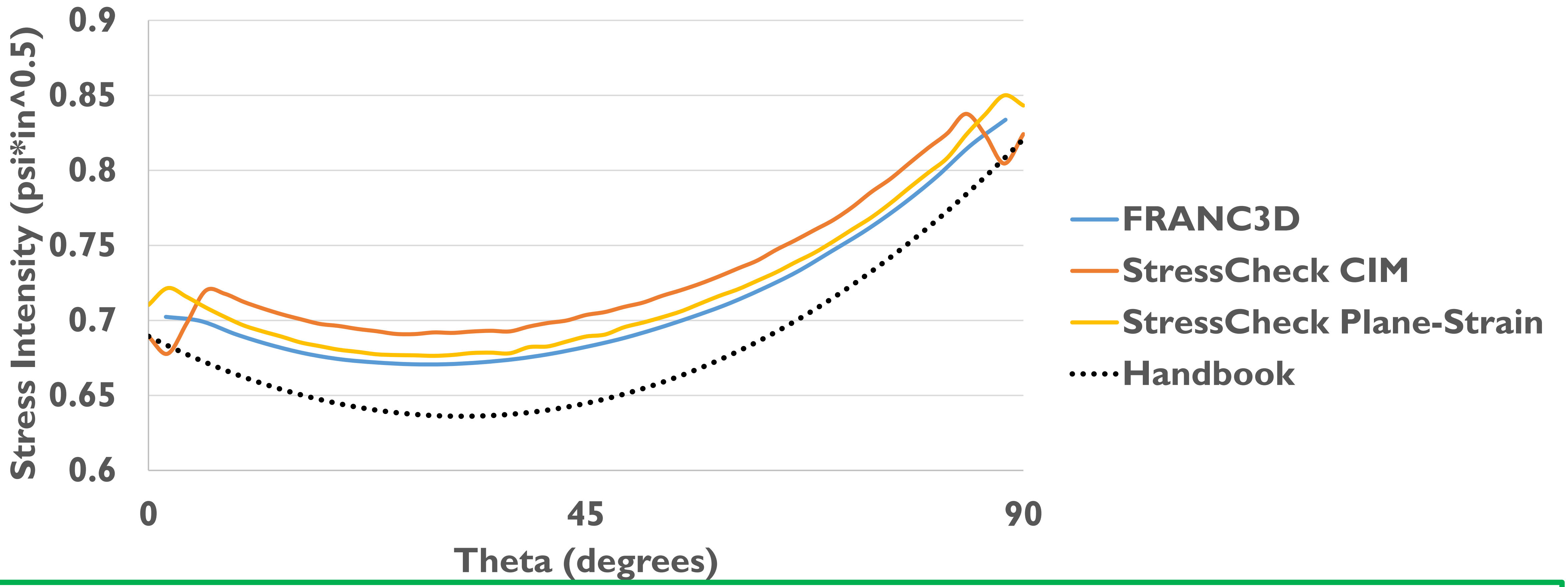
Quarter Circular Crack at Hole

StressCheck vs. FRANC3D vs. Handbook + SC J-Plane Strain



J-Plane Strain is closer to FRANC3D

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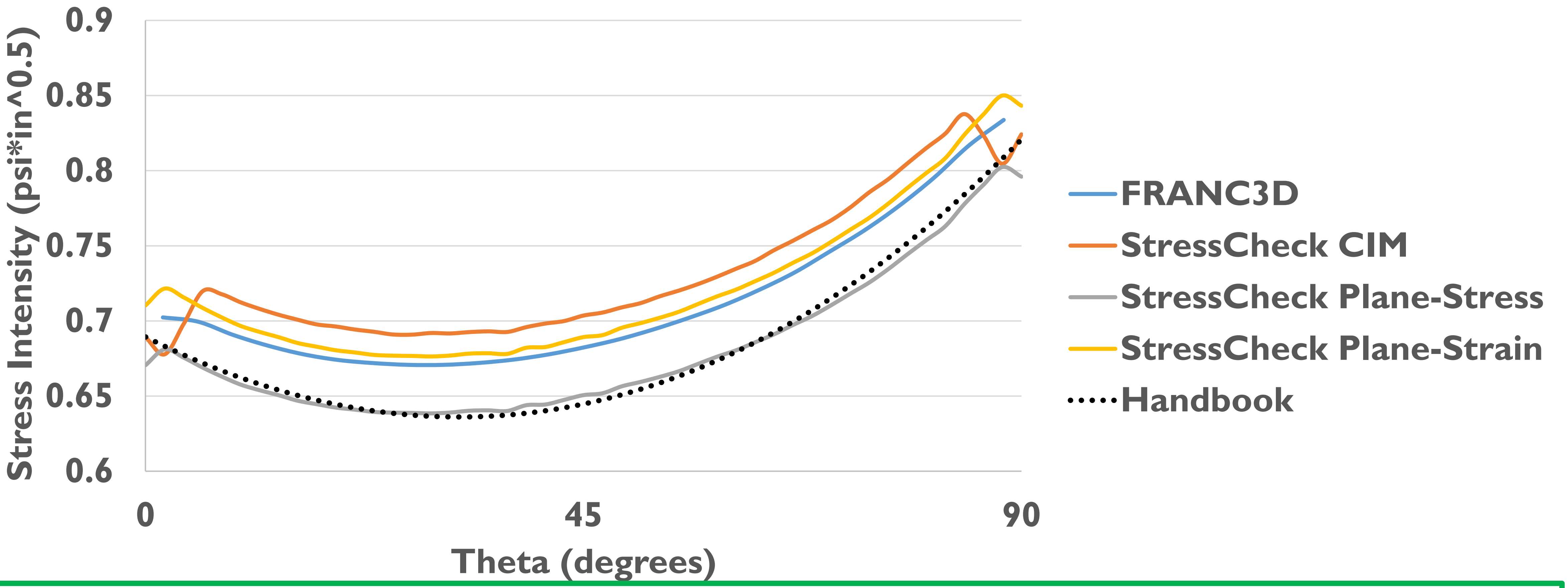
Quarter Circular Crack at Hole

StressCheck vs. FRANC3D vs. Handbook + SC J-Plane Strain + SC J-Plane Stress



J-Plane Stress is close to the Handbook solution

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Converting J to K

"The conversion… is valid for linear elastic materials under plane stress or plane strain conditions. For these cases it can be shown that the conversion of J to K produces the same result as directly computing K using the Contour Integral Method (CIM). However, for 3D part-thru (corner) cracks that are not purely **plane stress or strain**, the J to K conversion… produces values of K that may differ from those computed by CIM. Numerical studies have shown that the difference is problem-dependent but is typically limited to 5% or less"

StressCheck Master Guide, Advanced Guide, Chapter 3: Fracture Mechanics, Note about Converting J to K

Should plane strain and plane stress bound the CIM?

