Modification of Glinka’s Weight Function For Use at a Hole

THE VALUE OF PERFORMANCE.

NORTHROP GRUMMAN

2019 AFGROW Workshop

Wednesday, September 11, 2019

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Strike Division

Outline

- Who
  - Am I?
- What…
  - Are Weight Functions?
- Why…
  - Use Weight Functions?
  - Are Modification Needed?
- How…
  - Was the Modification Developed
  - ‘Good’ is it?

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What…

…Are Weight Functions

A method for determining the stress intensity present in a cracked body, under a given load, as a function of the stresses present in the uncracked body, subjected to the same load.

$$K = \int_0^a \sigma(x)\, \boxed{m(x,a)}\, dx$$

This part is the 'weight function'

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…Are Weight Functions

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There are numerous proposed forms of the weight function and methods for determining them.

$$m(x,a) = \frac{E'}{2K_I}\frac{\partial u(x,a)}{\partial a}$$

Petroski-Achenbach Method (PAM)

$$m(x,a) = \frac{2}{\sqrt{2\pi a\left(1-\frac{x}{a}\right)}}\left[1 + M_1\left(1-\frac{x}{a}\right) + M_2\left(1-\frac{x}{a}\right)^2\right]$$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[\sum_{i=0}^{n} M_i\left(1-\frac{x}{a}\right)^i\right] \quad \therefore \quad M_0 = 1$$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[\sum_{i=0}^{n} M_i\left(1-\frac{x}{a}\right)^{i/2}\right] \quad \therefore \quad M_0 = 1$$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{1/2} + M_2\left(1-\frac{x}{a}\right) + M_3\left(1-\frac{x}{a}\right)^{3/2}\right]$$

Direct Adjustment Method (DAM)

…Are Weight Functions

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$$m(x,a) = \frac{E'}{2K_I}\frac{\partial u(x,a)}{\partial a}$$

Petroski-Achenbach Method (PAM)

DON'T PANIC

$$m(x,a) = \frac{2}{\sqrt{2\pi a\left(1-\frac{x}{a}\right)}}\left[1 + M_1\left(1-\frac{x}{a}\right) + M_2\left(1-\frac{x}{a}\right)^2\right]$$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[\sum_{i=0}^{n} M_i\left(1-\frac{x}{a}\right)^{i}\right] \quad \therefore \quad M_0 = 1$$

We won't be doing any of that today.

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[\sum_{i=0}^{n} M_i\left(1-\frac{x}{a}\right)^{i/2}\right] \quad \therefore \quad M_0 = 1$$

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{1/2} + M_2\left(1-\frac{x}{a}\right) + M_3\left(1-\frac{x}{a}\right)^{3/2}\right]$$

Direct Adjustment Method (DAM)

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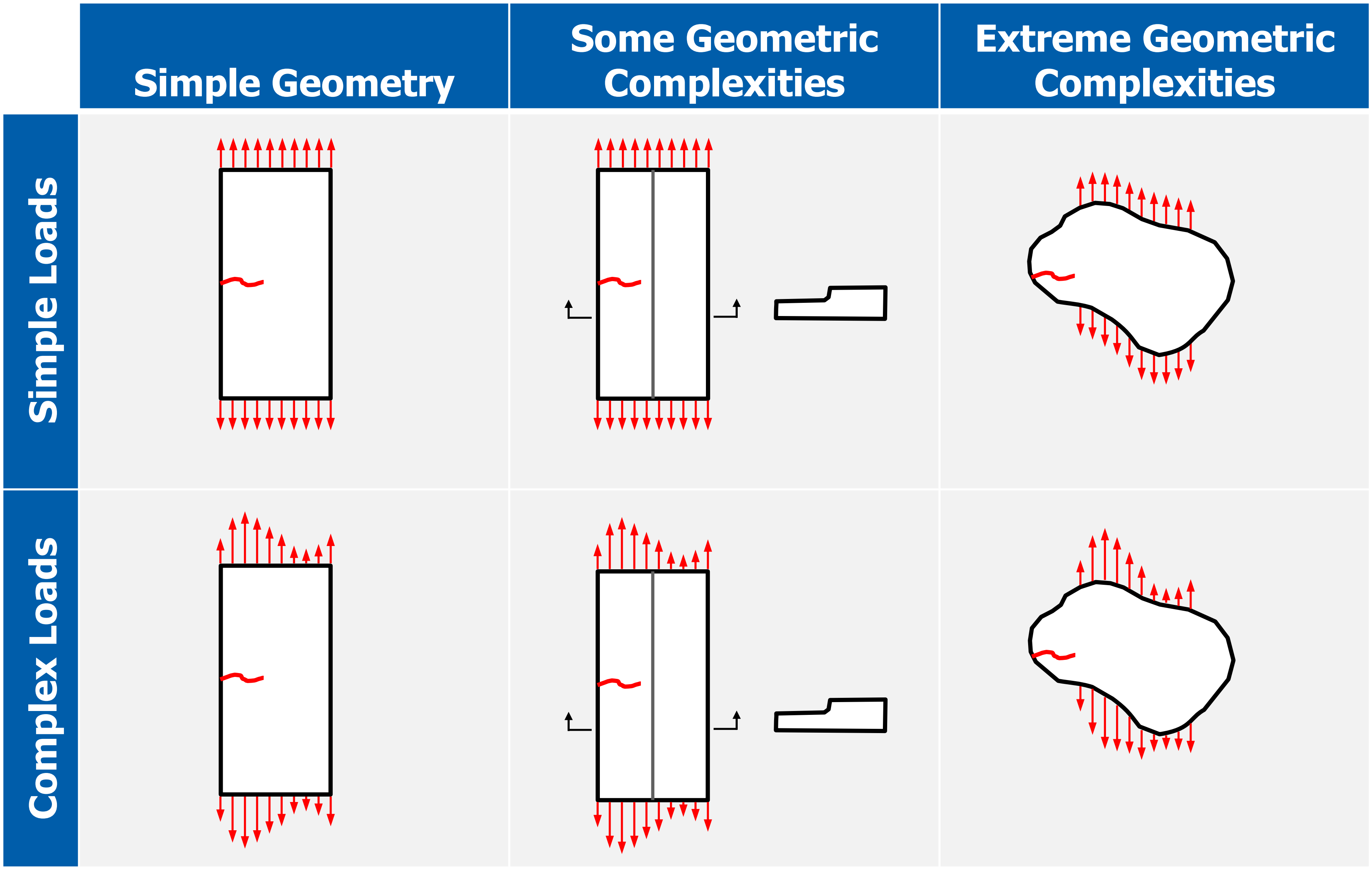
Why…

