

**FASTRAN**  
**AN ADVANCED NON-LINEAR**  
**CRACK-CLOSURE BASED LIFE-PREDICTION**  
**CODE**

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**Mississippi State University**

***AFGROW WORKSHOP***  
***Layton, Utah***  
***September 15, 2015***

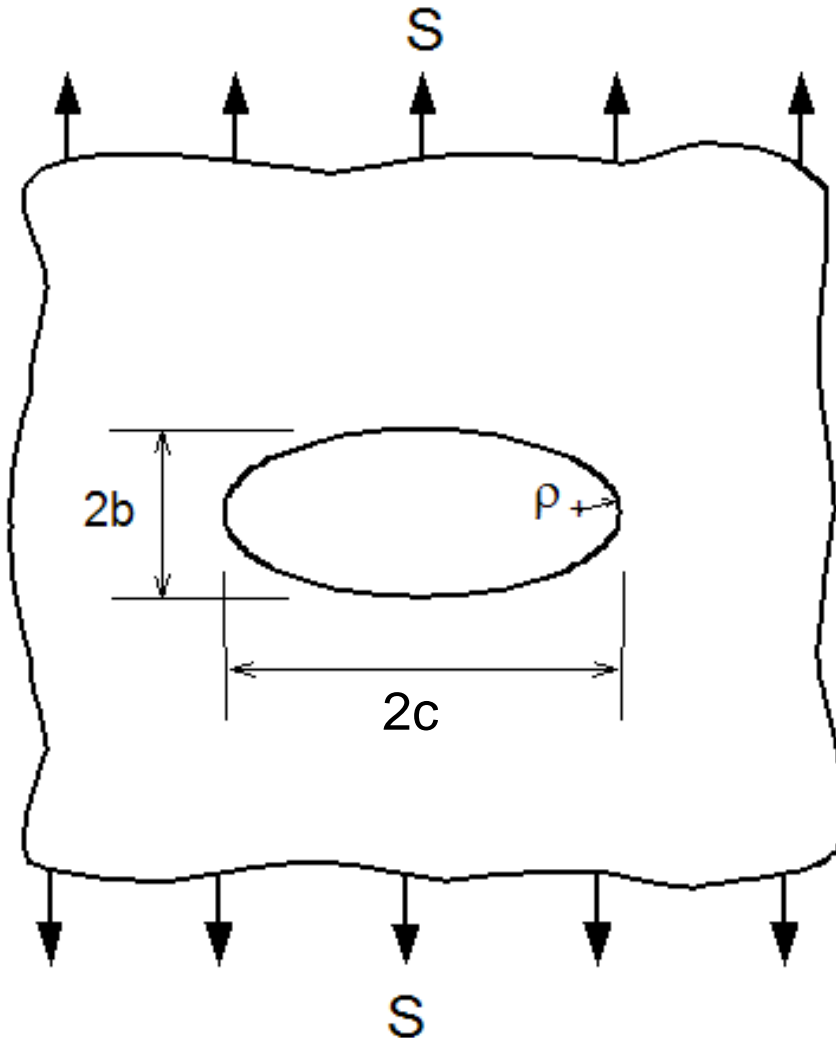
# OUTLINE OF PRESENTATION

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- **Brief History on Fatigue-Crack Growth**
- **Plasticity-Induced Crack-Closure Model**
- **Crack Initiation and Small-Crack Behavior**
- **Fatigue-Crack Growth and Fracture**
- **Concluding Remarks**

# Stress Concentration Factor for an Elliptical Hole in an Infinite Plate

Inglis (1913)



$$K_T = 1 + 2\sqrt{c/\rho}$$

$$\sigma_e = S K_T$$

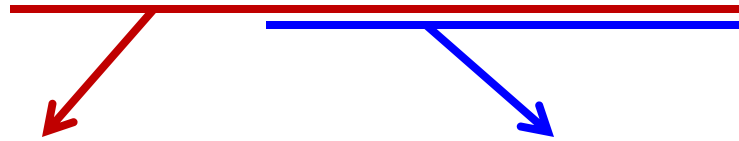
$$\sigma_e = S + 2S\sqrt{c/\rho}$$

# Notch Strength Analysis – Fracture Mechanics

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$$K_T = 1 + 2 \sqrt{c / \rho}$$