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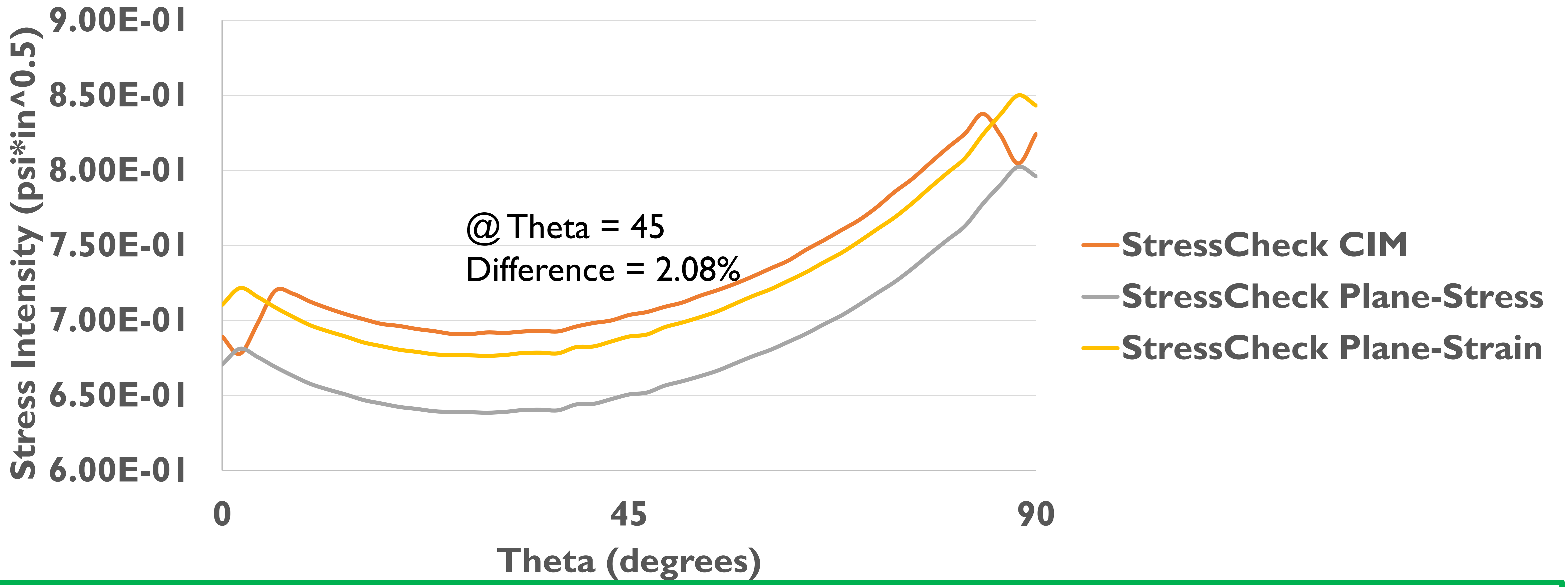
Quarter Circular Crack at Hole

StressCheck vs. SC J-Plane Strain vs. SC J-Plane Stress



CIM is outside the Plane Stress and Strain bounds

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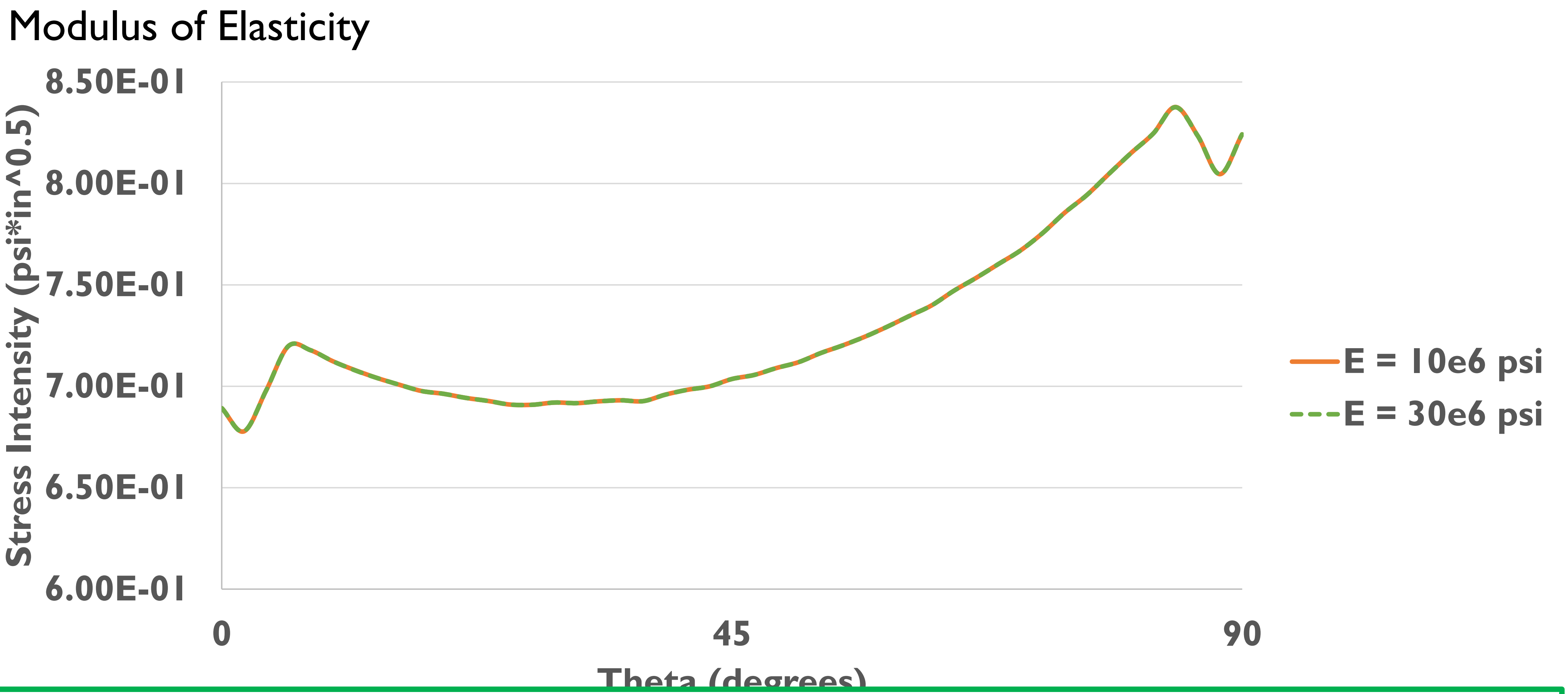
Material Sensitivity



