Stress Ratio Influence on da/dN with the Generalized Willenborg Model

Updated Material Data & Fitting Process

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Robert Pilarczyk
Group Lead – Structural Integrity
Hill Engineering, LLC
rtpilarczyk@hill-engineering.com
Phone: 801-391-2682

Dallen Andrew
Engineer Extraordinaire
Southwest Research Institute
dallen.andrew@swri.org
Phone: 210-522-5401

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http://californiasciencecenter.org/sites/default/files/styles/meta/public/media/image/northrop-t-38-talon.jpeg?itok=xTHR_qSd

Agenda

- Background
- Approach
- Test Matrix
- Material Fitting Process
- Re-Correlation of SOLR
- Updated Life Predictions
- Conclusions

http://warbirdalley.com

Background

- Previous studies by T-38 Analysis (Chad King, others) characterized typical T-38 spectra and the associated $R_{effective}$ based on the utilization of the Generalized Willenborg Retardation model
- Study indicated $R_{effective}$ was predominantly negative
  - Minimal existing crack growth rate data for R < 0
- Recommendations from this study included:
  - Complete additional crack growth rate testing at R < 0
  - Compare to existing "extrapolated material model"
  - Revisit R_{lo}
  - Re-correlate crack growth rate data
  - Sensitivity study of select FCLs to understand impact

Background

- **Generalized Willenborg Model**
  - Adjusts da/dN by reducing R to R_{eff}

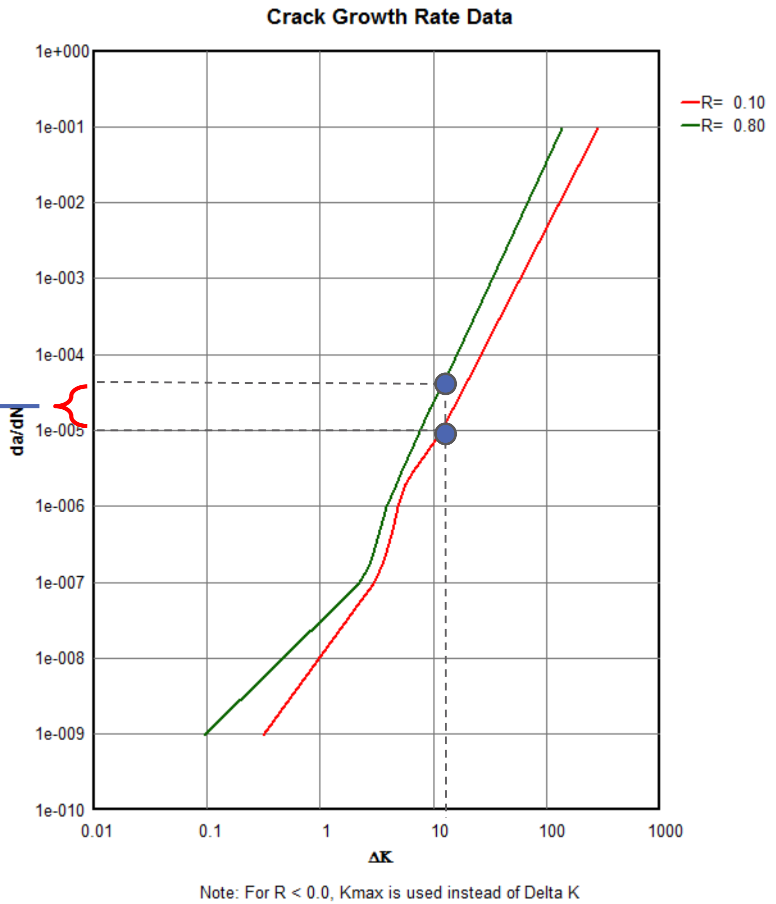
$$K_r = \frac{1-\frac{\Delta K_{th}}{K_{max}}}{(SOLR-1)}\left[K_{OL}\sqrt{1-\frac{x-x_{OL}}{r_{OL}}}-K_{max}\right]$$

$$K_{min,eff} = K_{min} - K_r$$

$$K_{max,eff} = K_{max} - K_r$$

$$R_{eff} = \frac{K_{min,eff}}{K_{max,eff}}$$

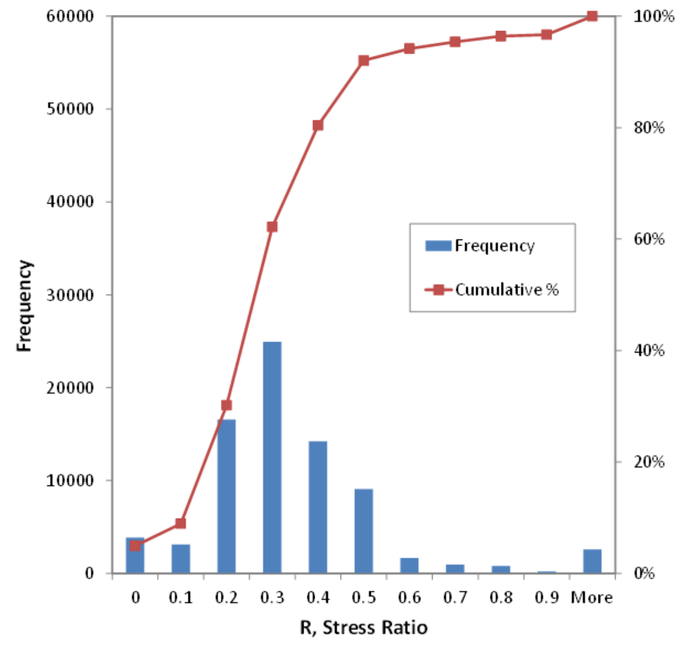
Retardation Effect:
Reduction in *da/dN*



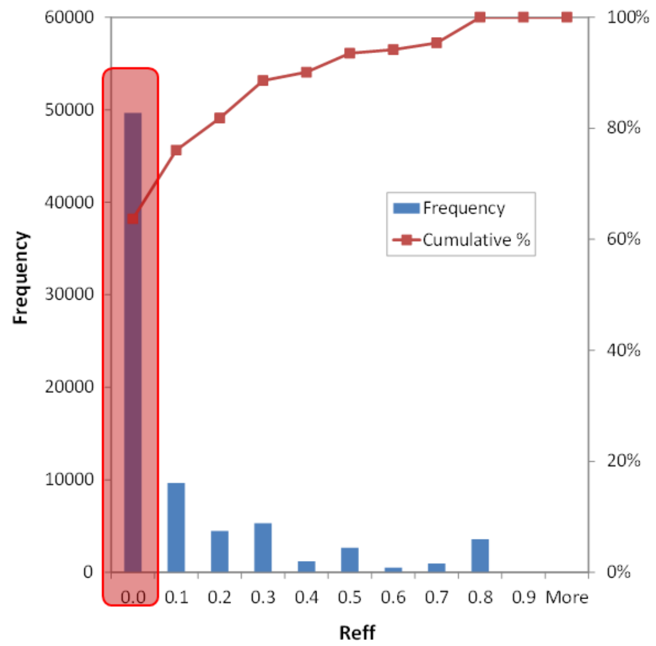
Ref: Lessons Learned: Stress Ratio Influence on *da/dN* when using the Generalized Willenborg Model, Chad King, AFGROW Workshop 2015.

Background

FCL A-1-29 IFF Usage



***R*, Stress Ratio**



R_{eff}, Effective Stress Ratio

1st 1000 hours

Ref: Lessons Learned: Stress Ratio Influence on *da/dN* when using the Generalized Willenborg Model, Chad King, AFGROW Workshop 2015.

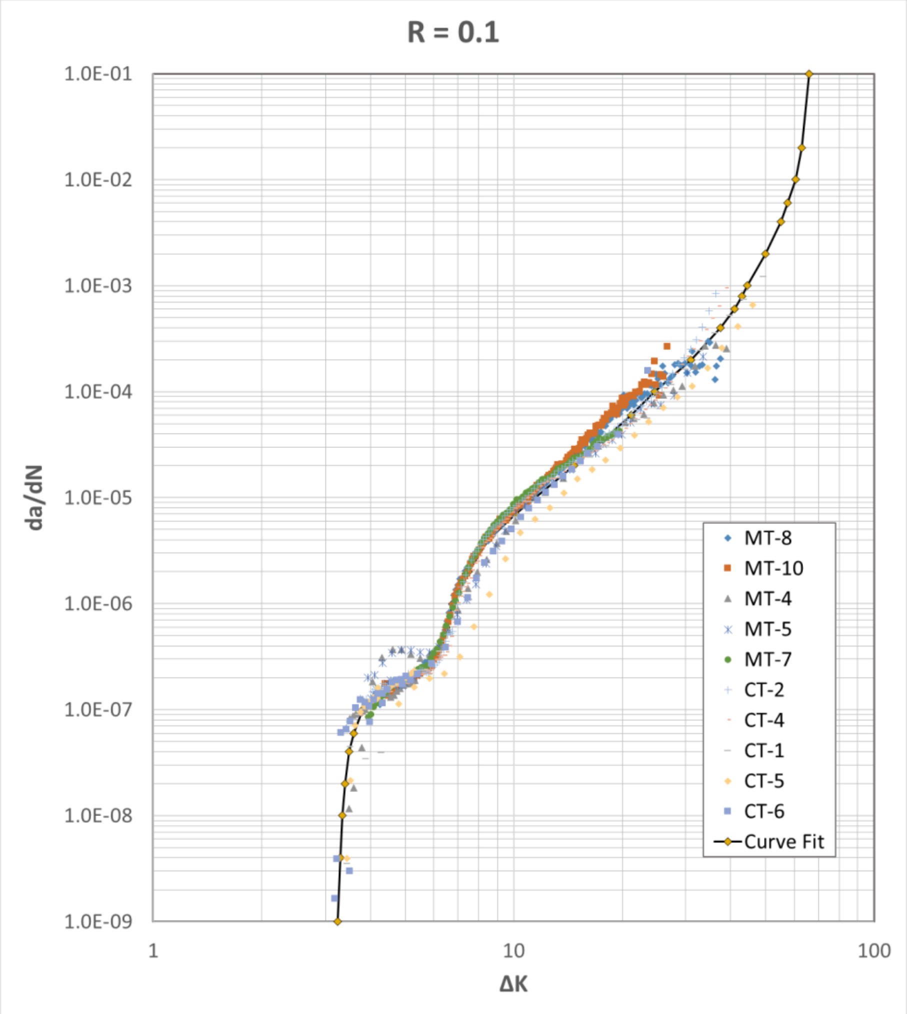
Approach

- Identify necessary negative R data
- Complete new Fatigue Crack Growth Rate (FCGR) testing
- Re-correlate tabular lookup files
  - Initial focus – 7075-T7351, 7475-T7351
  - Incorporate new test data
  - Follow disciplined material fitting process
- Select (2) FCLs (A-10a, A-15):
  - Re-correlate SOLRs
  - Recalculate damage tolerance life
- Characterize impacts of negative R data and new fitting process
- Provide recommendations