…Use Weight Functions

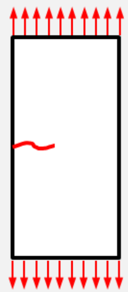
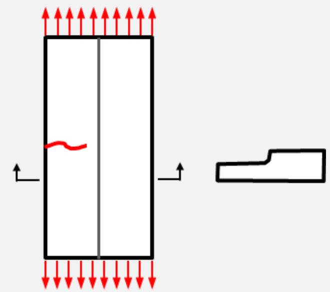
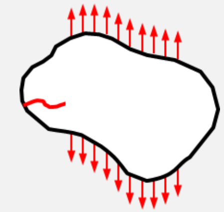
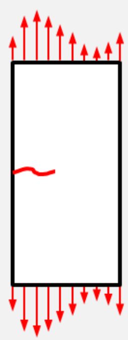
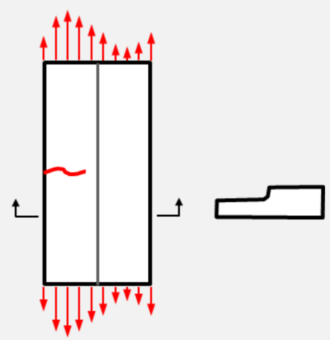
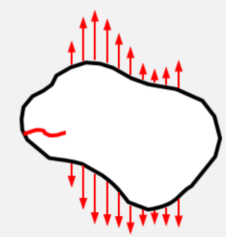
Weight functions have a distinct advantage over many closed-form solutions in that they can capture arbitrary loading.

- Non-elementary loads
  - In-plane bending
  - Non-parallel bearing/bypass
- Nearby geometric features
  - Shadowing/Channeling of Stresses Along Crack Path
- Leverage Results of Parametric Models

…Use Weight Functions



…Use Weight Functions

	Simple Geometry	Some Geometric Complexities	Extreme Geometric Complexities
Simple Loads	 Closed Form (C.F)	 C.F. with Beta Compounding	 Strong Assumptions or FEM
Complex Loads	 Weight Functions (W.F.)	 W.F. with Beta Compounding	 Strong Assumptions or FEM

…are Modifications Needed

Development of ‘new’ Weight Functions is non-trivial and the resulting complexity increases with the number of geometric factors included.

$$K_I = \int_0^a \sigma(x)\left(\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + 0.0719768\left(1-\frac{x}{a}\right)^{1/2} + 0.246984\left(1-\frac{x}{a}\right) + 0.514465\left(1-\frac{x}{a}\right)^{3/2}\right]\right) dx \cdot F_w$$

Is easier to develop and work with than:

$$K_I = \int_0^a \sigma(x)\left(\frac{2}{\sqrt{2\pi(a-x)}}\left[1 + M_1\left(1-\frac{x}{a}\right)^{1/2} + M_2\left(1-\frac{x}{a}\right) + M_3\left(1-\frac{x}{a}\right)^{3/2}\right]\right) dx$$

$Where:$

$$M_1 = 0.0719768 + \sum_{i=1}^{10} C_i \cdot \left(\frac{a}{W}\right)^i$$

$$M_1 = 0.246984 + \sum_{i=1}^{10} C_i \cdot \left(\frac{a}{W}\right)^i$$

$$M_1 = 0.514465 + \sum_{i=1}^{10} C_i \cdot \left(\frac{a}{W}\right)^i$$

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How…

…Was the Modification Developed

Simple Algebra...

if…

$$K_I = \int_0^a \sigma(x) m(x,a)\, dx \cdot F_{cor}$$

then…

$$F_{cor} = \frac{K_I}{\int_0^a \sigma(x) m(x,a)\, dx}$$

(Repeat ad nauseam to cover geometric parameters of interest and fit a function)

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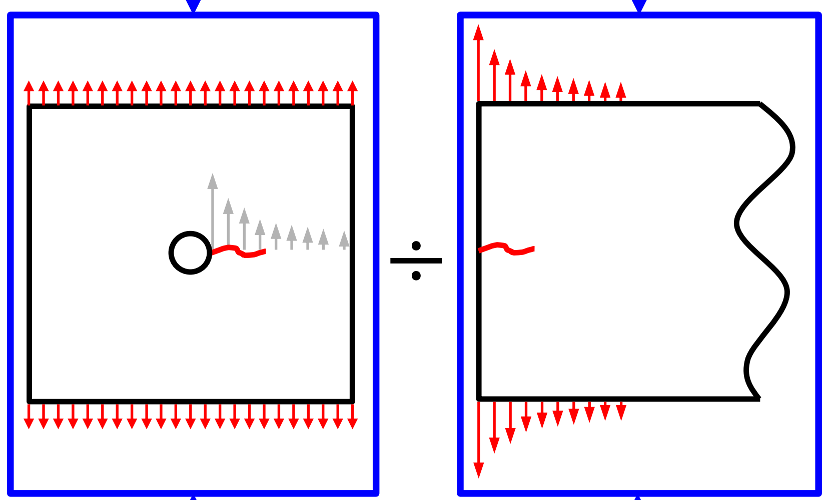
Or...