No change for Modulus of Elasticity

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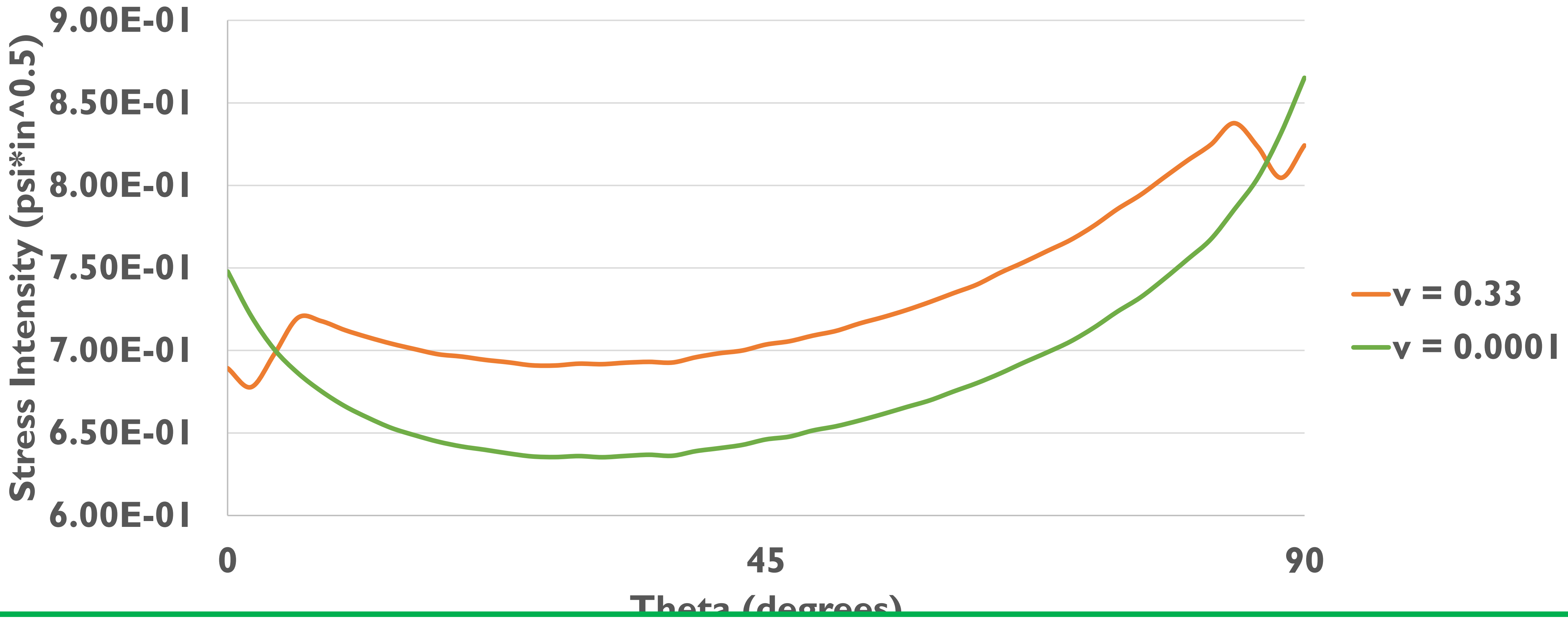
Material Sensitivity

Poisson's ratio



Sensitive to Poisson's Ratio

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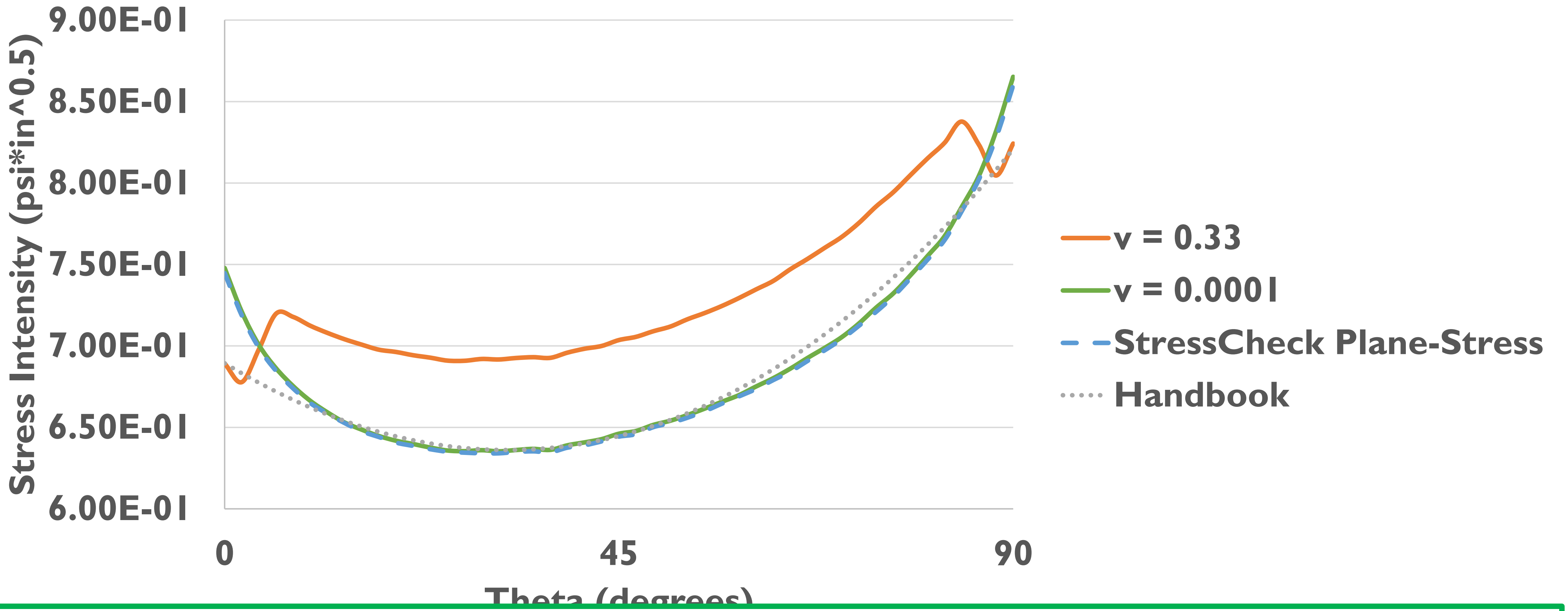
Material Sensitivity

Poisson's ratio, J – Plane-Stress, Handbook



Solutions collapse onto a single curve close to Handbook

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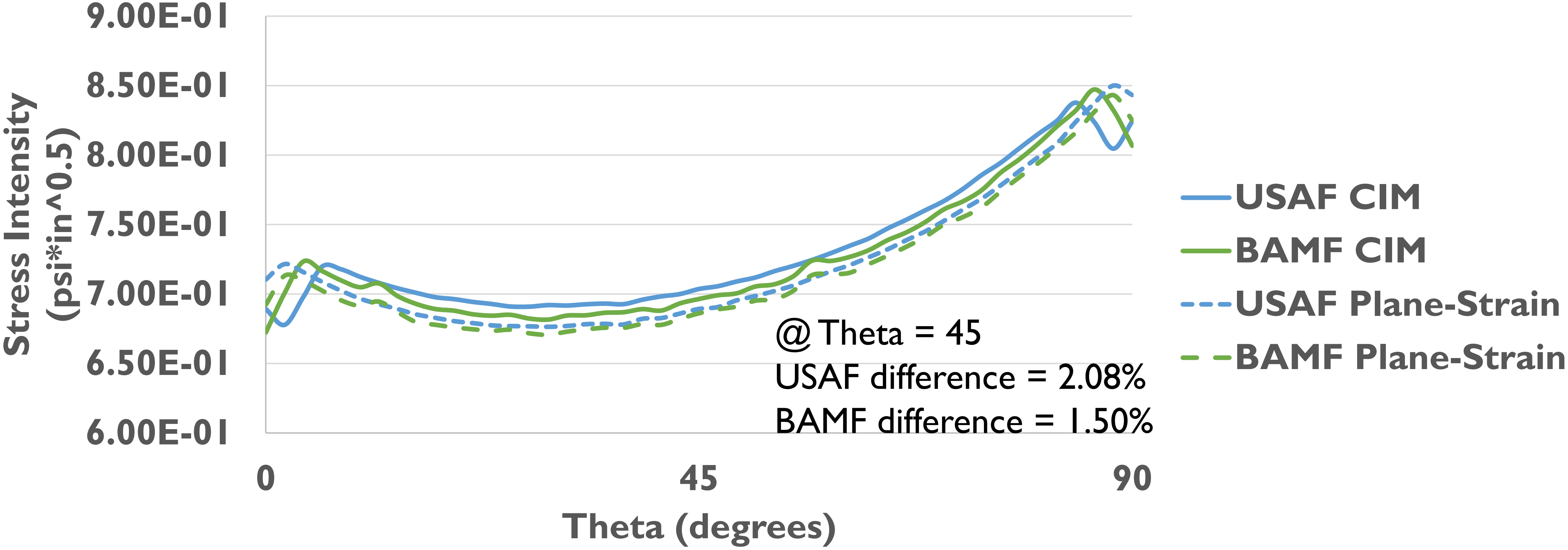
USAF/BAMF Ground Rules