Material	Stress Ratios
7075-T7351	R=0.5, 0.1, 0.05, -0.3, -0.5, -1.0
7475-T7351	R=0.8, 0.5, 0.1, 0.05, -0.3, -0.5, -1.0
4330M	R=0.1
4130	R=0.7, -0.2, -0.5
7075-T73511	R=-0.2, -0.5

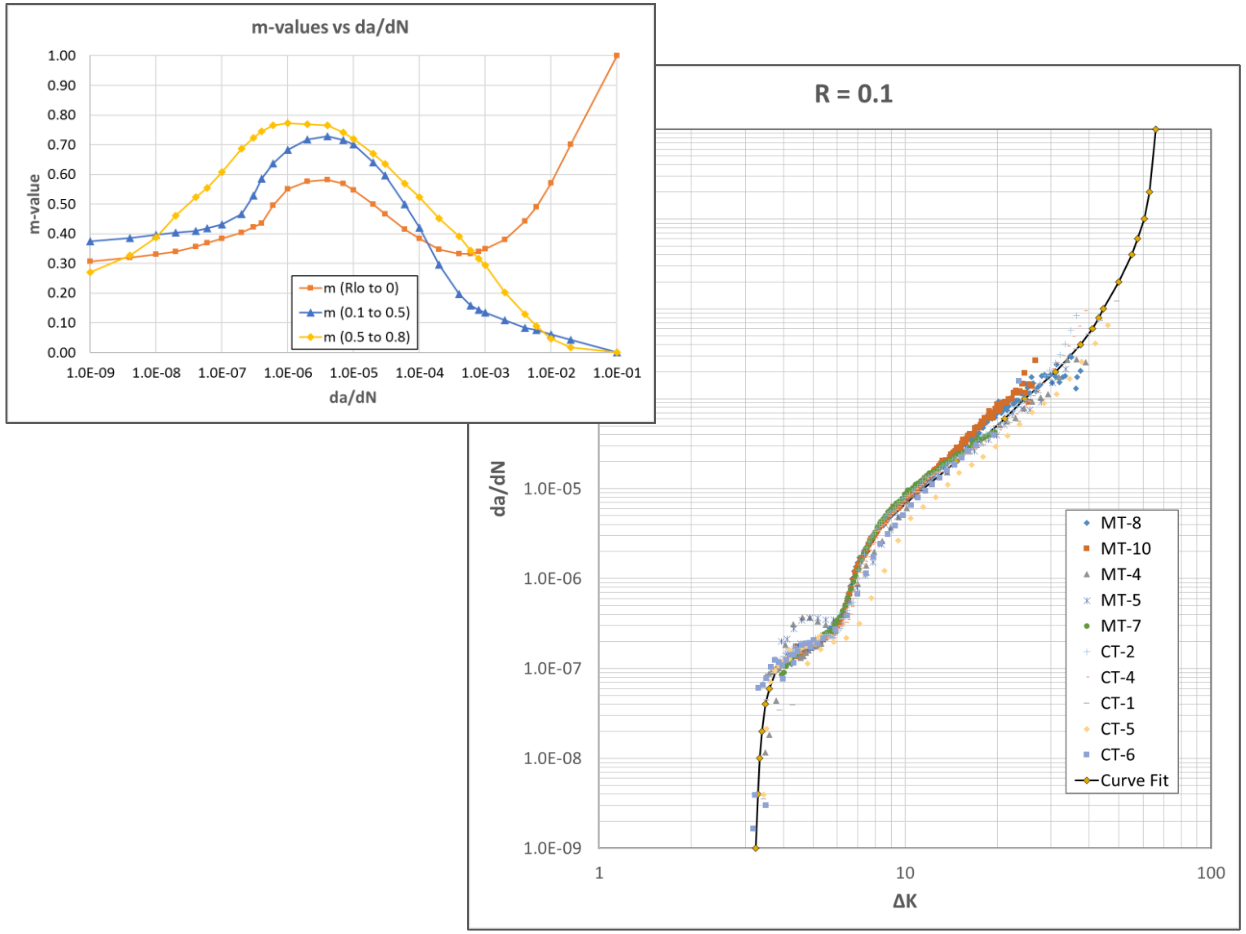
Material Fitting Process

- Discipline process presented at AFGROW training courses
- Based on Harter-T method
  - Shifts da/dN curves for any R
  - Applies Walker Equation (point-by-point basis)
- General process:
  - Step 1: Initial fit of data at each R
  - Step 2: Calculate Walker equation “m” values
  - Step 3: Define max ΔK for each R
  - Step 4: Smooth m curves for positive R values
  - Step 5: Fit negative R data & determine Rlo



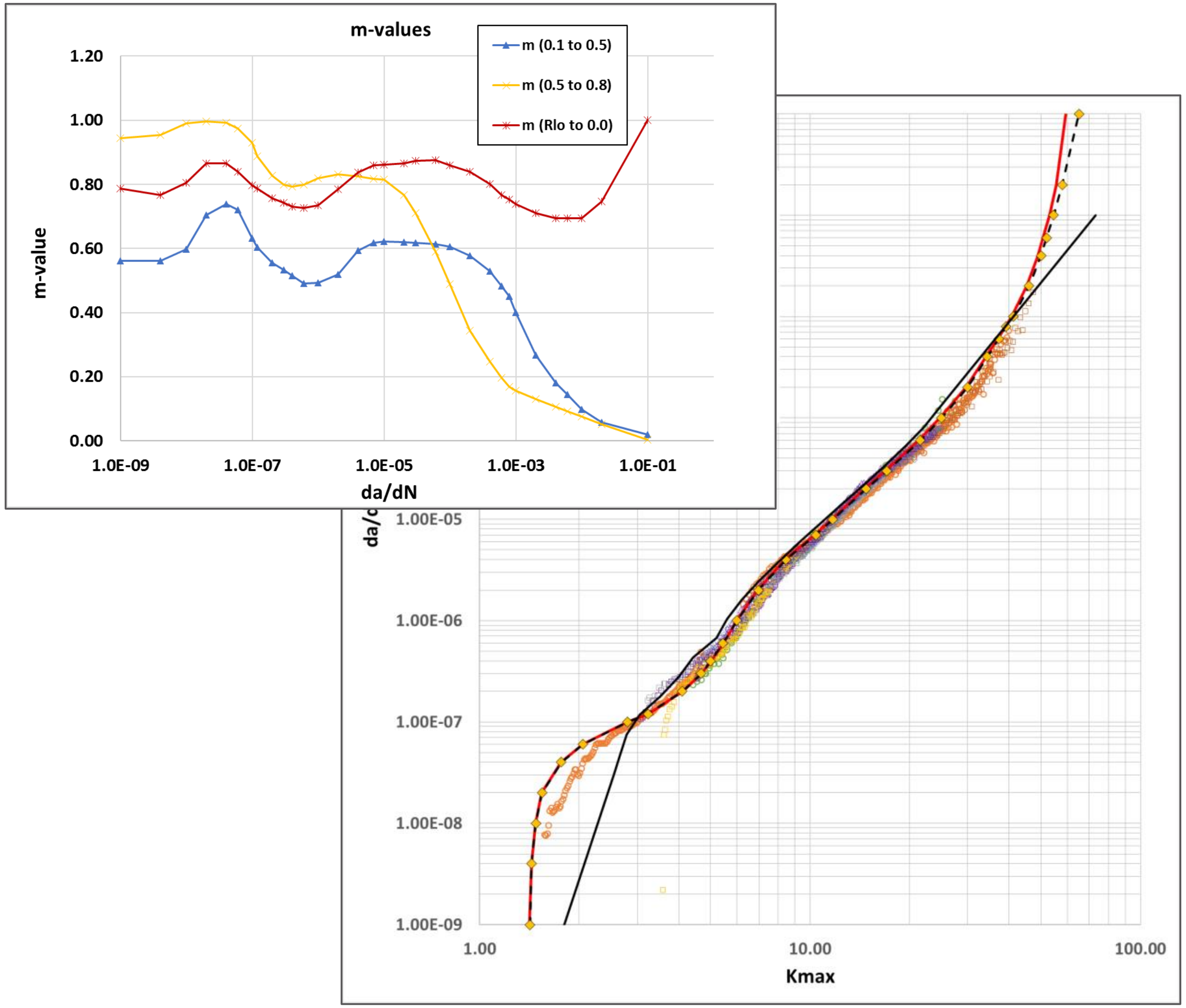
Material Fitting Process

- General process:
  - Step 1: Initial fit of data at each R
  - Step 2: Calculate Walker equation "m" values
    - Based on Harter-T method and plot for adjacent R values
    - See AFGROW users manual for additional info
  - Step 3: Define max ΔK for each R
    - ΔK at R=0 should match Kc as long as coupon is in plane stress
    - Other ΔK max values = (1-R)*Kc
  - Step 4: Smooth m curves for positive R values
    - Work from smallest to largest R
    - Priority – (1) match data, (2) smooth m, (3) similar m-curve shapes
    - Positive R m-curves must trend to 0 at highest da/dN



Material Fitting Process

- **General process:**
  - Step 5: Fit negative R data & determine Rlo
    - Fit negative R data
    - Generate negative R curve based on m-values from adjacent positive R
    - Adjust R until the shifted curve matched the fit of the negative R data
      - The resulting R is the R_{lo} value
  - General rules:
    - Rlo must go to R=0 curve at highest point
    - Positive R curves cannot cross each other
    - R_{hi} is typically 0.7-0.8
    - Negative R shift relative to R=0 must be less than the shift for the same positive R



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Material Fitting Process

Walker Equation:

da/dN = C[ΔK(1-R)^(m-1)]^n

At a given da/dN, the relationship reduces to:

ΔK =ΔK(R=0)(1-R)^(1-m), For R>=0.0

ΔK =ΔK(R=0)(1-R)^(m-1), For R<0.0 - Note: Here, ΔK is Kmax

Although not algebraically correct, it is important that the proper trend in **R** shift be maintained. This trend is that as **m** increases, the **R shift** decreases.

Note, that this method is simply a way to interpolate/extrapolate data in log-log scale by using the exponential form. This method has given very good results over the years.