Stress Intensity of a Through Crack at a Hole Centered in a Finite Width Plate

Edge Crack in a Semi-Infinite Plate Weight Function Solution

$$F_{cor} = \text{[left panel]} \div \text{[right panel]}$$



StressCheck 2D Crack Path

Glinka's Weight Function in AFGROW

…Was the Modification Developed

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Also I cheated a little bit...

$$m(x,a) = m_{ec}\, \Phi_1\, \Phi_2\, \Phi_3$$

where

$$m_{ec} = (a-x)^{-1/2}\ \left[1 + 0.6147\left(1-\frac{x}{a}\right) + 0.2502\left(1-\frac{x}{a}\right)^2\right]\ \left[\sqrt{2/\pi}\right]$$

Bueckner's Weight Function for an Edge Crack in a Semi-Infinite Plate

$$\Phi_1 = 1 - 0.6449\left(\frac{a}{R}\right) + 0.8964\left(\frac{a}{R}\right)^2 - 0.7327\left(\frac{a}{R}\right)^3 + 0.3335\left(\frac{a}{R}\right)^4 - 0.0781\left(\frac{a}{R}\right)^5 + 0.0073\left(\frac{a}{R}\right)^6$$

That sweet correction factor we're looking for

$$\Phi_2 = 1 \qquad \text{(single crack)}$$

$$\Phi_2 = \left\{ \frac{\sqrt{R+a}\left\{1 - \frac{2}{\pi}\tan^{-1}\left[\frac{R}{\sqrt{(R+a)^2 - R^2}}\right]\right\}}{\frac{1}{2}\sqrt{R+\frac{a}{2}}\left\{1 - \frac{2}{\pi}\tan^{-1}\left[\frac{2R-a}{\sqrt{(2R+2)^2-(2R-a)^2}}\right] + \frac{2}{\pi}\sqrt{1-\left(\frac{2R-a}{2R+a}\right)^2}\right\}} \right\} \qquad \text{(double crack)}$$

Single vs. Double Through Crack at a Hole Correction

$$\Phi_3 = \sqrt{\frac{\sec\left\{\left(\frac{2R+a}{W-a}\right)\left(\frac{\pi}{2}\right)\right\}\sin\left\{\left(\frac{2R+a}{W-a}\right)\left(\frac{2a}{W}\right)\right\}}{\sec\left\{\left(\frac{2R}{W}\right)\left(\frac{\pi}{2}\right)\right\}\left\{\left(\frac{2R+a}{W-a}\right)\left(\frac{2a}{W}\right)\right\}}} \qquad \text{(single crack)}$$

$$\Phi_3 = \sqrt{\frac{\sec\left\{\left(\frac{2R+2a}{W}\right)\left(\frac{\pi}{2}\right)\right\}}{\sec\left\{\left(\frac{2R}{W}\right)\left(\frac{\pi}{2}\right)\right\}}} \qquad \text{(double crack)}$$

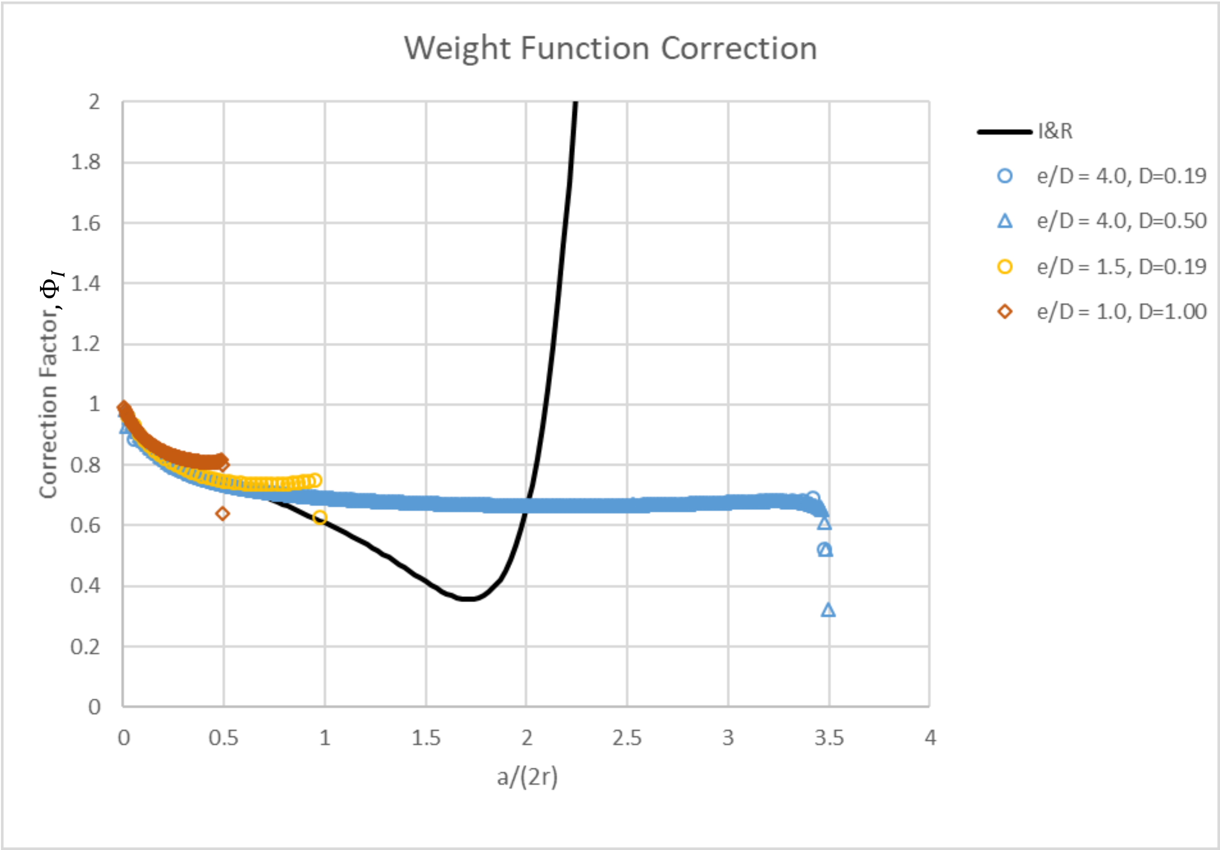
Finite Width Correction

REFERENCE: Impellizzeri, L. F. and Rich, D. L., "Spectrum Fatigue Crack Growth in Lugs," *Fatigue Crack Growth Under Spectrum Loads, ASTM STP 595,* American Society for Testing and Materials, 1976, pp. 320-336.