$$\sigma_e = S + 2 S \sqrt{c / \rho}$$



**Paul Kuhn**

**Notch Strength Analysis  
(Neuber )**

**George Irwin**

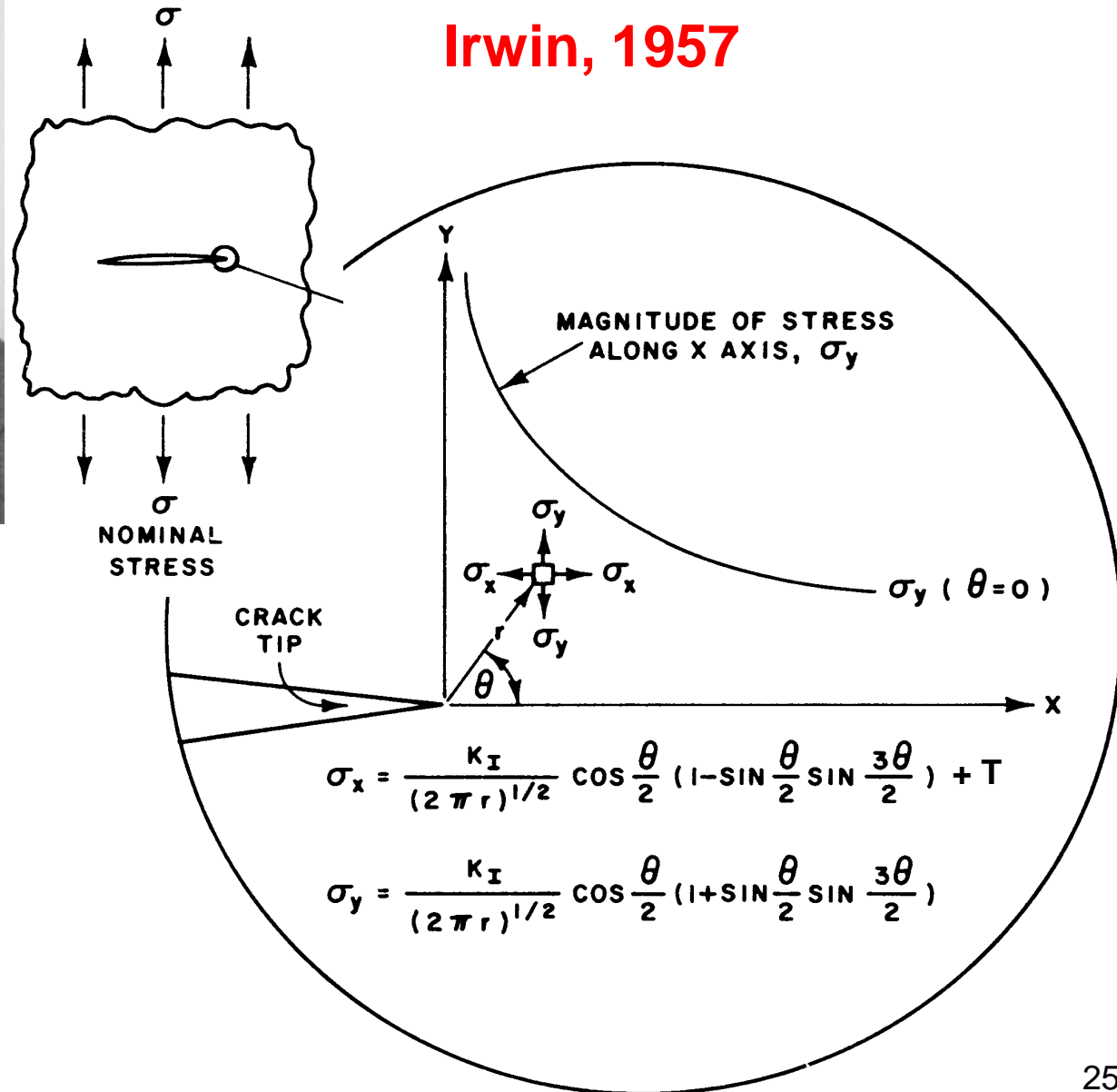