USAF: Layers = 3, To = 0.0225*c, Ttotal = 0.2475*c, Krad = 0.06*c

BAMF: Layers = 2, To = 0.0225*c, Ttotal = 4*To, Krad = 1.3*To



Similar behavior between USAF and BAMF

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Radius of Integration

"In general for 3D crack problems the contour integral is not path-independent, in particular for curved cracked fronts. … the radius of integration includes the effect of adjacent points… Using an integration radius as small as possible will decrease the influence of adjacent points when computing SIFs… the SIFs must be evaluated in the **limit when the radius of integration goes to zero**… Ideally, the radius should be the smallest possible radius outside the first layer of refinement… Numerical evidence suggests that an integration radius in the range **R/a = 0.025** – 0.030 is enough to obtain accurate results" when Ri = 0.0225*a

StressCheck Master Guide, Advanced Guide, Chapter 3: Fracture Mechanics, A note about the use of the CIM in 3D

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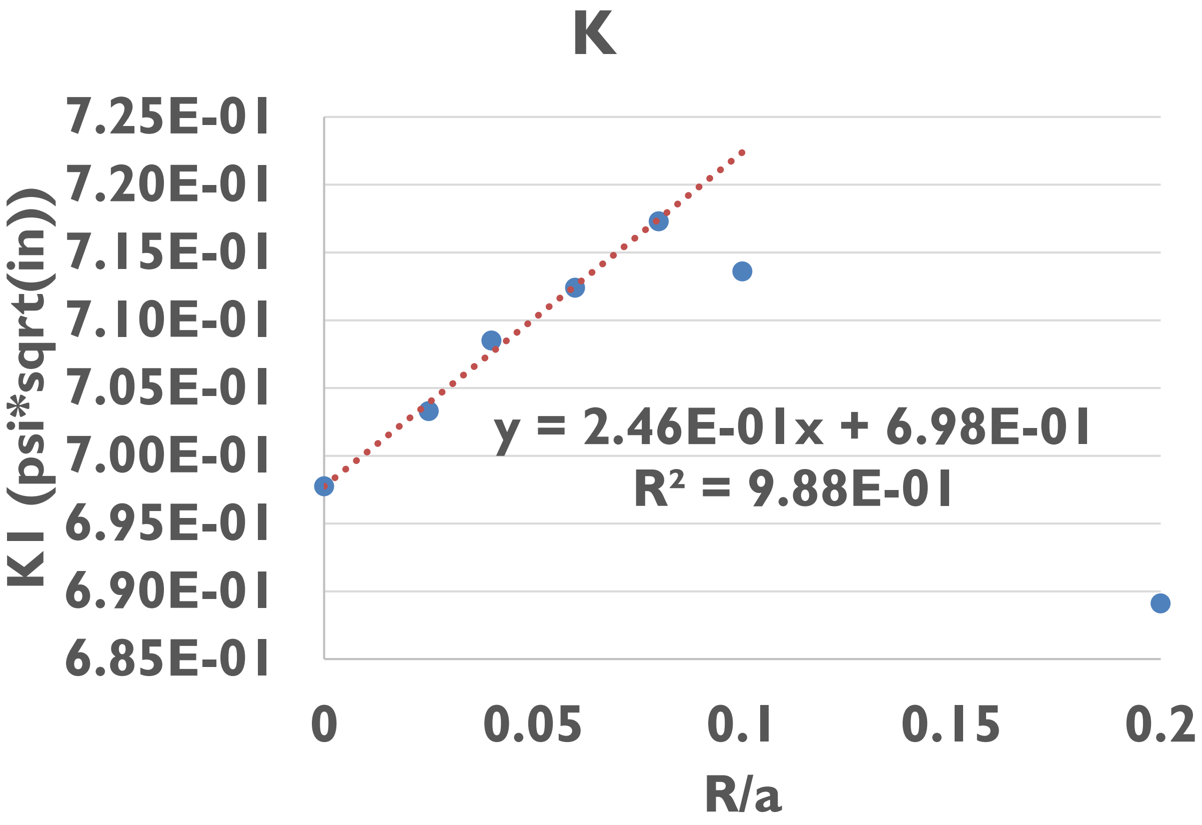
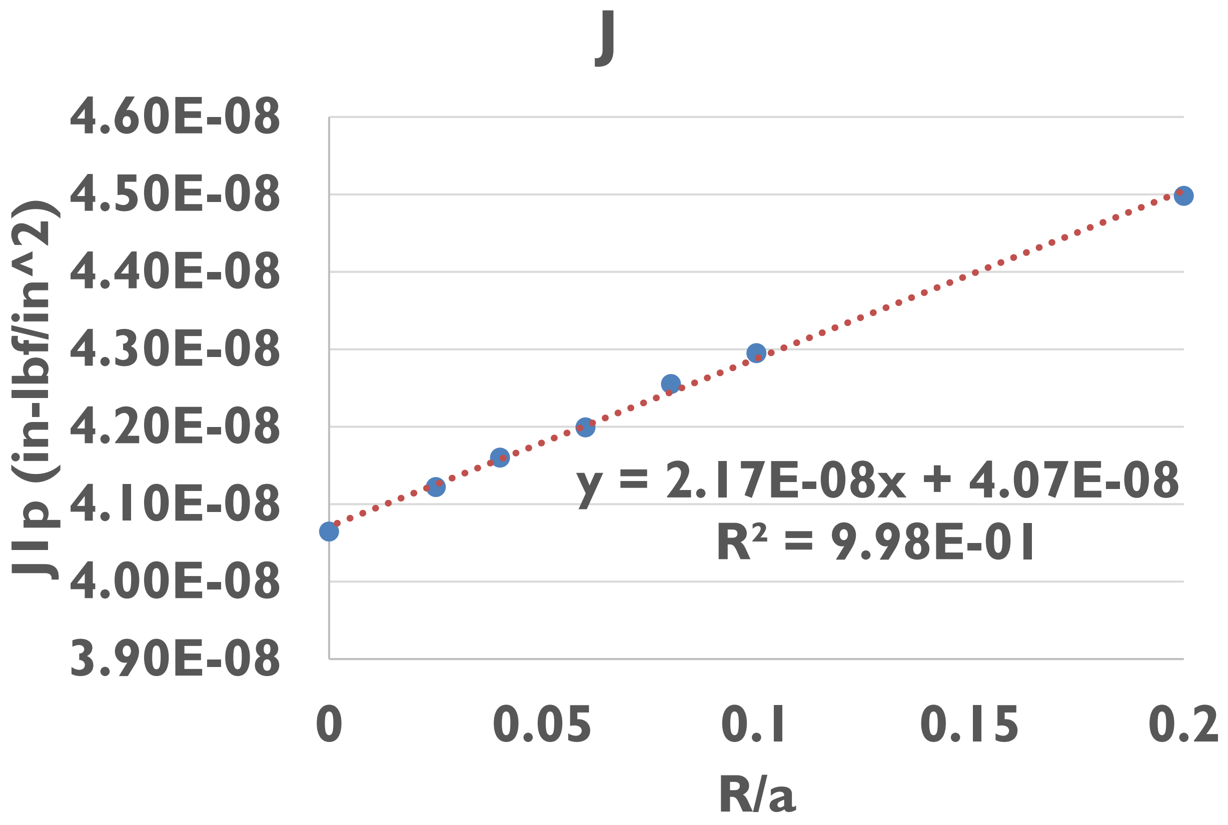
Radius of Integration