So using Impellizzeri and Rich's Nomenclature...

$$\Phi_I = \frac{K_I}{\Phi_{II} \Phi_{III} \int_0^a \sigma(x) m(x,a)\, dx}$$

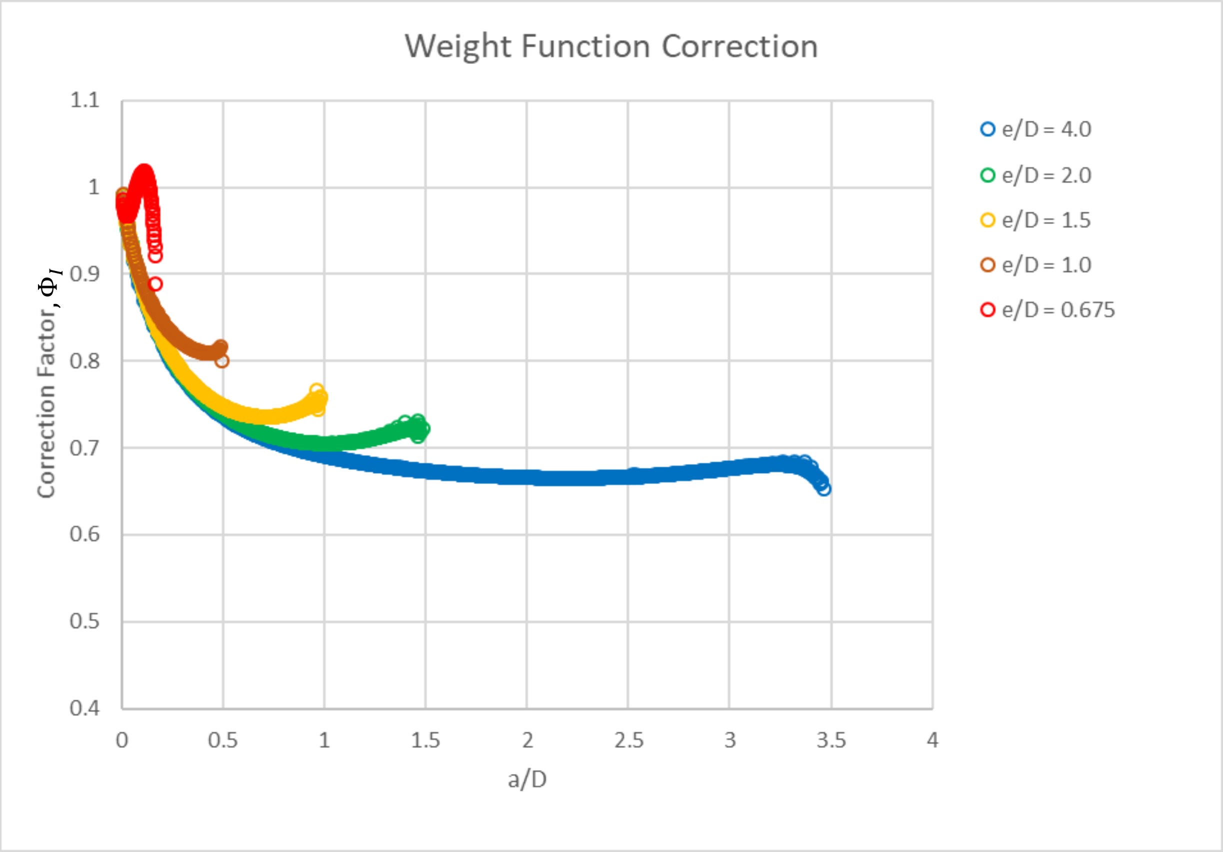
So let's see if we're done...



(Used a simple bypass stress at a hole)

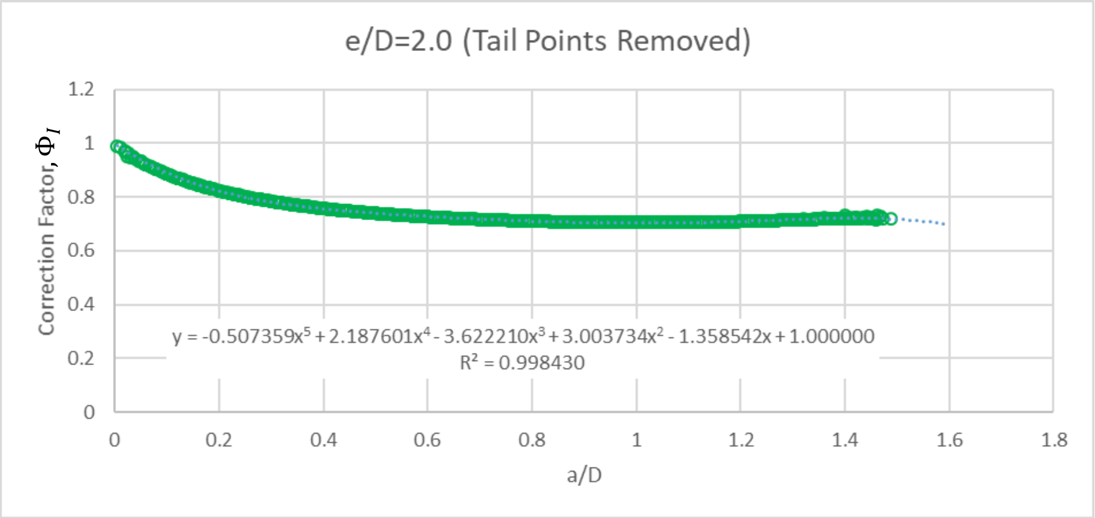
…Was the Modification Developed

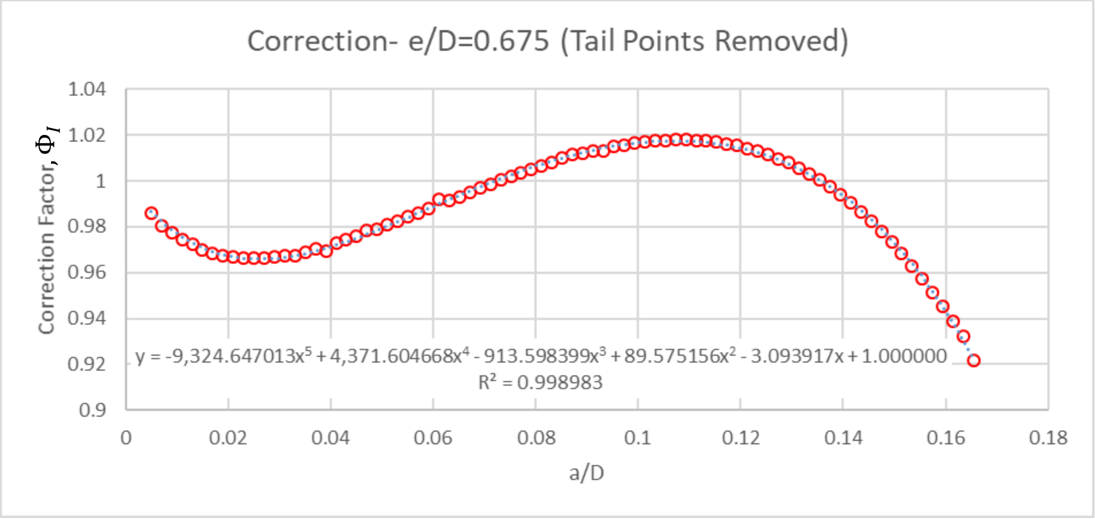
Nope, so let's run some more edge distances and sizes...