**Fracture Mechanics  
(Griffith)**

# Father of “Modern” Fracture Mechanics

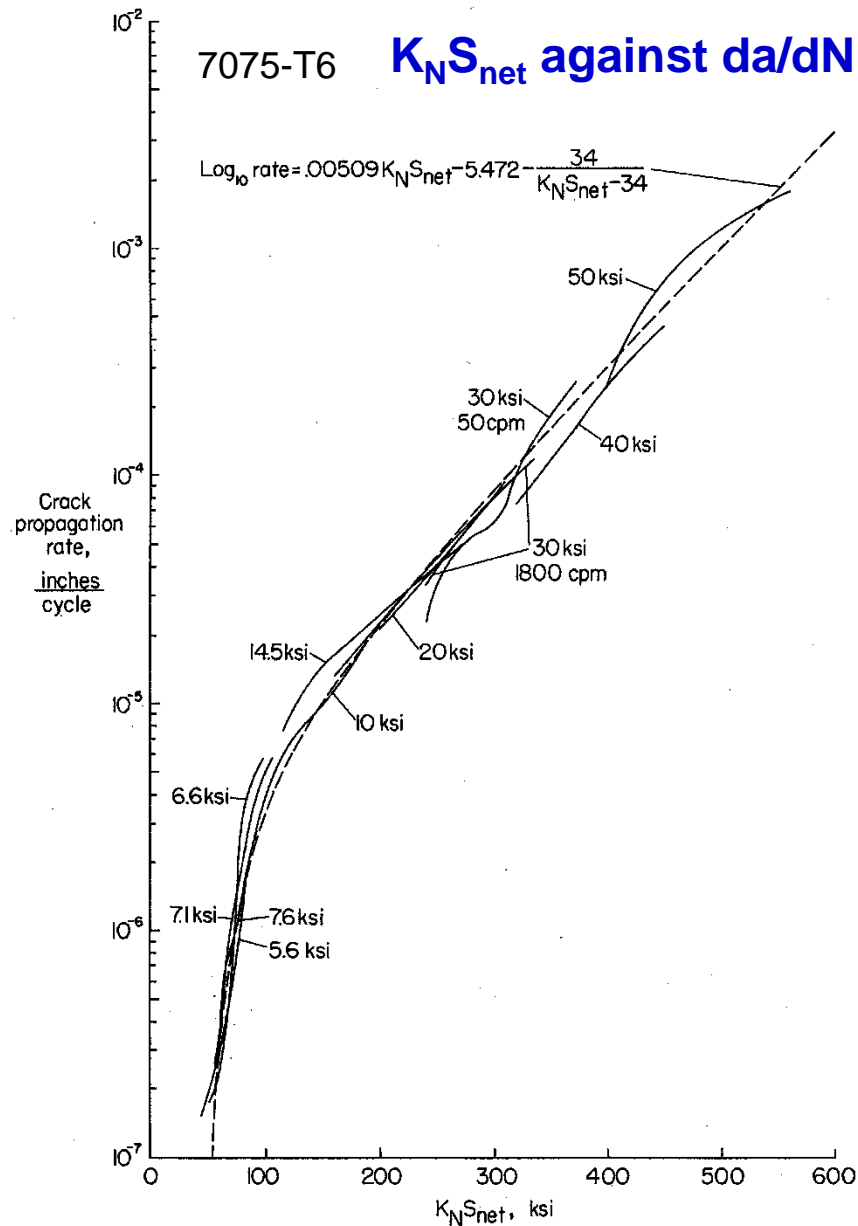


George Rankin Irwin  
(1907-1998)