Since **da/dN = C[ΔK(1-R)^(m-1)]^n** , (Walker equation)

At a given **da/dN**, data at any two positive **R**-values becomes:

ΔK1(1-R1)^(m-1) = ΔK2(1-R2)^(m-1)

Solving for **m** yields:

m = 1 + [(log10(ΔK1 / ΔK2)) / log10((1-R2)/(1-R1))]

For the reasons stated in the Harter T-Method reference the method to handle negative stress ratios simply involves switching the exponent for the negative R as follows:

ΔK1(1-R1)^(1-m) = ΔK2(1-R2)^(m-1), Where **R1** is negative and **R2** is positive

Solving for **m** yields:

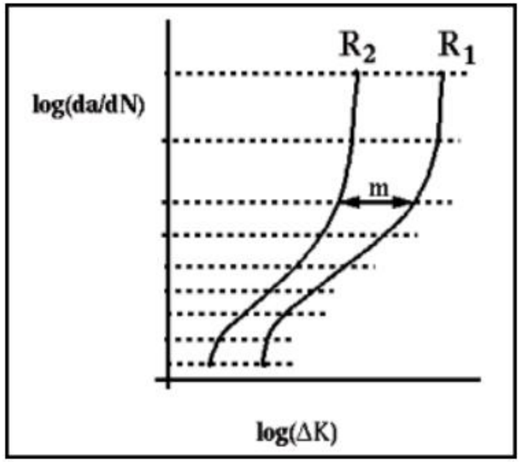
m = 1 + [(log10(ΔK1 / ΔK2)) / log10((1-R1)(1-R2))]

For two negative R values, the relationship becomes:

ΔK1(1-R1)^(1-m) = ΔK2(1-R2)^(1-m)

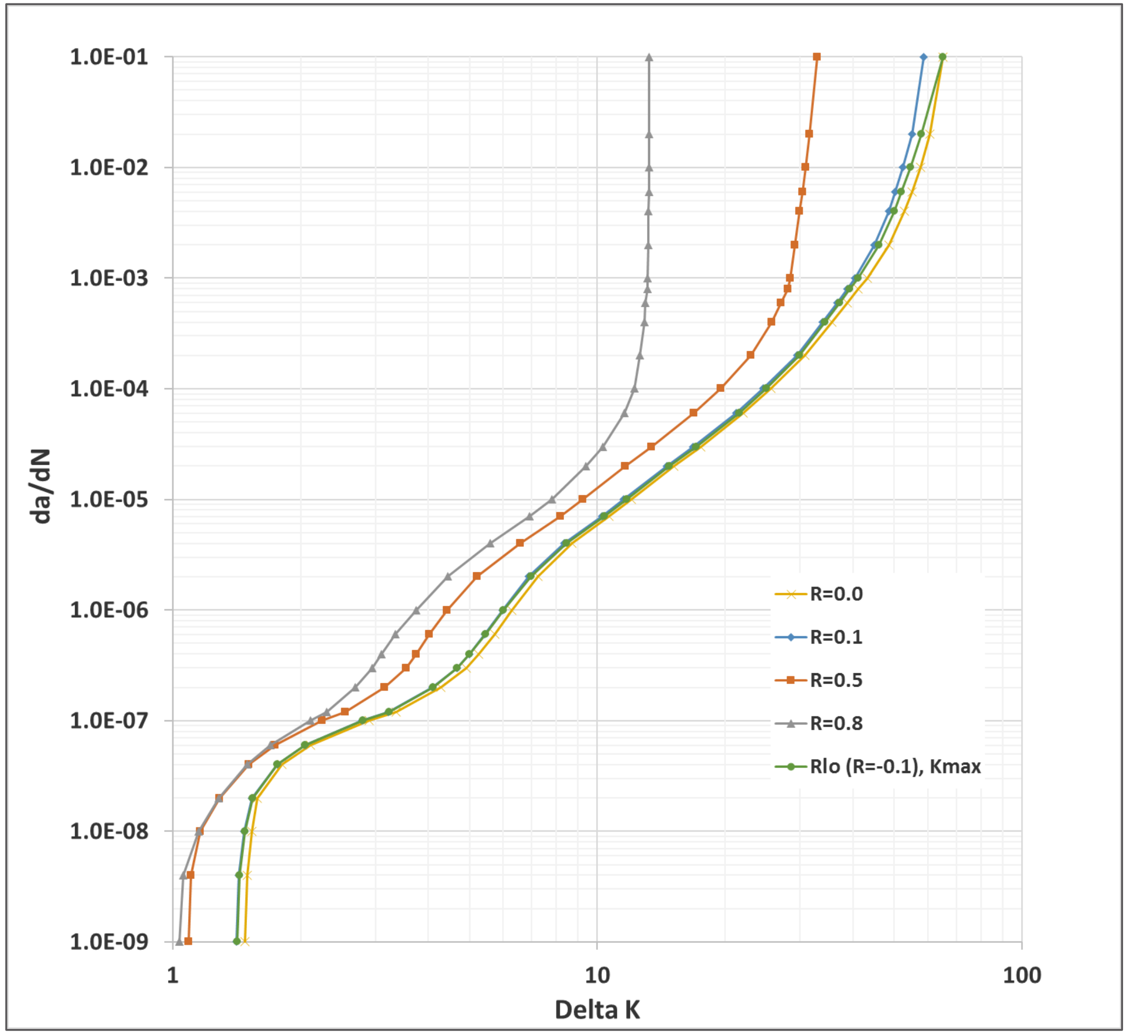
Solving for m yields:

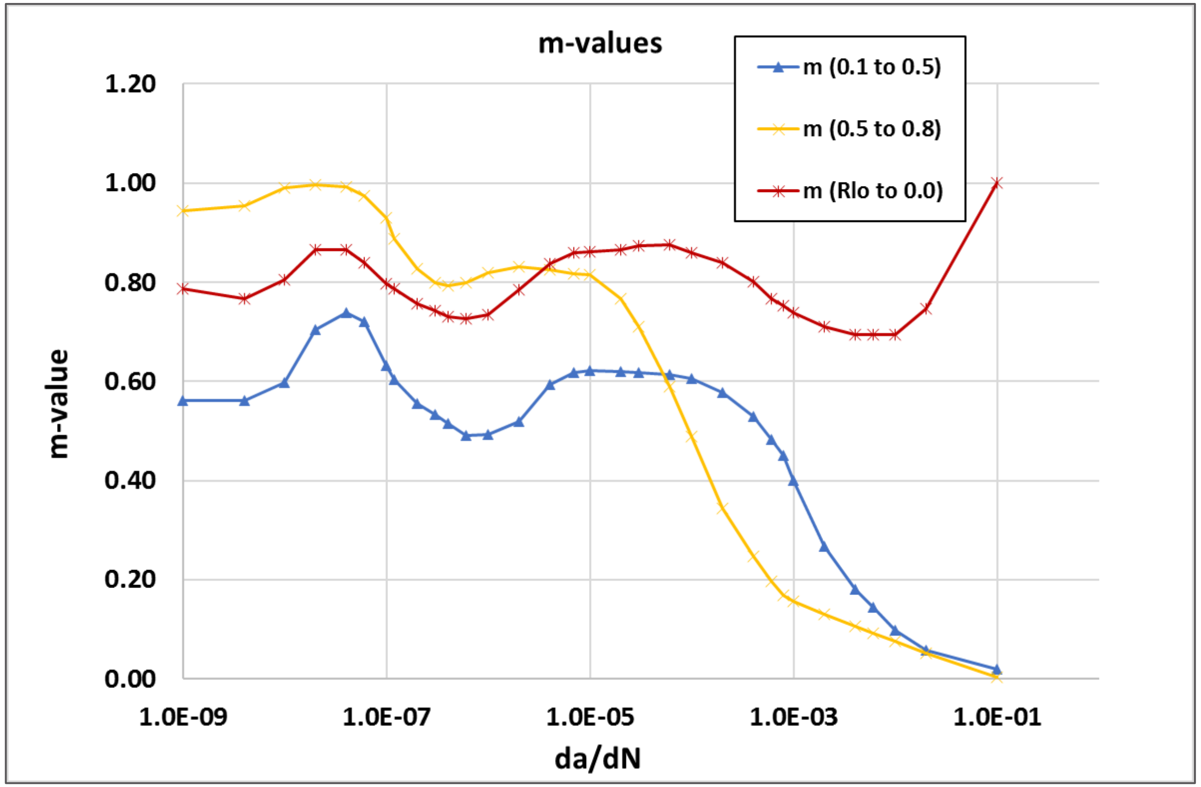
m = 1 - [(log10(ΔK1 / ΔK2)) / log10((1-R2)/(1-R1))]



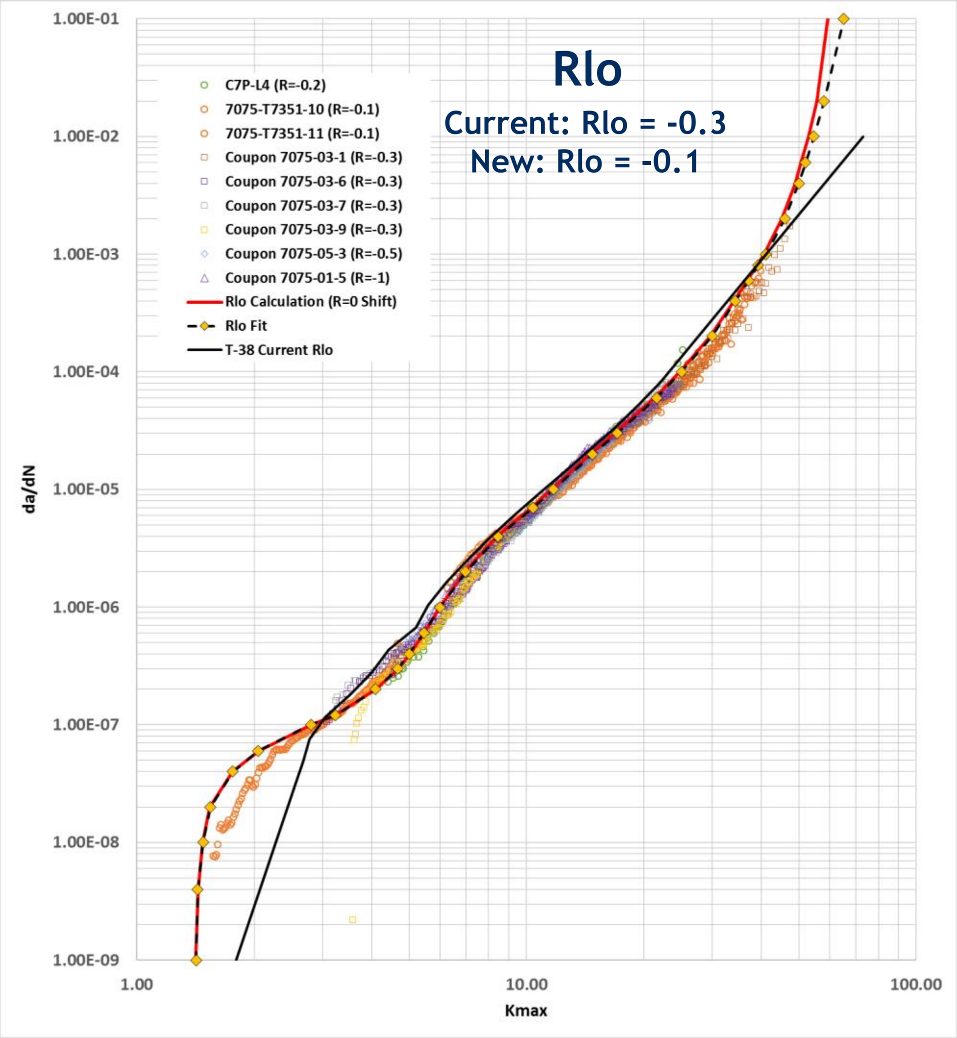
- **m is NOT the slope of anything of interest**
- **m is NOT a distance or stress intensity**
- **m has no dimensions**
- **m in not part of the candy team that melts in your mouth, not in your hand**
- **The value of m is merely a mathematical means of controlling the shift of the crack growth rate data as a function of stress ratio (R)**