…Was the Modification Developed

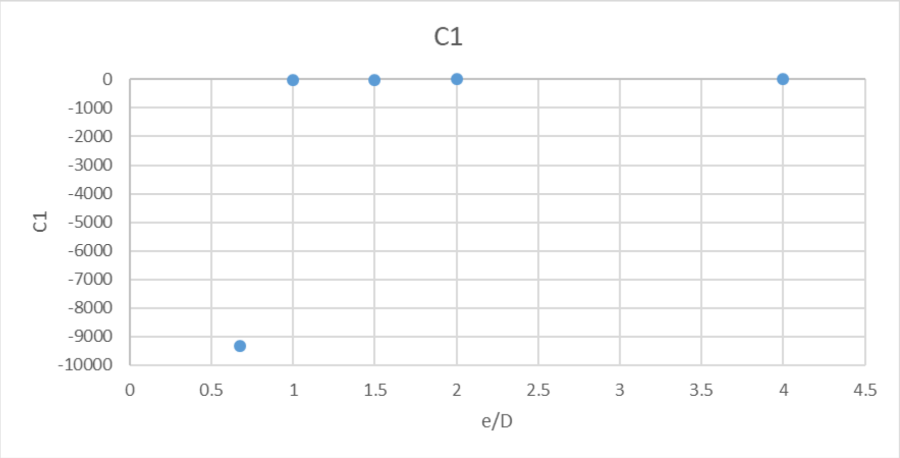
…and let's perform some fits to that

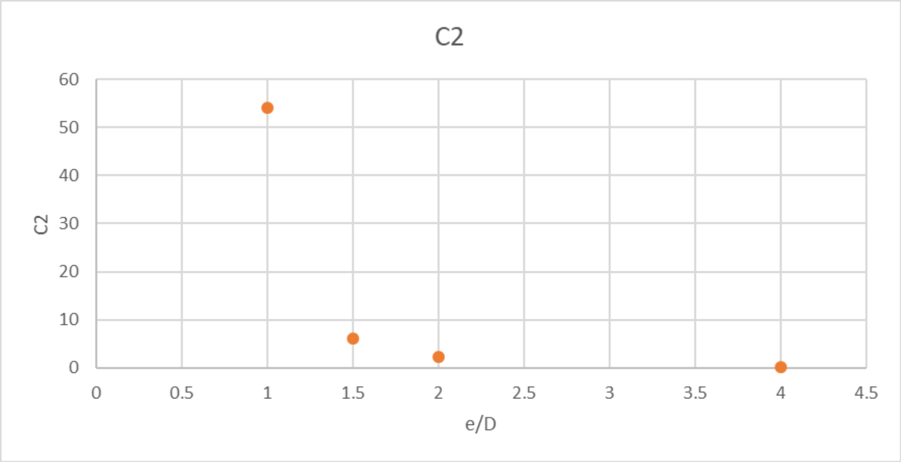


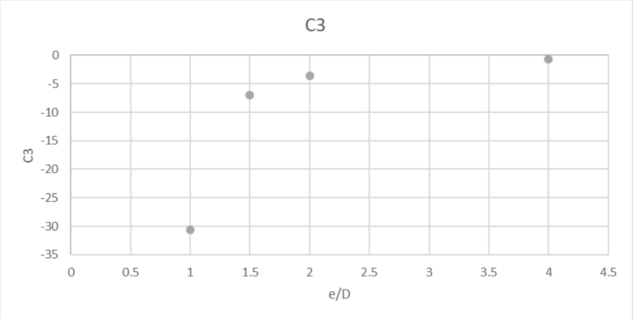


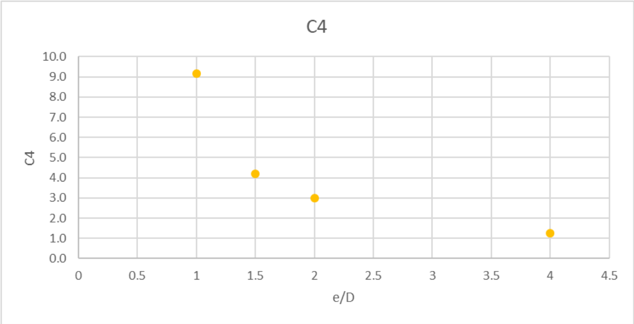
…Was the Modification Developed

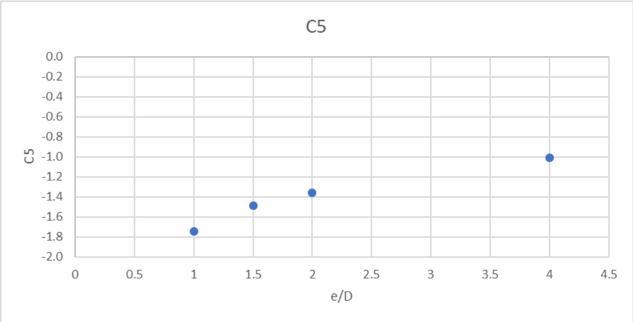
…and look at the polynomial coefficients