Irwin, 1957

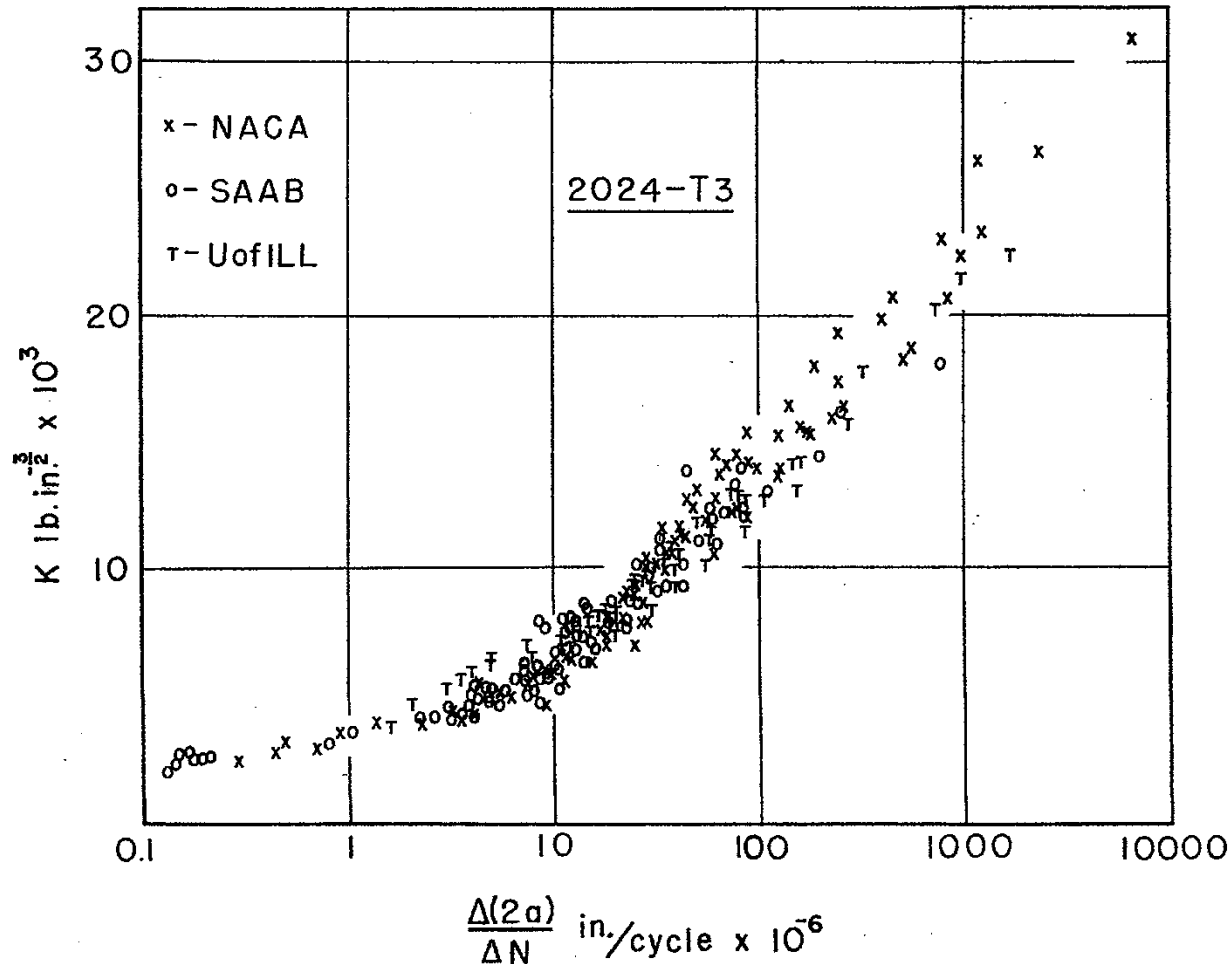


# Notch-Strength Analyses: McEvily and Illg (LaRC), NACA TN-4394, 1958



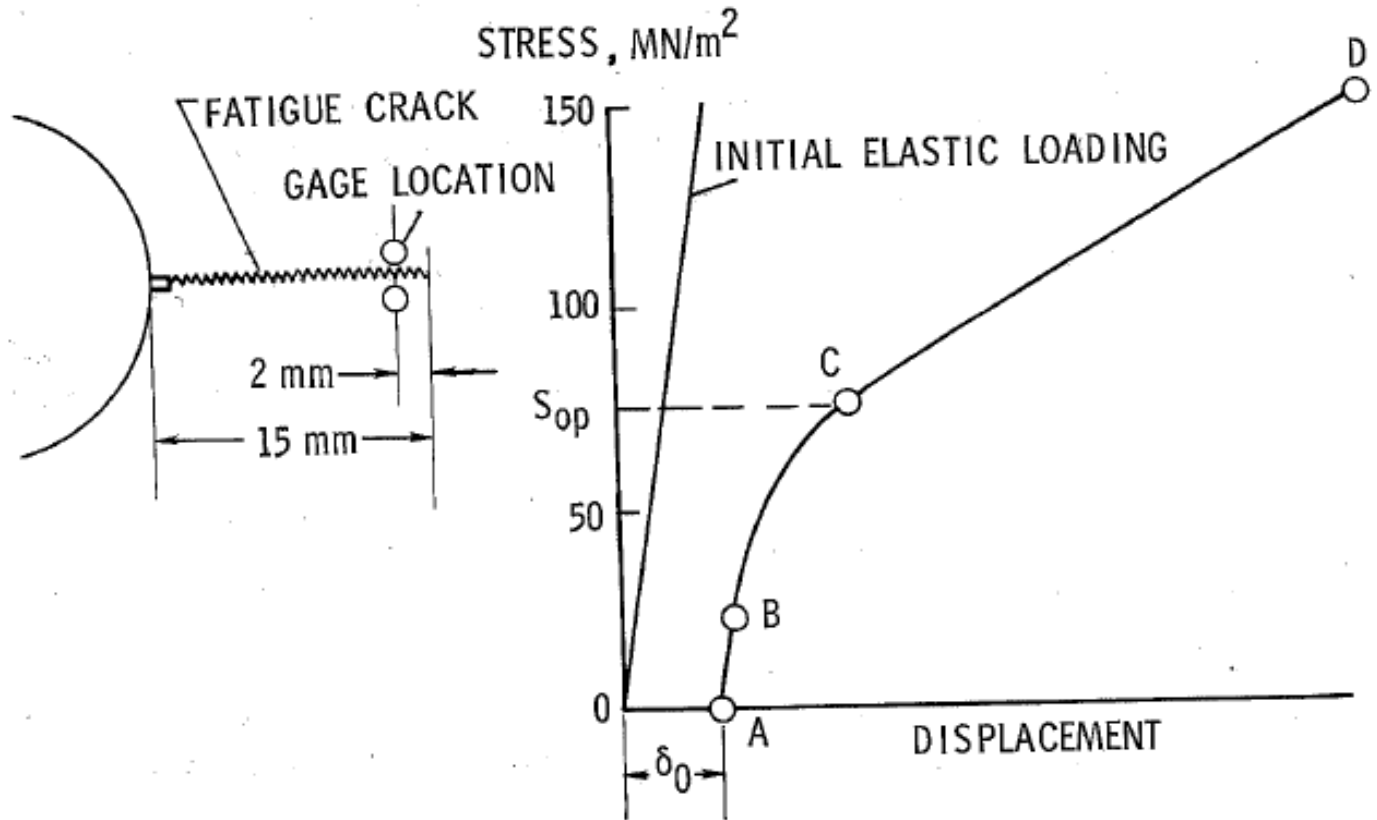
# Fracture Mechanics: Paris, Gomez, and Anderson, Trends in Engineering, Seattle, WA, 1961

## LEFM: K against $d(2a)/dN$



Paris (1970):  
 $K_N S_{net} \sim K_{max}$

# Plasticity-Induced Fatigue-Crack Closure: Elber, 1968



(a) CRACK CONFIGURATION AND GAGE LOCATION

(b) APPLIED STRESS - DISPLACEMENT RELATION



# DOMINANT MECHANISMS OF FATIGUE-CRACK CLOSURE

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Plastic wake



Elber, 1968  
Newman, 1976  
(FASTRAN)