Convergence





Both K and J decrease linearly with decreasing radius

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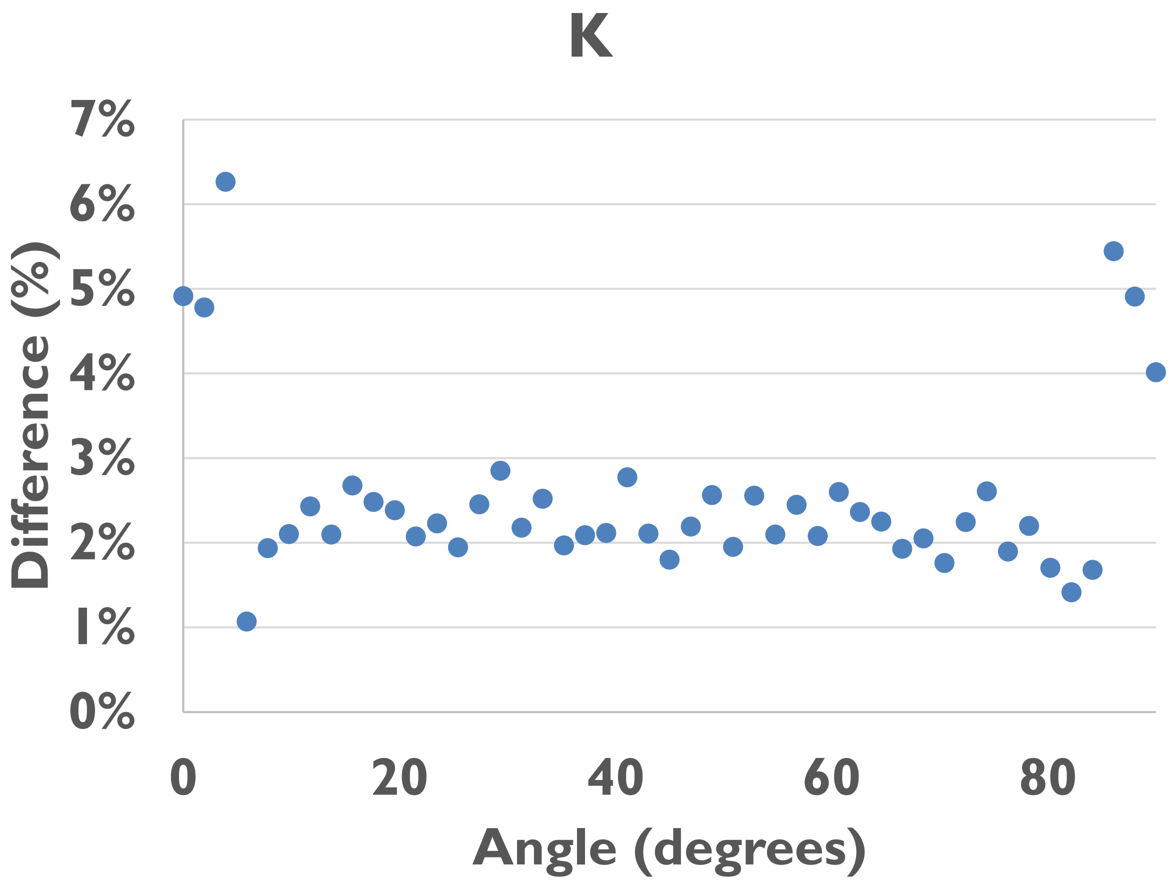
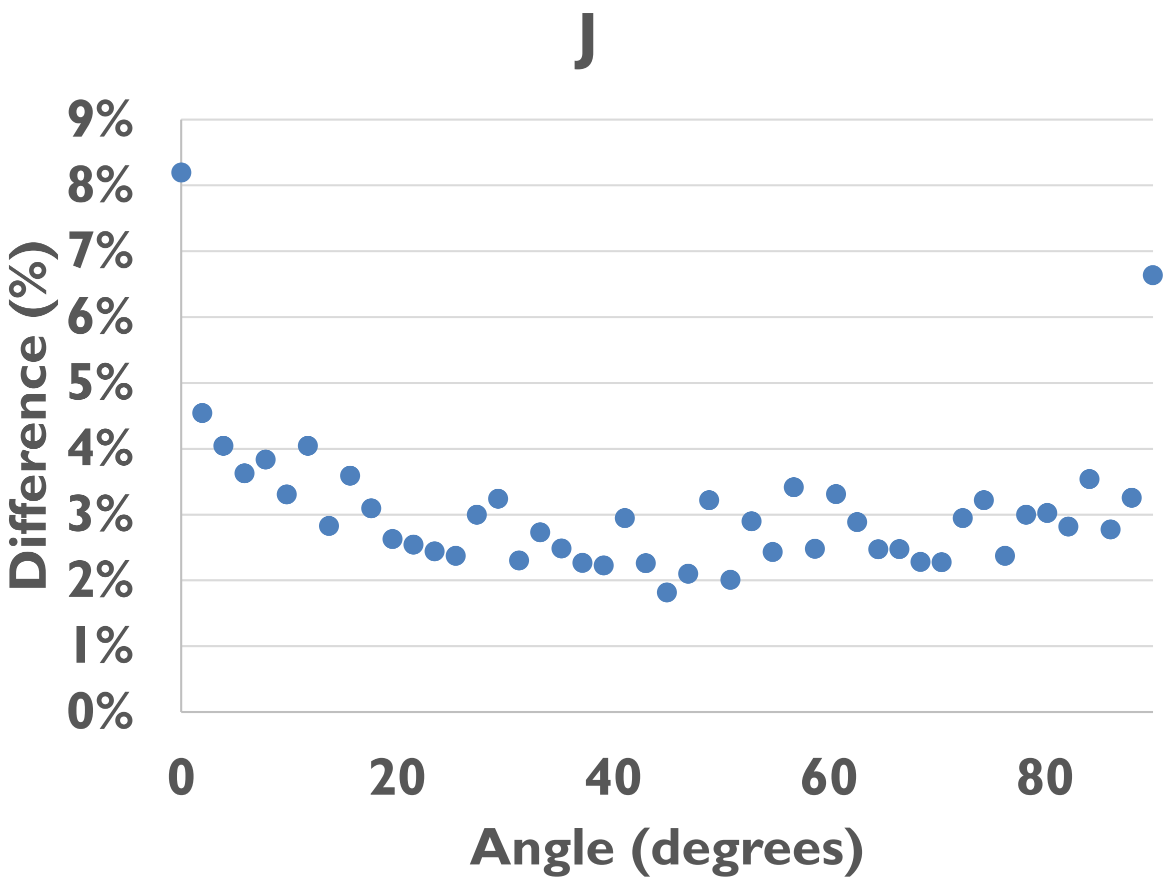
Radius of Integration