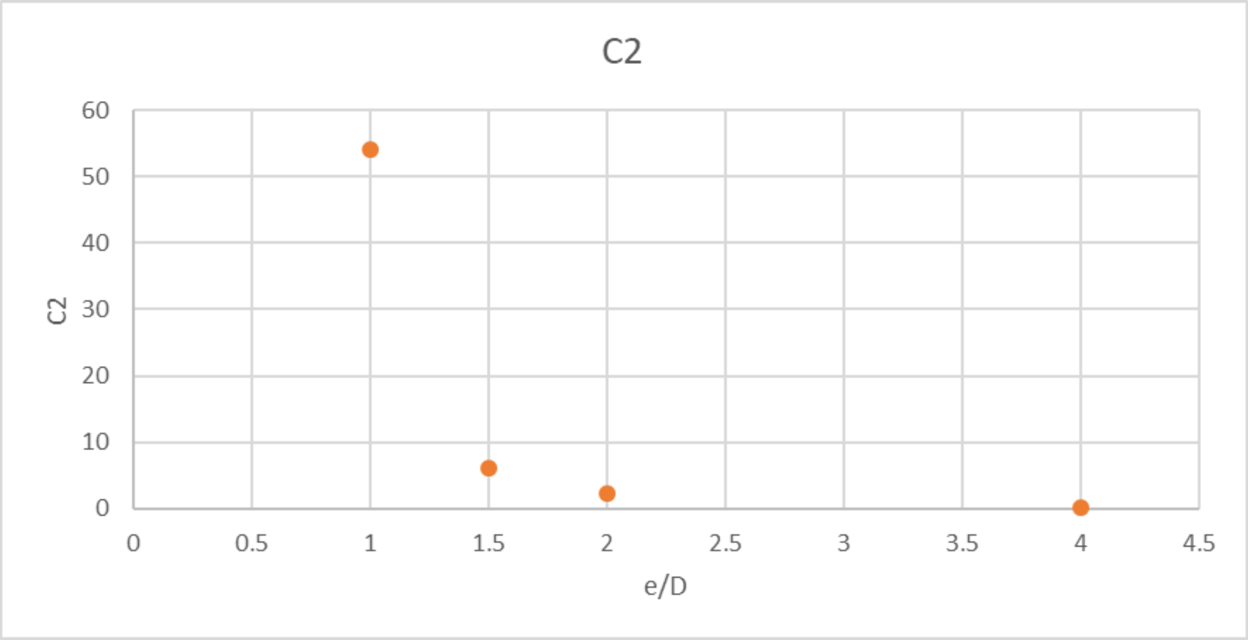






Asymptotic with values approach extremes at short edge distances.

MatLab to the Rescue!



All polynomial coefficients were fit to a rational equation with a third degree numerator as a function of (e/D)

…Was the Modification Developed

Resulting in the following:

$$K_I = \int_0^a \sigma(x) m(x,a)\, dx \cdot \varphi_{FW} \cdot \varphi_G$$

Where:

$$m(x,a) = \frac{2}{\sqrt{2\pi(a-x)}}\left(1 + 0.0719768\left(1-\frac{x}{a}\right)^{1/2} + 0.246984\left(1-\frac{x}{a}\right)^{1} + 0.514465\left(1-\frac{x}{a}\right)^{3/2}\right)$$

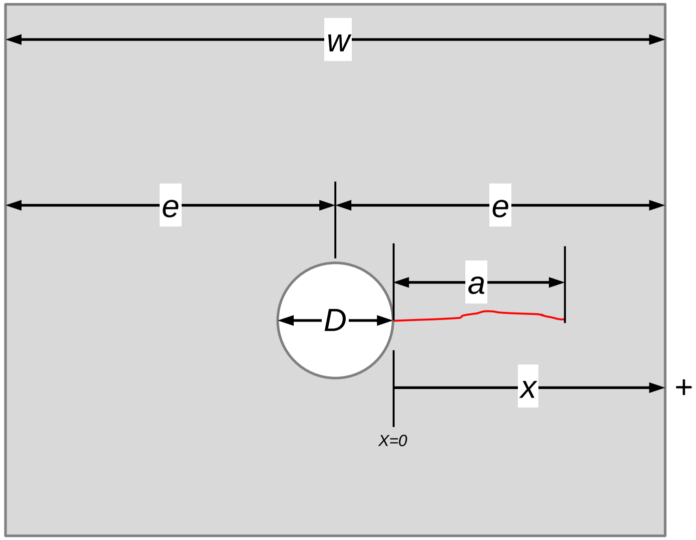
$$\varphi_{FW} = \sqrt{\frac{\sec\left(\left(\frac{D+a}{W-a}\right)\left(\frac{\pi}{2}\right)\right)\cdot \sin\left(\left(\frac{D+a}{W-a}\right)\left(\frac{2a}{W}\right)\right)}{\sec\left(\left(\frac{D}{W}\right)\left(\frac{\pi}{2}\right)\right)\cdot\left(\left(\frac{D+a}{W-a}\right)\left(\frac{2a}{W}\right)\right)}}$$

$$\varphi_G = C_1 \cdot \left(\frac{a}{D}\right)^5 + C_2 \cdot \left(\frac{a}{D}\right)^4 + C_3 \cdot \left(\frac{a}{D}\right)^3 + C_4 \cdot \left(\frac{a}{D}\right)^2 + C_5 \cdot \left(\frac{a}{D}\right) + 1$$

Where:

C_1, C_2, C_3, C_4, C_5 *take the form:*

$$C_i = \frac{p_{i,1}\cdot\left(\frac{e}{D}\right)^3 + p_{i,2}\cdot\left(\frac{e}{D}\right)^2 + p_{i,3}\cdot\left(\frac{e}{D}\right) + p_{i,4}}{\left(\frac{e}{D}\right) + q_i}$$



…Was the Modification Developed

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Resulting in the following:

$$C_i = \frac{p_{i,1} \cdot \left(\frac{e}{D}\right)^3 + p_{i,2} \cdot \left(\frac{e}{D}\right)^2 + p_{i,3} \cdot \left(\frac{e}{D}\right) + p_{i,4}}{\left(\frac{e}{D}\right) + q_i}$$

i	$p_{i,1}$	$p_{i,2}$	$p_{i,3}$	$p_{i,4}$	q_i
1	6.10405502470467	-46.4481550524899	108.076022862816	-79.865541898636	-0.672190254598611
2	-6.89816384032163	52.9502068790586	-125.662520470743	97.6255134276986	-0.667037879902161
3	2.35285083987837	-17.9585627512209	43.0487275715412	-37.950267640094	-0.657102491885296
4	-0.239801885314716	1.44396422365637	-1.87449822502427	3.98219724090163	-0.638147518068896
5	-0.002341760045582	0.161267325315714	-1.66694681626203	0.794022439863378	-0.591478237880012

First…

Can the weight function approximate the FEM results used to develop the correction?