Beevers, 1979  
Suresh & Ritchie, 1982

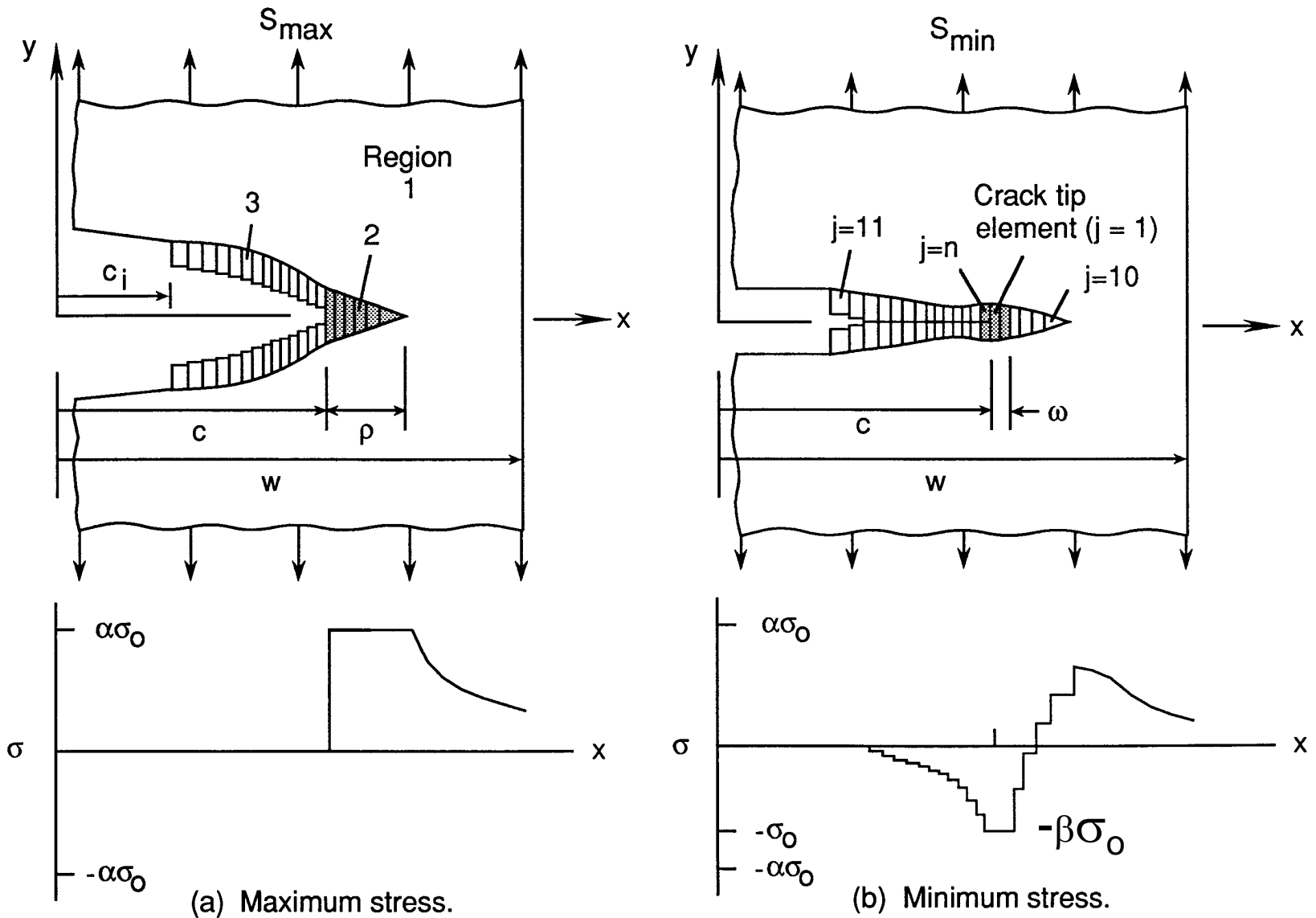
Paris et al., 1972  
Suresh & Ritchie, 1981

# OUTLINE OF PRESENTATION

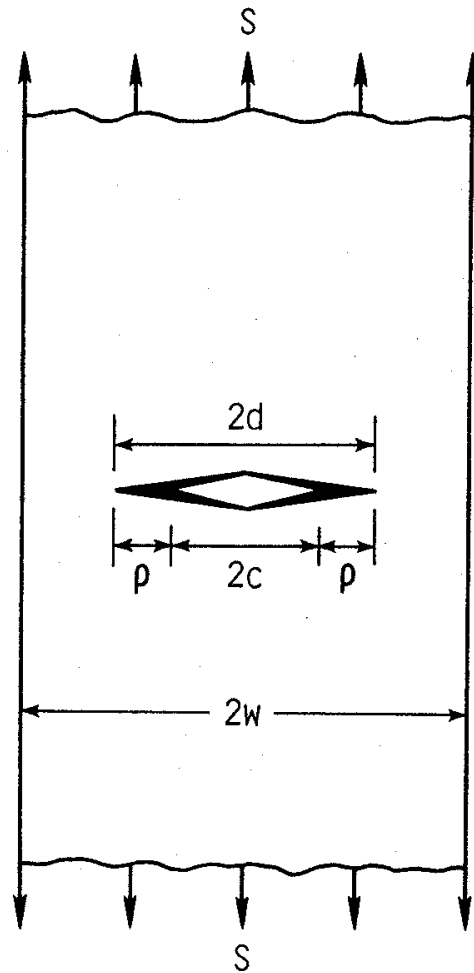
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- Brief History on Fatigue-Crack Growth
- **Plasticity-Induced Crack-Closure Model**
- **Crack Initiation and Small-Crack Behavior**
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# FASTRAN – Crack-Closure Based Life-Prediction Code