Error





2-3% error for K, 2-4% error for J

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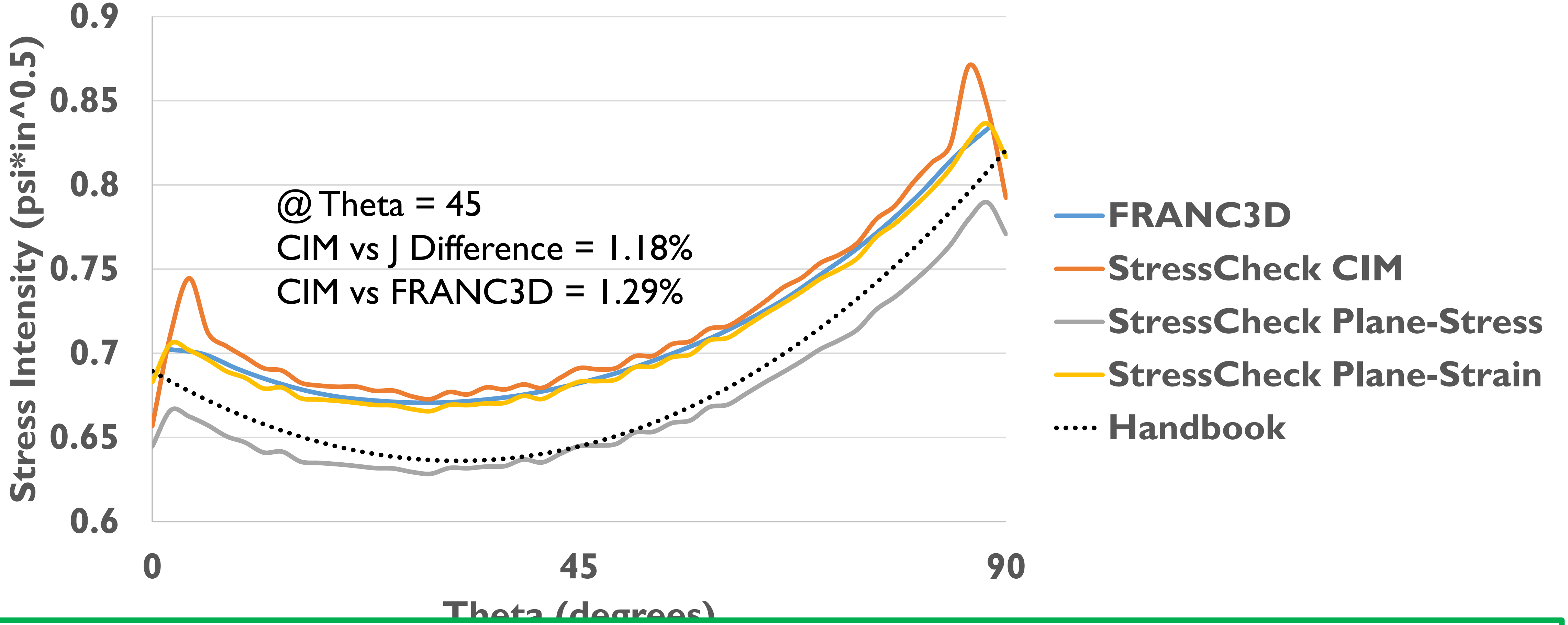
Radius of Integration

Limit as radius of integration goes to zero



Spread is decreasing, noise is increasing

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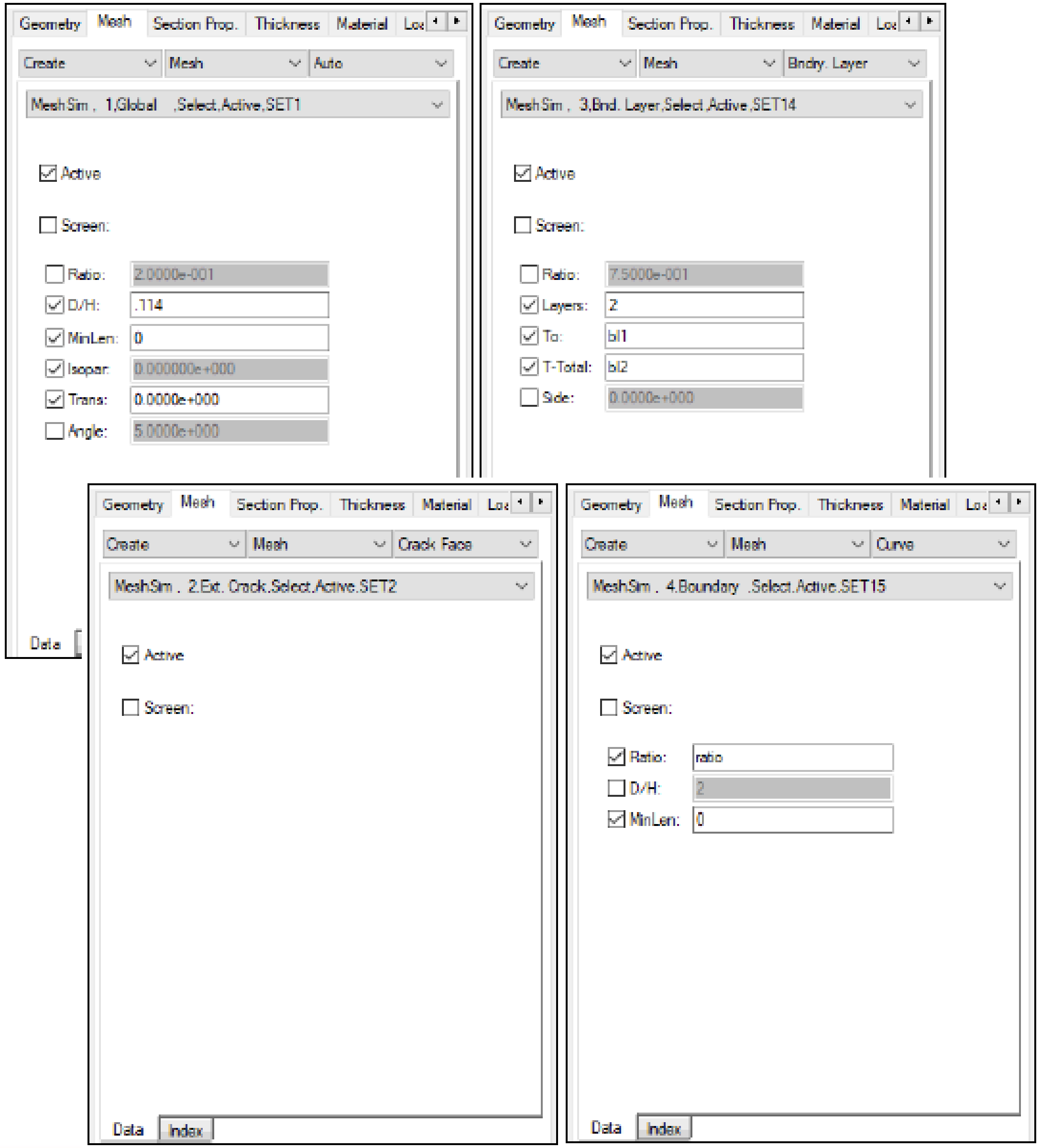
StressCheck Ground Rules