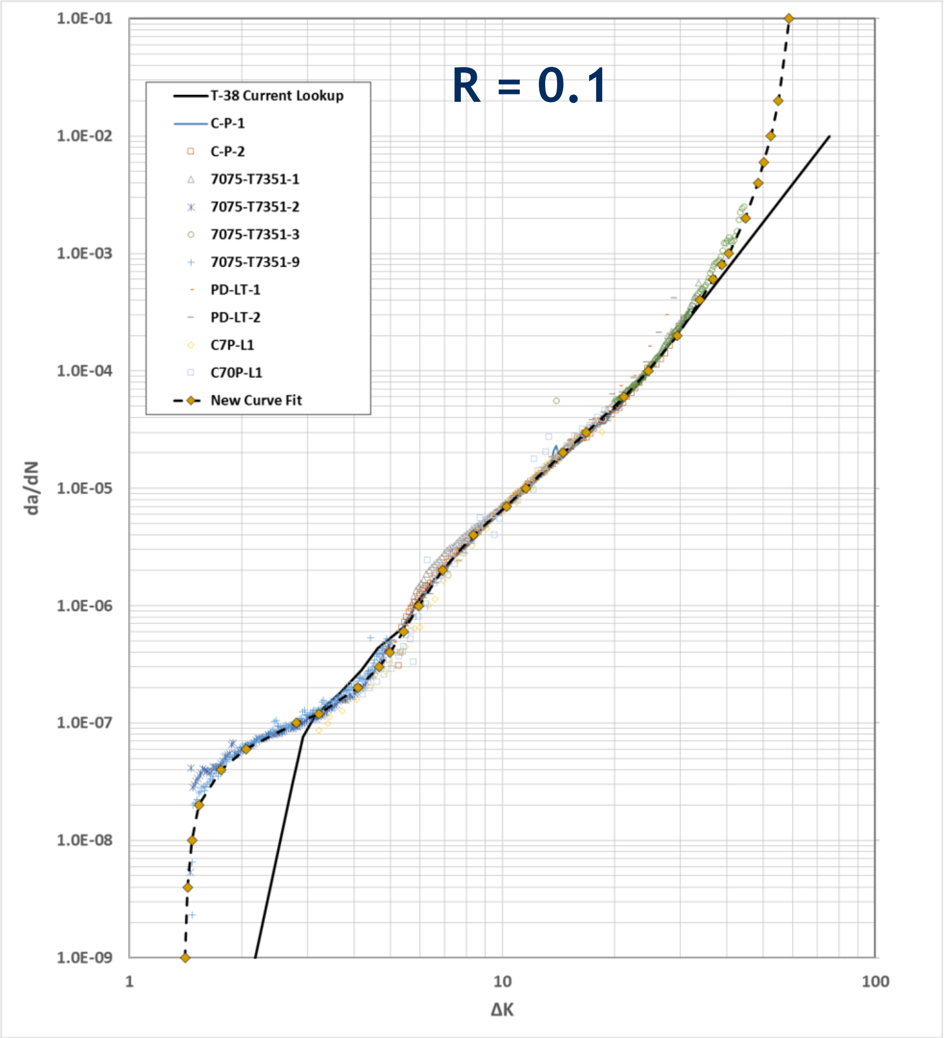
Material Fitting Process



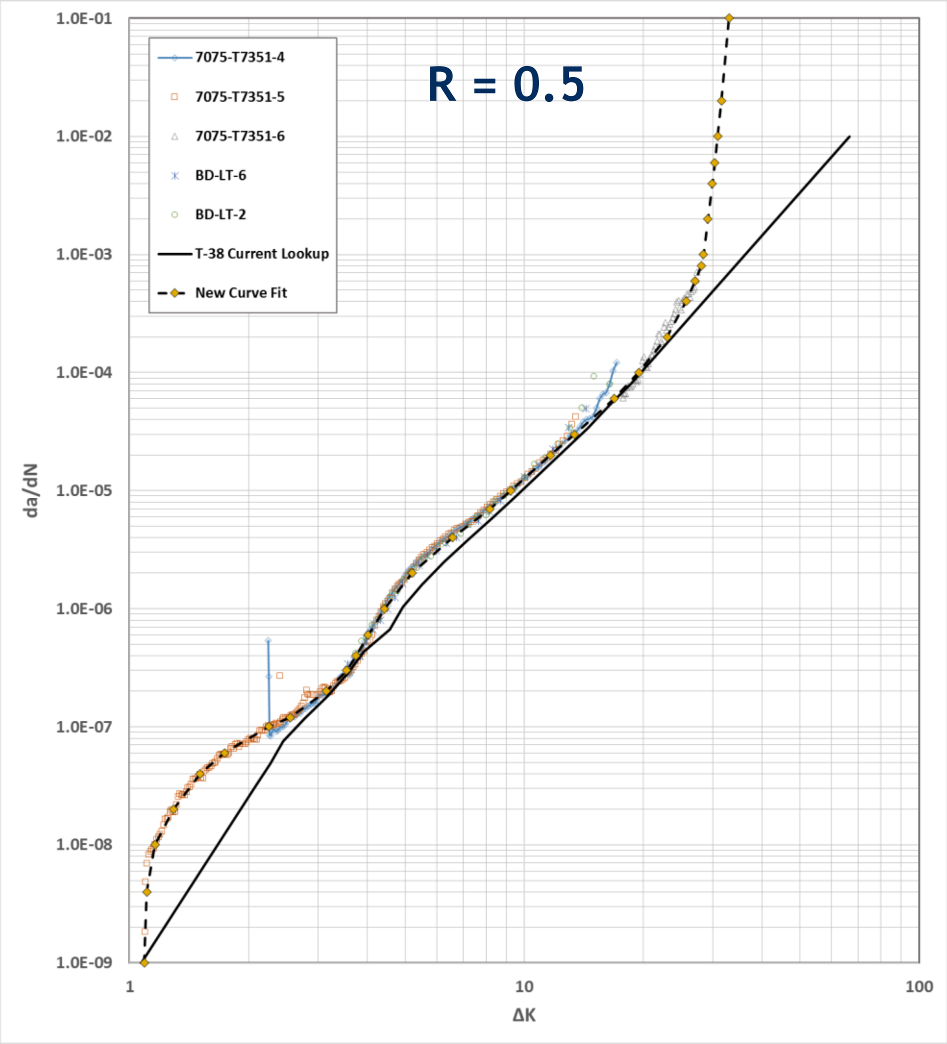


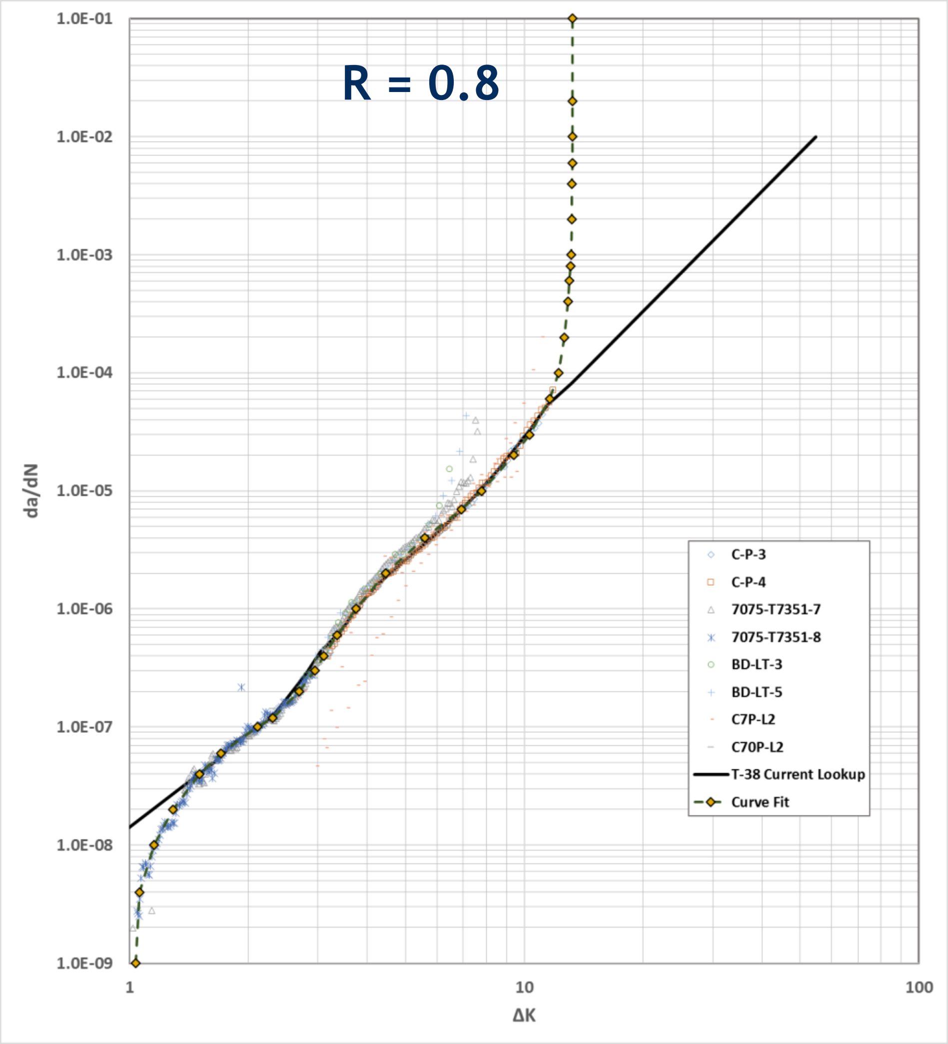
7075-T7351 Material Fit



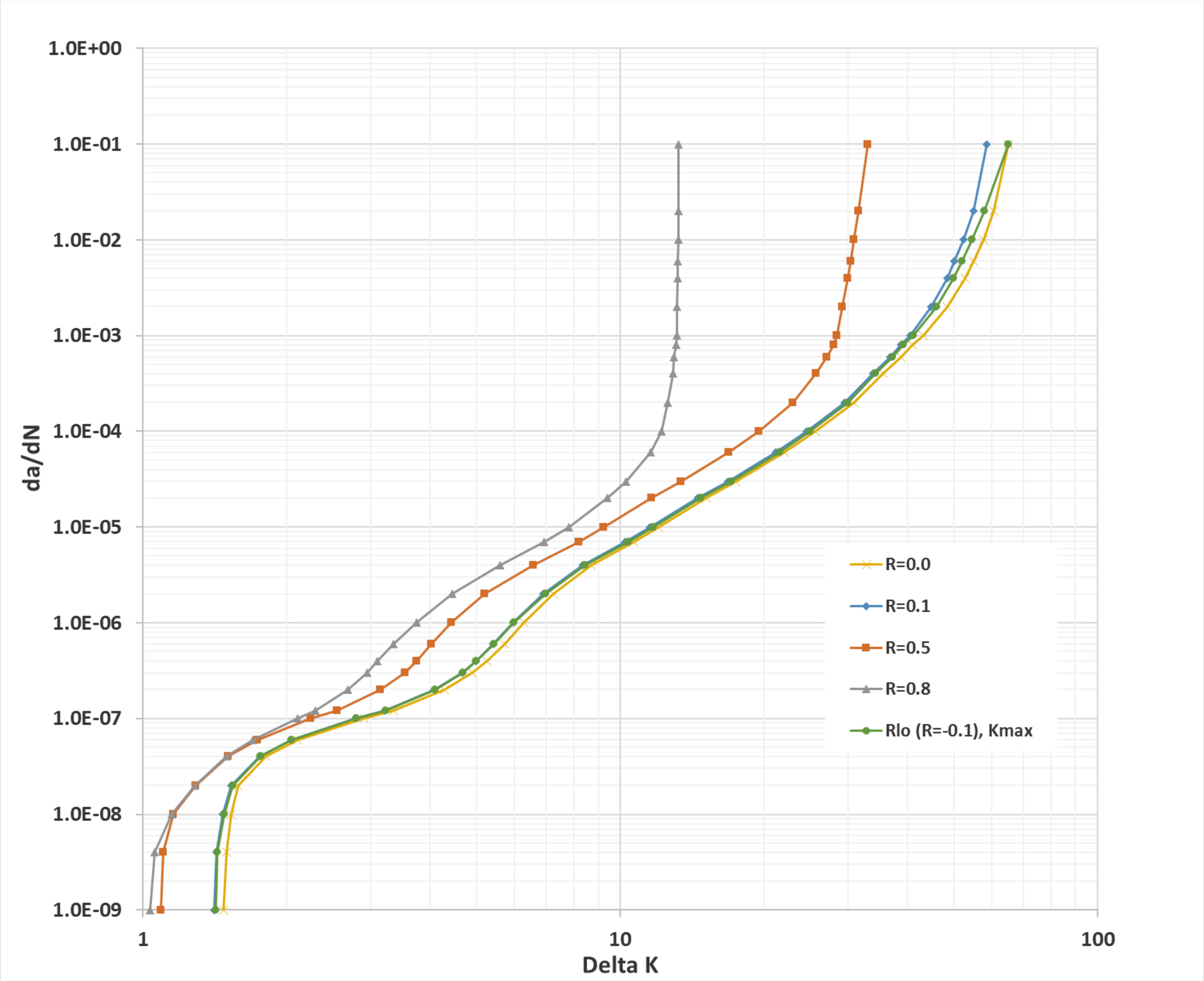


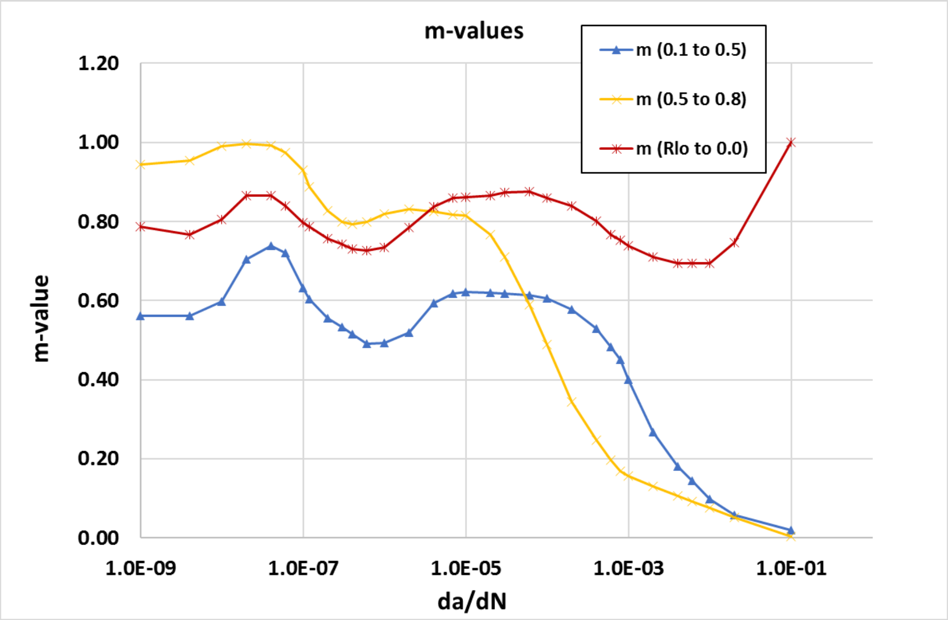
7075-T7351 Material Fit



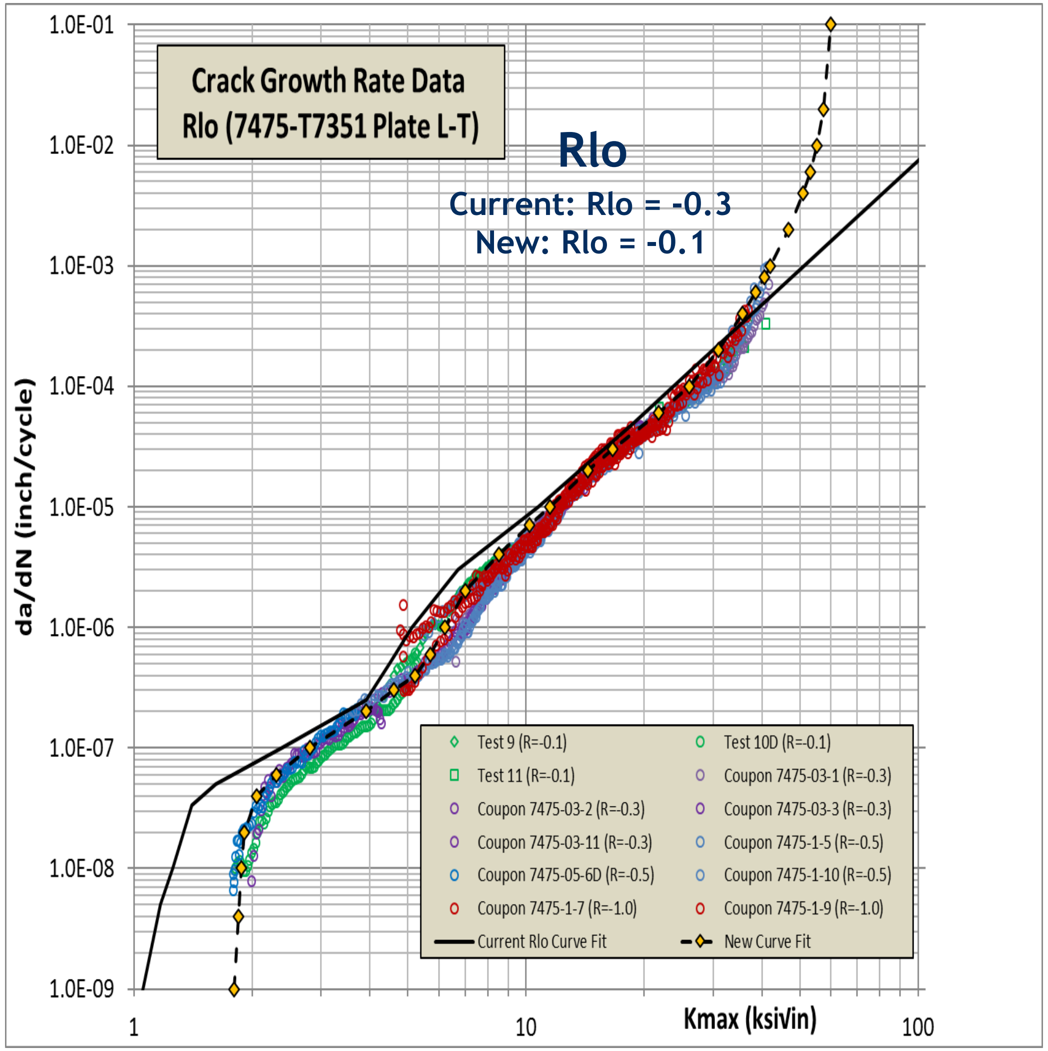


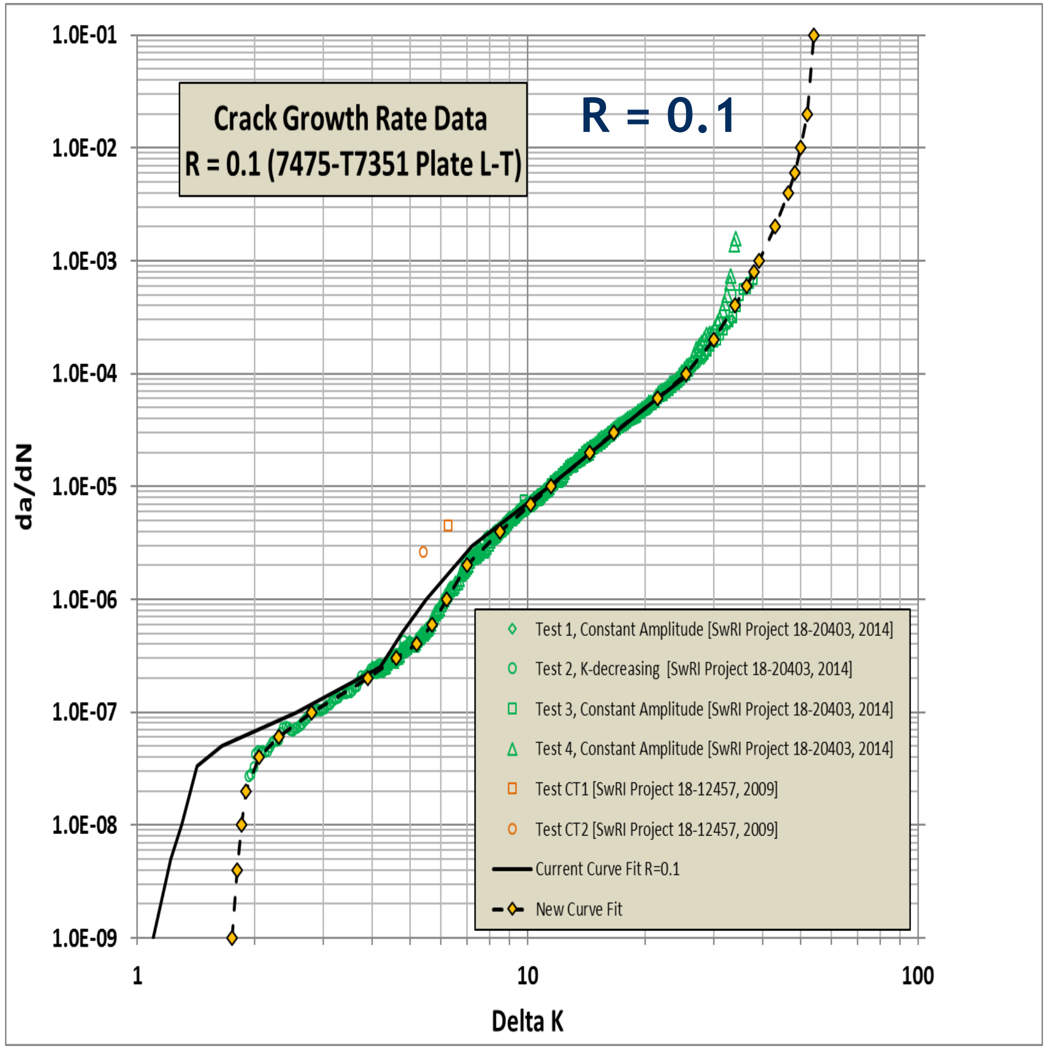