…Good is it?

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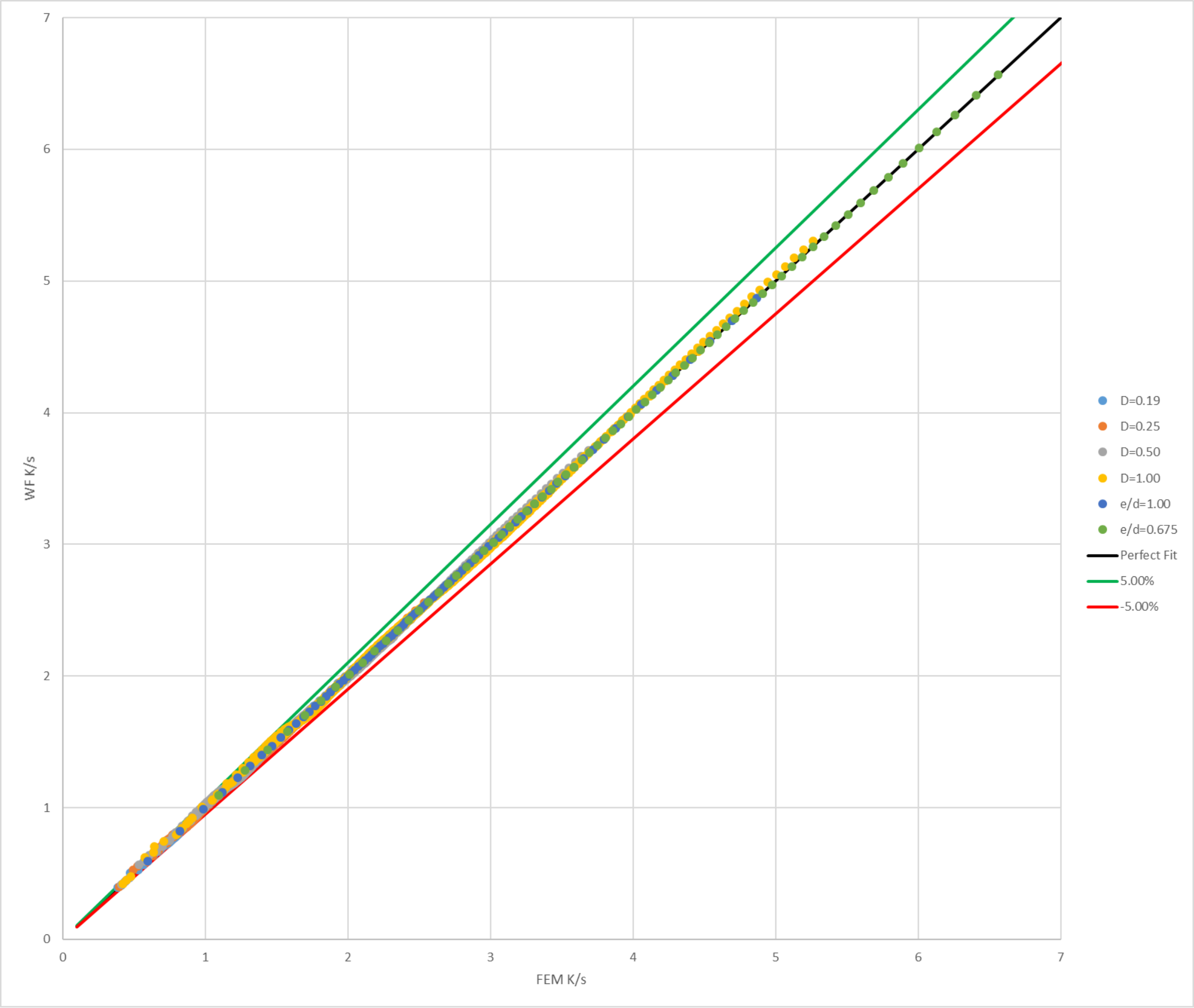
First…

Can the weight function approximate the FEM results used to develop the correction?

YES!!!



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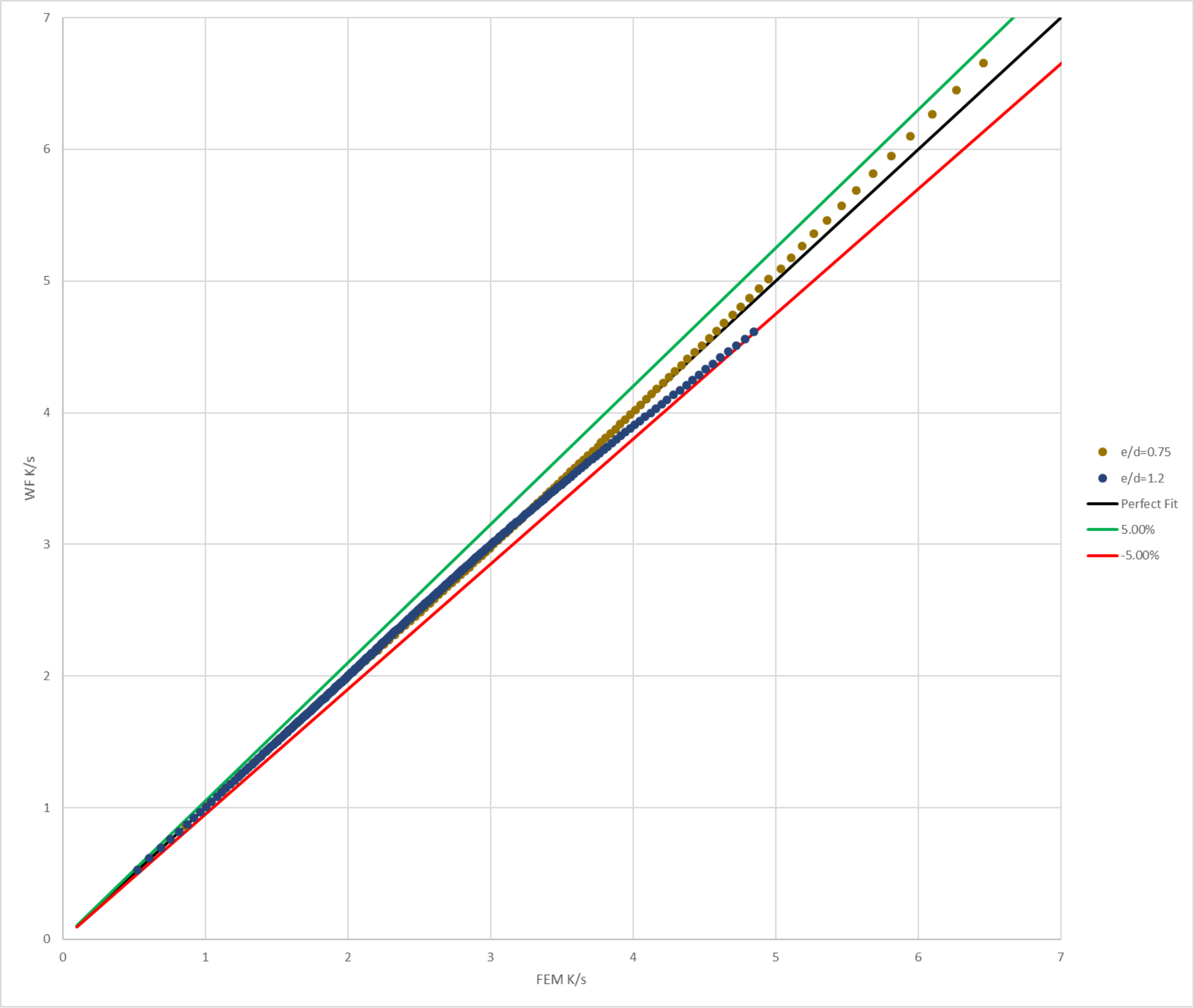
…Good is it?