Elliptical Crack Meshing Guidelines

Rev 3 (May 2019)

- Additional curve refinement along crack front
- Higher mesh density, To can be decreased to 0.15*0.15*0.15*c

Parameter	Expression
bl1	0.15^2*min(a,c)
bl2	2*bl1
curvelen	pi*sqrt((a^2+c^2)/2)/2
densityFactor	0.5
ratio	bl1*max(a,c)/curvelen^2/densityFactor



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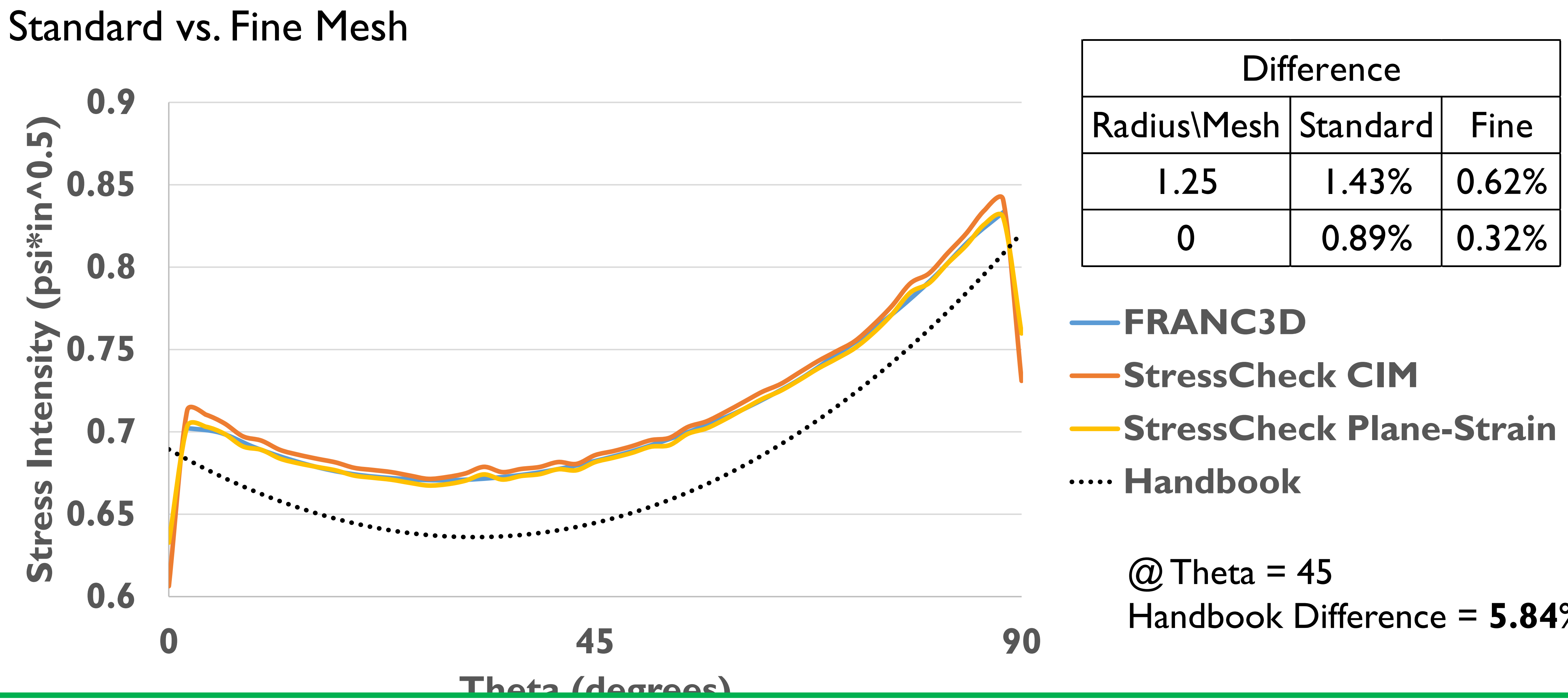
StressCheck Ground Rules