7075-T7351 Material Fit



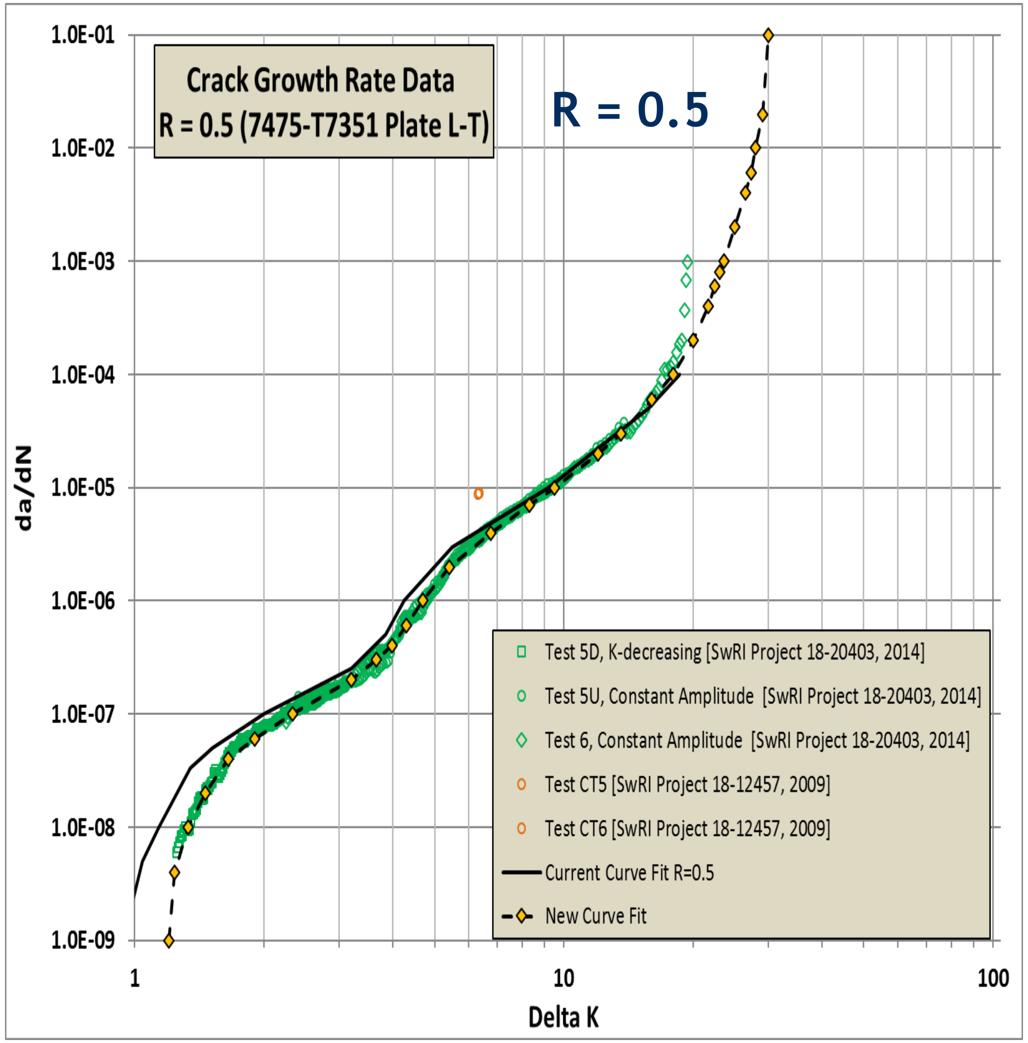


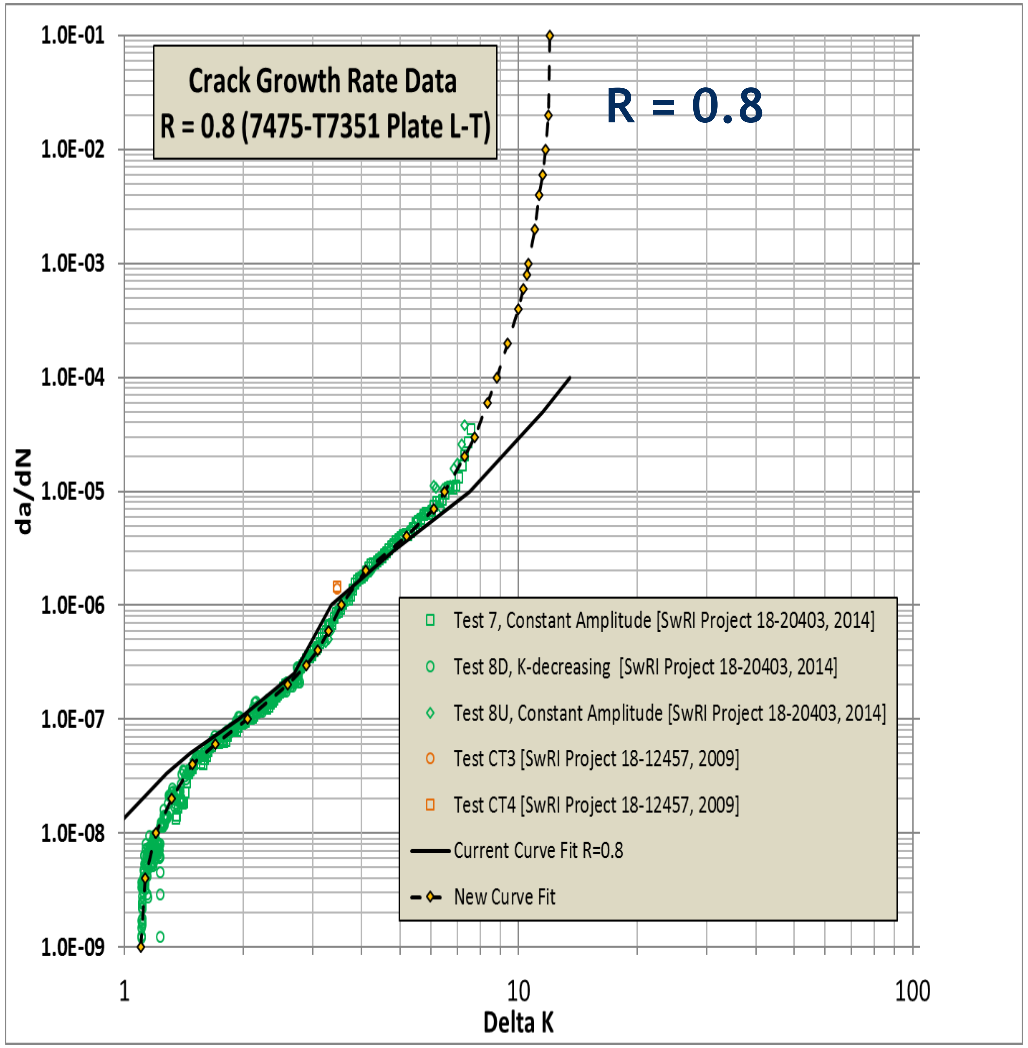
7475-T7351 Material Fit



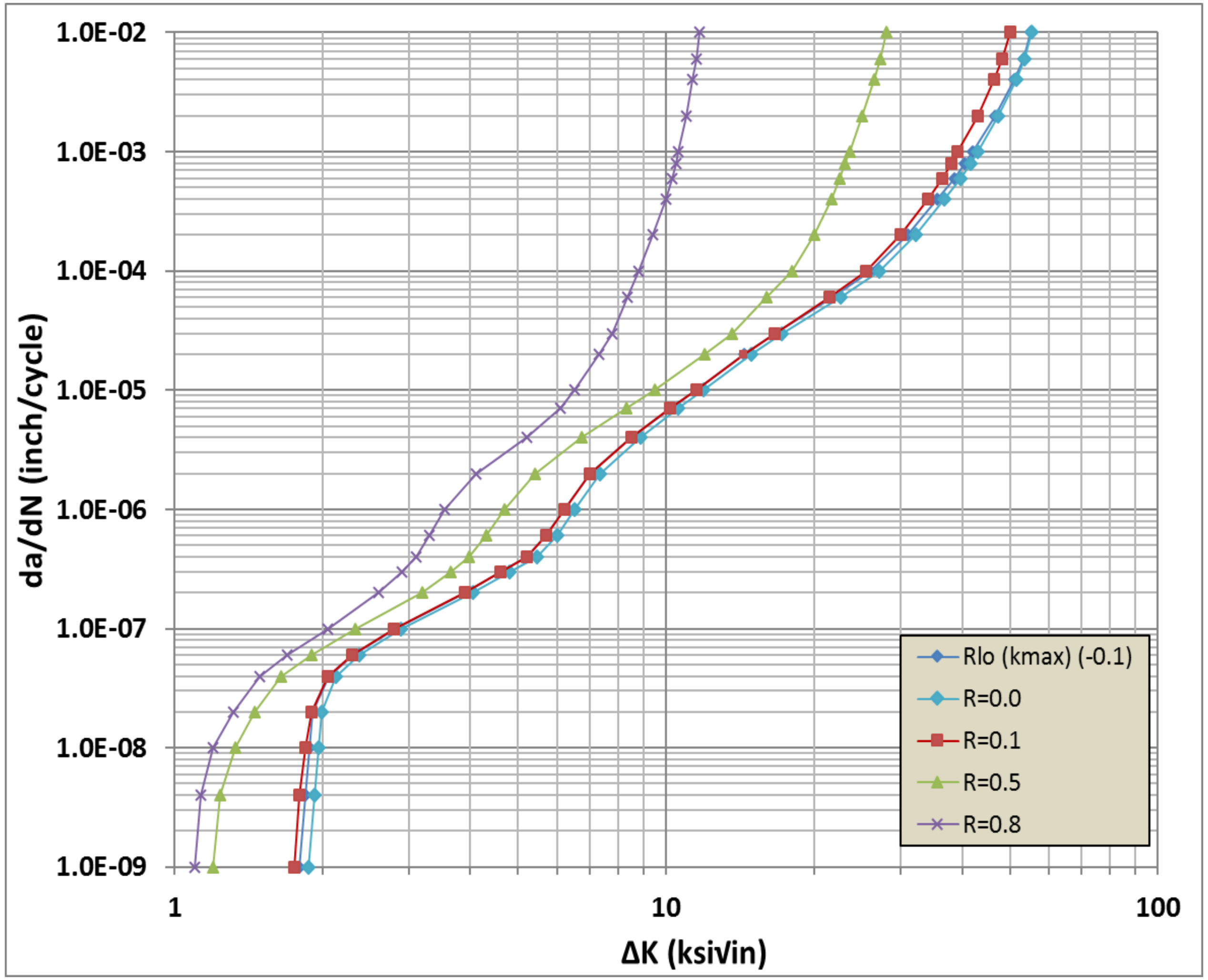


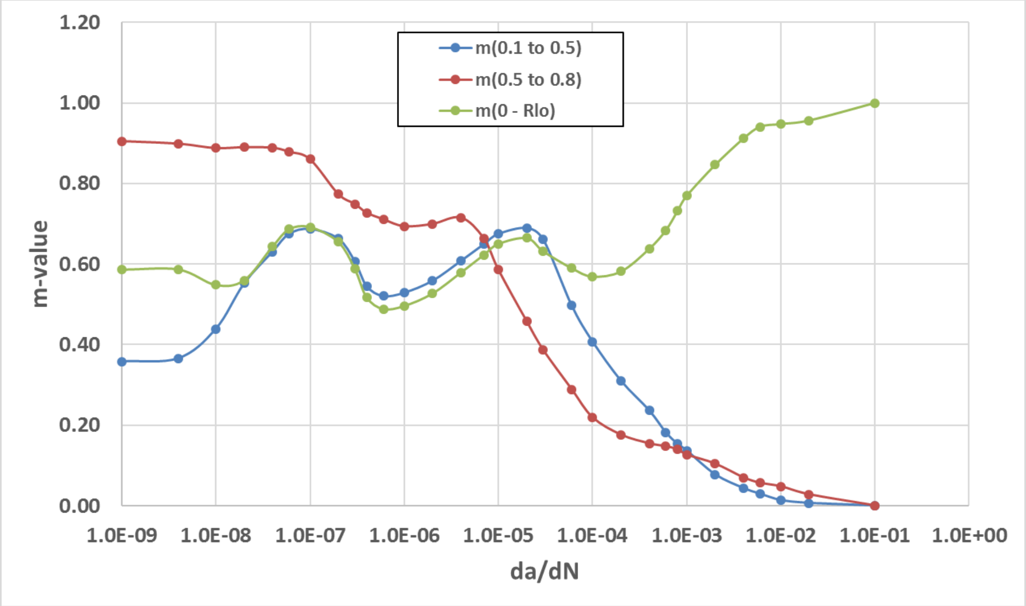
7475-T7351 Material Fit





7475-T7351 Material Fit





7075-T7351, Overall Summary

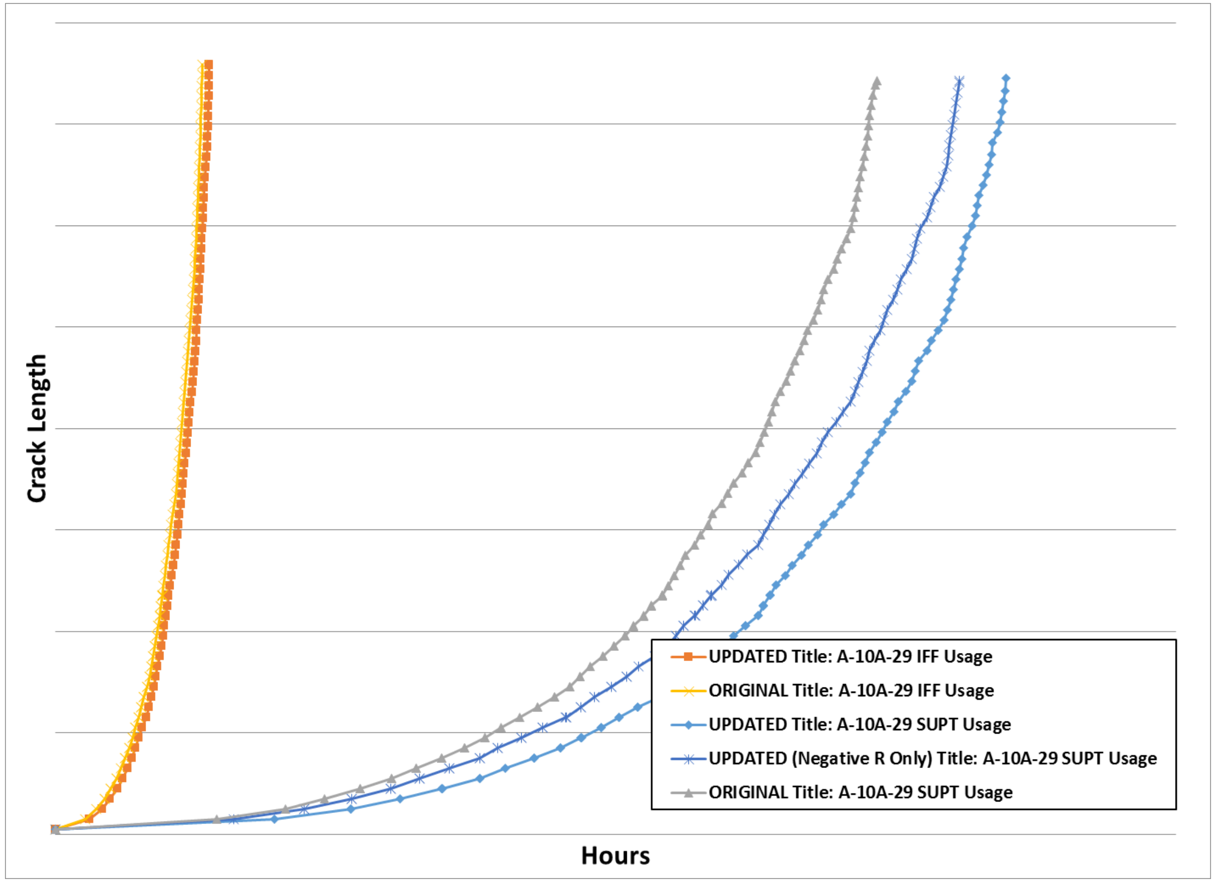