Second…

Can the weight function approximate FEM results not used to develop the correction?

Yes

But less enthusiastically



…Good is it?

Cursory checks have been promising, but limited.

Need to evaluate

- **Additional e/D's**
  - Check Interpolation
  - Check Extrapolation beyond e/D = 4
- **Additional Load Types**
  - Developed using bypass loads, which begs the question:
    If my gradient doesn't look like a bypass load, how well will it do?
- **How to handle part-through cracks**
  - Slice synthesis techniques
  - Determine an 'equivalent' bearing bypass combination for the gradient and use closed form solutions to determine the ratio of β_a to β_{thru} and β_c to β_{thru}.

Still Work to Be Done!

QUESTIONS?

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Backup

What is Slice Synthesis?

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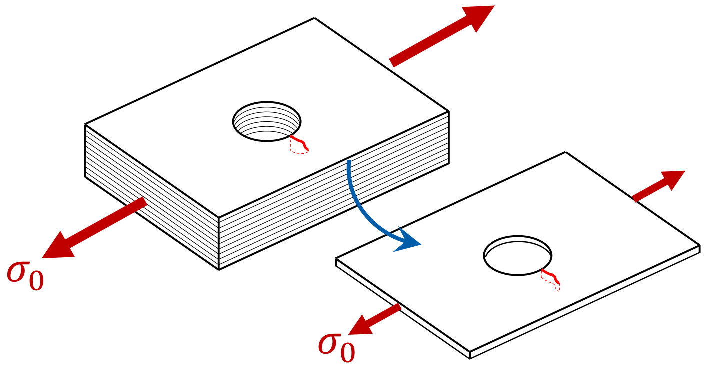
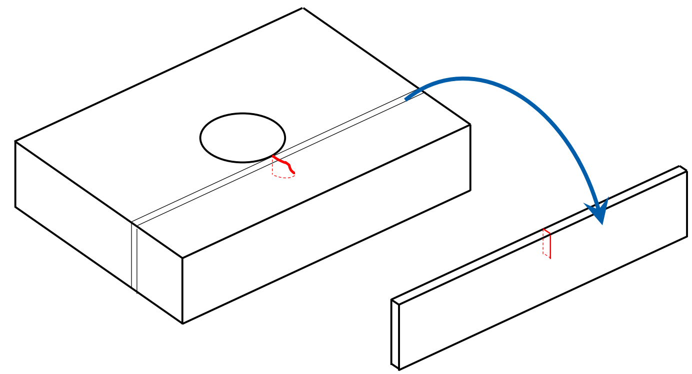
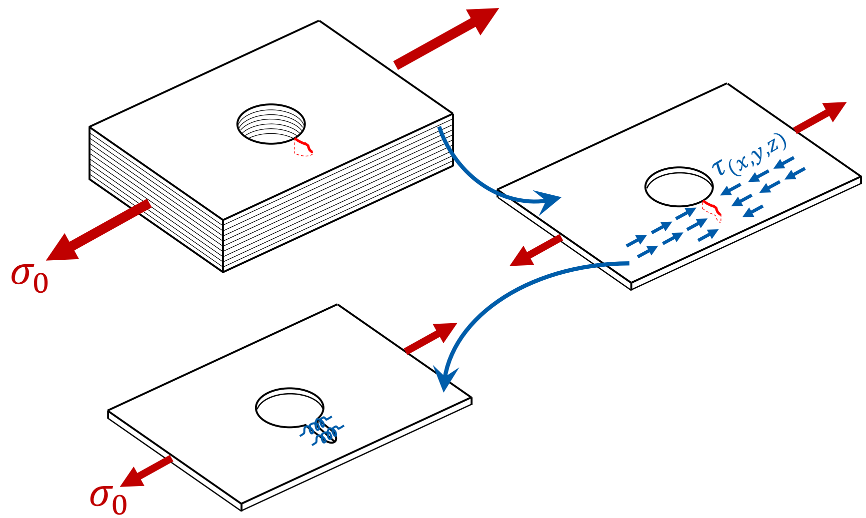


Structure is broken down into a series of ‘slices’ represented by through cracks



Vertical slices are used to determine the mechanical properties of line springs



REFERENCE: Fujimoto, W. T., “Determination of Crack Growth and Fracture Toughness Parameters for Surface Flaws Emanating from Fastener Holes," *17th Structures, Structural Dynamics, and Materials Conference*, May 1976, King of Prussia, PA, USA.