Difference		
Radius\Mesh	Standard	Fine
1.25	1.43%	0.62%
0	0.89%	0.32%

@Theta = 45
Handbook Difference = **5.84%**

Improved difference with new ground rules

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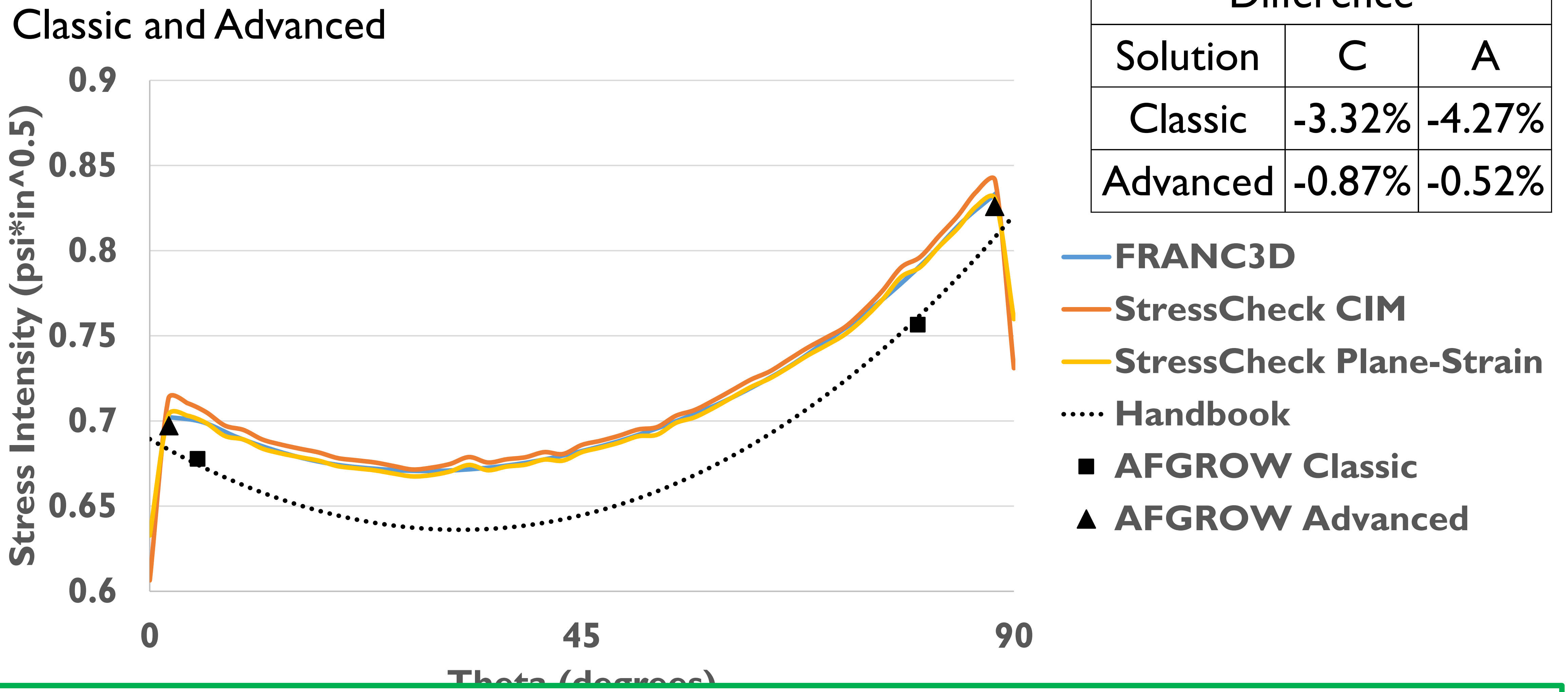
AFGROW Solutions



Difference		
Solution	C	A
Classic	-3.32%	-4.27%
Advanced	-0.87%	-0.52%

Advanced solution is closest to FEM solutions

So What?

AFGROW used with different levels of uncertainty on select T-38 locations

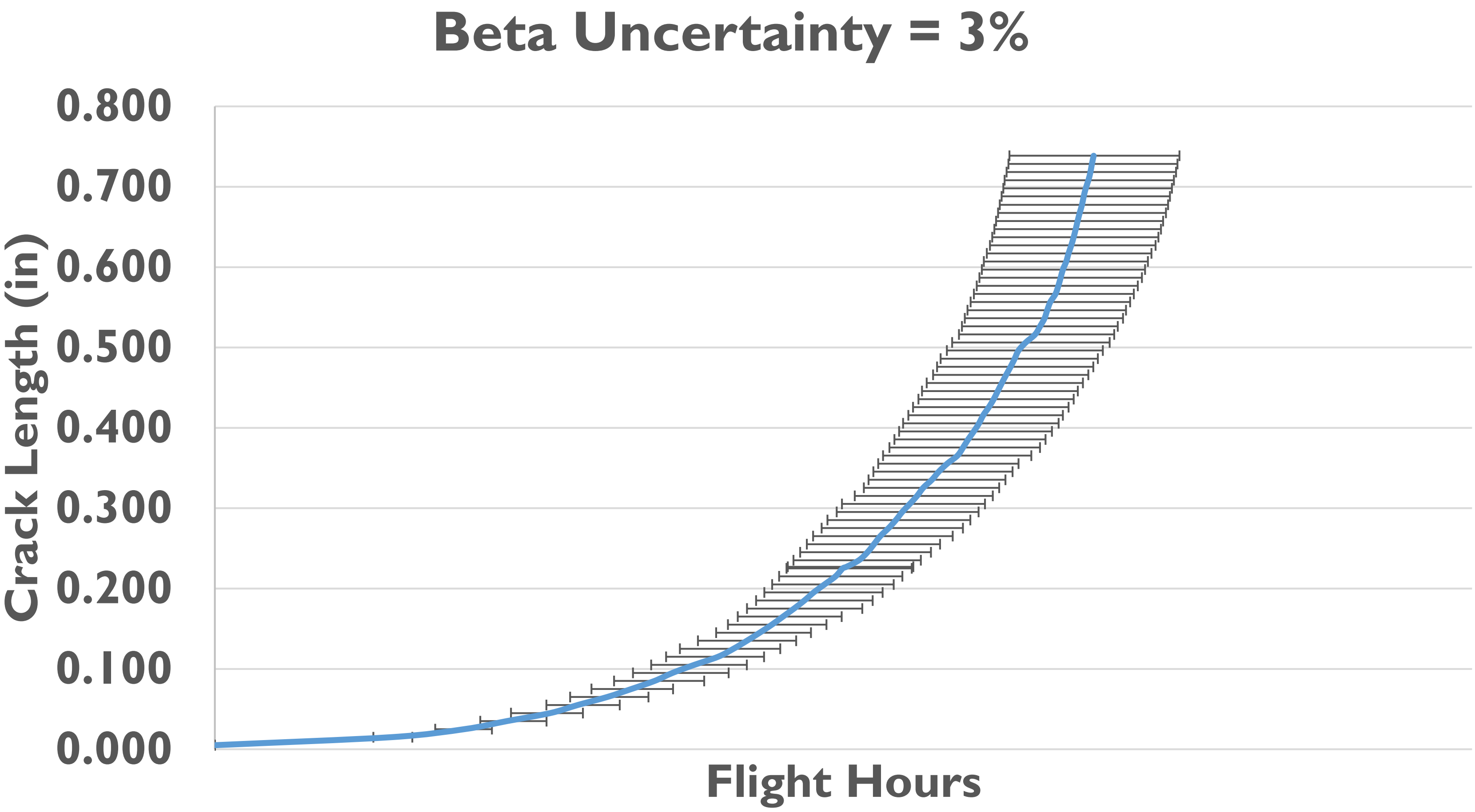
Uncertainty	
Beta	Life
3.00%	10.00%
1.50%	4.00%
0.50%	1.70%

Implications on:

Crack growth rate

Crack shape

SOLR correlations



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Summary/Conclusions

- Verification errors should be less than Validation errors
- K solutions are dependent on material
- Plane-strain conditions at crack front due to stress triaxiality
- Handbook solutions do not account for material
- K solution is dependent on radius of integration
- Current USAF/BAMF ground rules result in 3% uncertainty
- StressCheck's meshing guidelines produce the least uncertainty
- Small beta uncertainty can lead to large life uncertainty

- Extra Verification: Compare to J integral derived SIF

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Recommendations

USAF

Revise SIAG-2010-10884 "Procedure for Developing Stress Intensity Factor (SIF or K) Solutions in StressCheck"

- Solution is dependent on Poisson's ratio
- Update meshing ground rules with StressCheck's recent guidelines
- Include information on radius of integration

ESRD

- Incorporate automated radius of integration convergence
- Implement calculation of K from J integral

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