FCL	Material	Specimen ID	Usage	SOLR CORRELATION DETAILS						CRACK GROWTH LIFE RESULTS	
				ORIGINAL ANALYSIS		UPDATED MATERIAL FIT + -R DATA		NEGATIVE R DATA ONLY		UPDATED MATERIAL + -R	-R ONLY
				Correlated SOLR	SOLR for Analysis	Correlated SOLR	SOLR for Analysis	Correlated SOLR	SOLR for Analysis		
A-10a-29	7075-T7351 plate	P-A10A-1	IFF	2.50	2.5	2.65	2.65	N/A	N/A	105%	N/A
		P-A10A-2		1.85		1.95					
		P-A10A-3		1.83		1.91					
		P-A10A-4	SUPT	2.23	2.3	2.36	2.36	2.3	2.30	116%	110%
		P-A10A-5		2.12		2.24		2.18			
		P-A10A-6		2.35		2.47		2.42			
A-15-29	7075-T7351 plate	P-A15-1	IFF	2.4	2.35	2.6	2.56	N/A	N/A	100%	N/A
		P-A15-2		2.2		2.35					
		P-A15-3		2.35		2.56					
		P-A15-4	SUPT	2.4	2.6	2.65	2.91	2.47	2.68	95%	106%
		P-A15-5		2.3		2.47		2.36			
		P-A15-6		2.6		2.91		2.68			

7475-T7351, Overall Summary

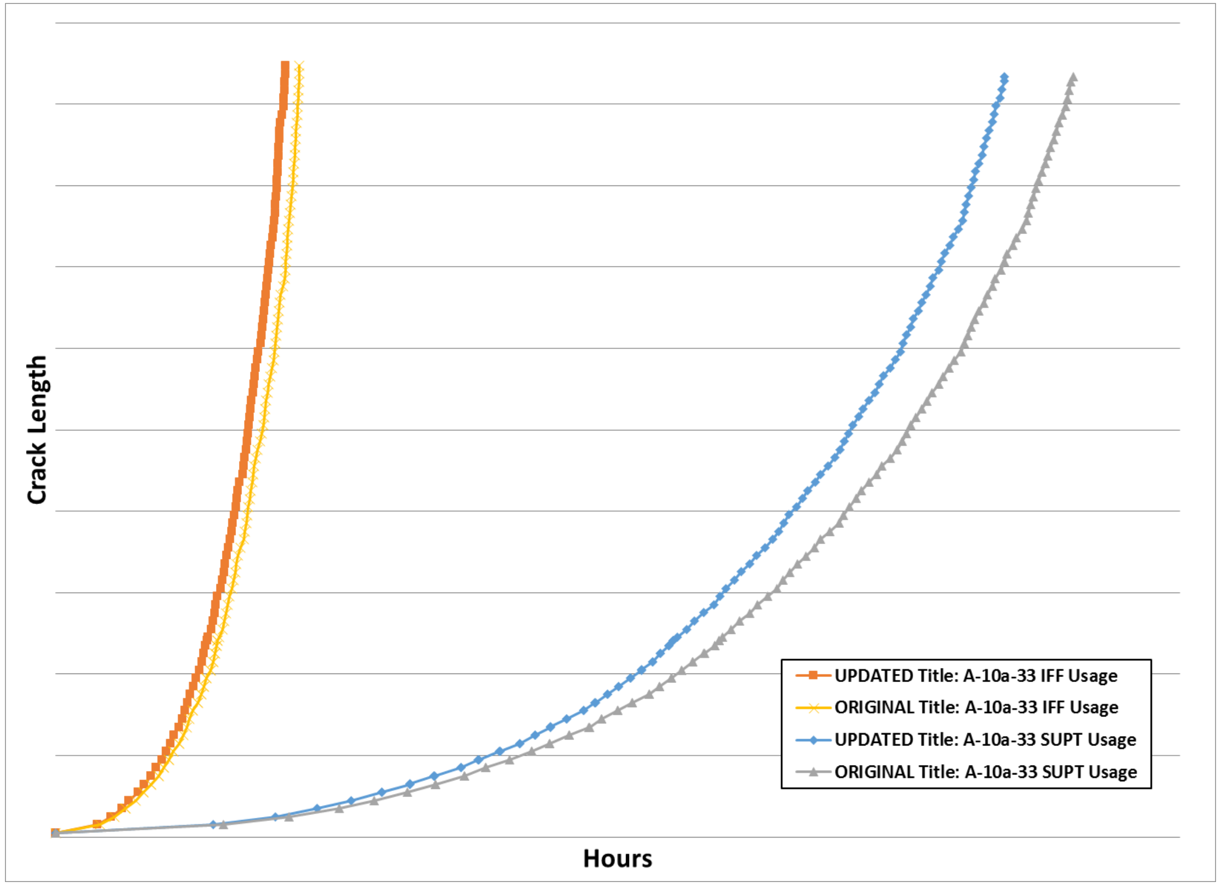
FCL	Material	Specimen ID	Usage	SOLR CORRELATION DETAILS				CRACK GROWTH LIFE RESULTS
				ORIGINAL ANALYSIS		UPDATED MATERIAL FIT + -R DATA		UPDATED MATERIAL + -R
				Correlated SOLR	SOLR for Analysis	Correlated SOLR	SOLR for Analysis	
A-10a-33	7475-T7351 plate	A10a-7475-1	IFF	1.94	1.94	2.01	1.99	94%
		A10a-7475-2		1.91		1.99		
		A10a-7475-3		1.93		1.99		
		A10a-7475-4	SUPT	2.14	2.19	2.21	2.24	93%
		A10a-7475-5		2.12		2.18		
		A10a-7475-6		2.19		2.24		
		A10a-7475-7		2.16		2.22		
A-15-33	7475-T7351 plate	A15-7475-1	IFF	1.73	1.79	1.76	1.83	94%
		A15-7475-2		1.76		1.79		
		A15-7475-3		1.79		1.83		
		A15-7475-4	SUPT	2.03	2.03	2.08	2.08	94%
		A15-7475-5		1.99		2.04		
		A15-7475-6		2.03		2.07		

A-10a Crack Growth Plot

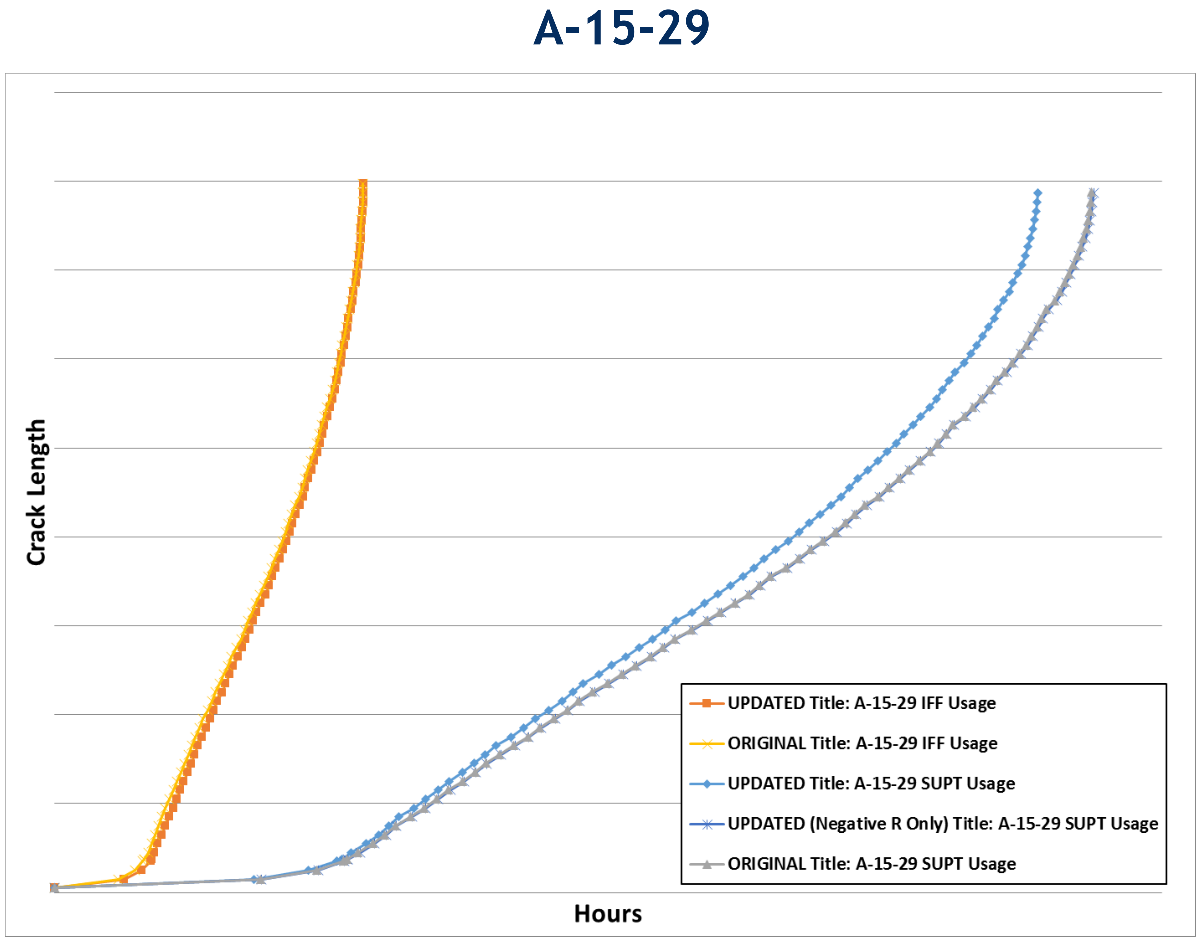
A-10a-29

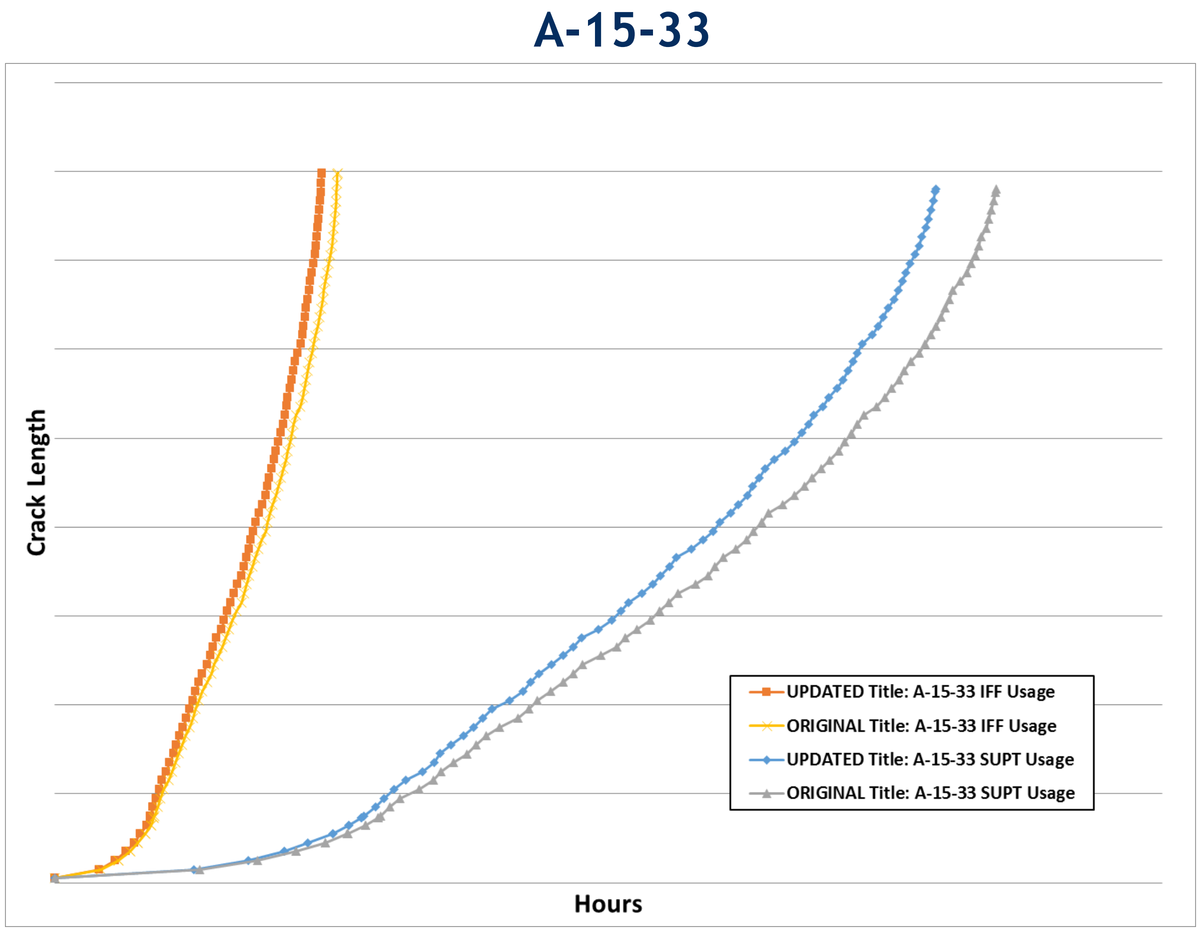


A-10a-33



A-15 Crack Growth Plot





Summary

- New FCGR data developed to fill in gaps
- New fitting process incorporated
- Significant improvement in “fits” relative to test data
- Extrapolated negative R from original fits reasonable, however, Rlo significantly different for new fits (-0.3 → -0.1)
- Moderate damage tolerance and inspection interval results for FCLs updated

Greater accuracy of tabular fits with the inclusion of new negative R data mitigates “translation errors” across range of crack growth rates

Tradeoff

Across the board updates drive downstream changes to SOLR correlations, DTAs, and inspection intervals

Questions?

Robert Pilarczyk
Principal Engineer, Hill Engineering, LLC
rtpilarczyk@hill-engineering.com
Cell: 801-391-2682