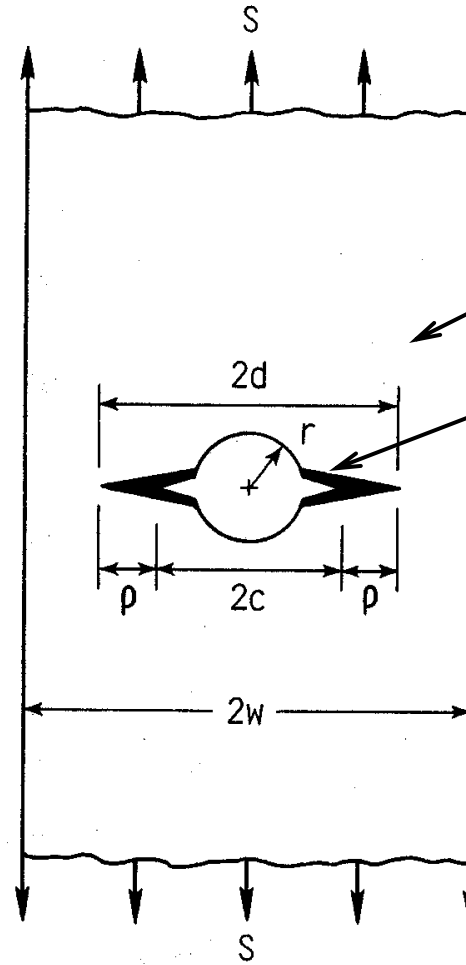


# MODIFIED DUGDALE MODELS IN FASTRAN



(a) Center crack

**NTYP = 1**



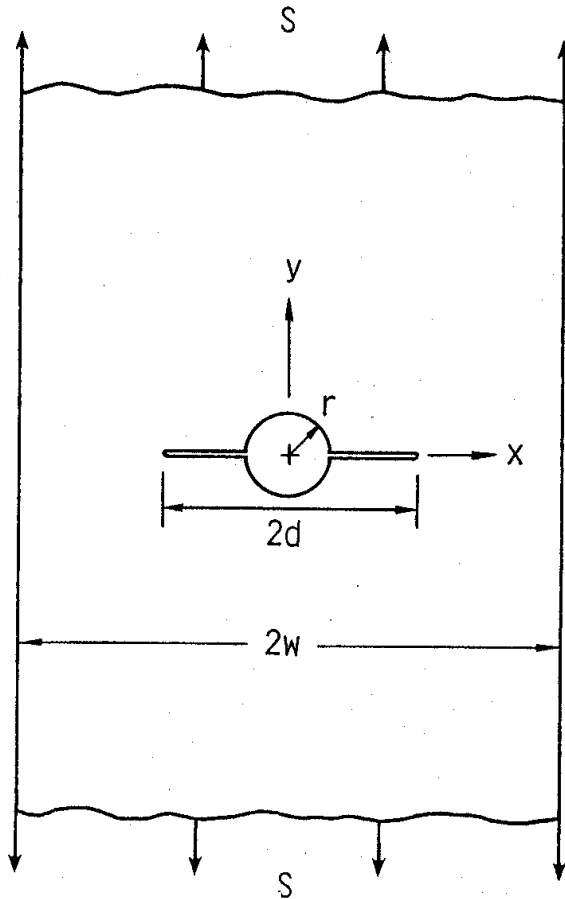
(b) Cracks from hole

**NTYP = -4**

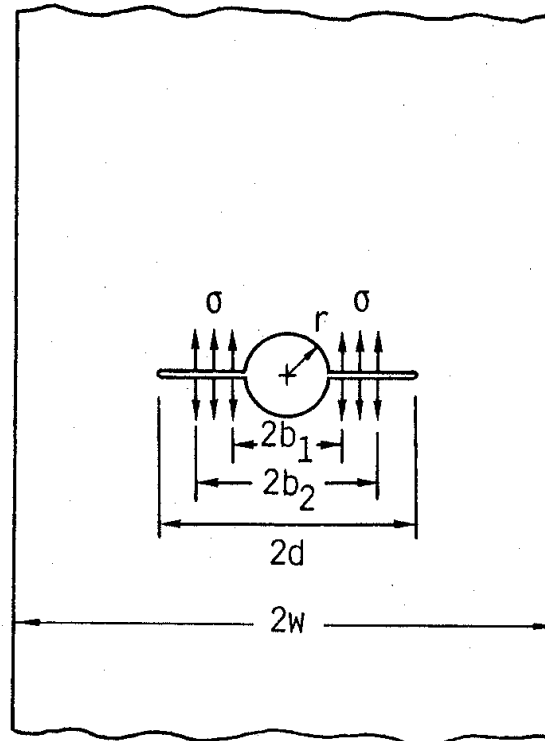
Elastic continuum

Bar elements

# BASIC CRACK SOLUTIONS REQUIRED FOR CLOSURE MODEL



(a) Uniform stress



(b) Partially-loaded crack

Crack solutions:

$$K_s = f_s(S, d, r, w)$$

$$V_s = g_s(S, d, r, w, x)$$

$$K_\sigma = f_\sigma(\sigma, d, r, w, b_i, x)$$

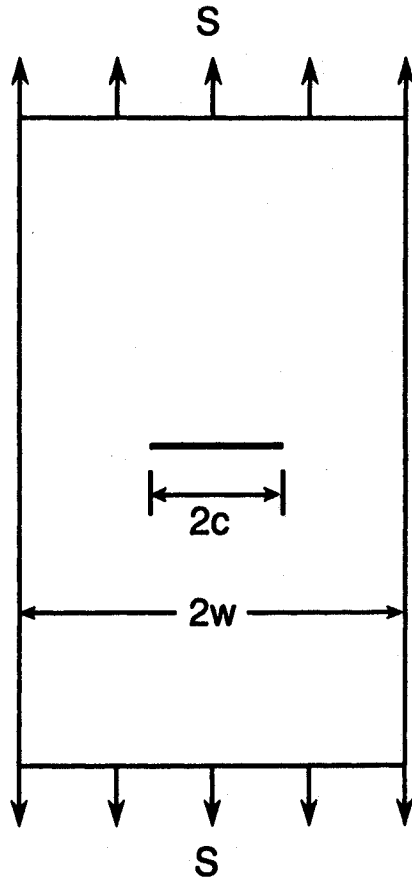
$$V_\sigma = g_\sigma(\sigma, d, r, w, b_i, x)$$

# FASTRAN Version 5.4+

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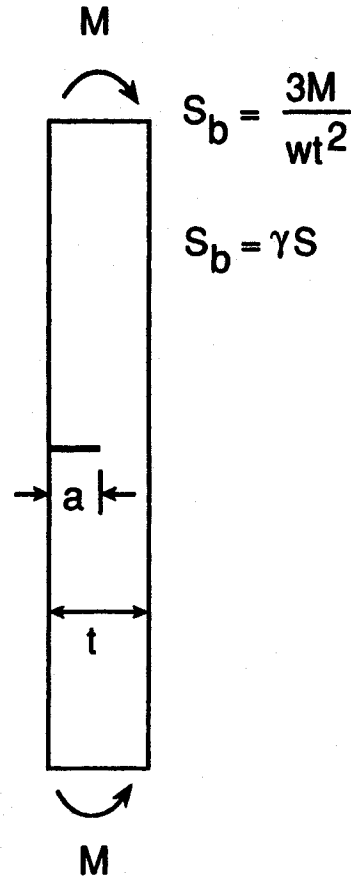
- Plastic-zone region refined (**20 elements** in plastic zone instead of **10 elements**, like STRIPY model in NASGRO)
- Crack-growth increments ( $\Delta c^*$ ) reduced to **5% of cyclic-plastic-zone size** instead of **20%** (only used for  $NMAX > 100$ )
- New **crack-opening-stress function** developed to fit the refined model (slight increase in crack-opening stresses) for steady-state constant-amplitude loading
- **NMAX** input (normally set to 300 to 1000), but enables **cycle-by-cycle** calculations with **NMAX = 1 (recommended)**
- **K-analogy** activated for all 2D and 3D crack configurations
- Residual strength ( $S_n/\sigma_u$ ) output as function of crack length
- Spectrum loading ( $NFOPT = 8, 9$  and  $10$ ) output in cycles and blocks or flights for NREP (repetitions of flight schedule)

# CRACK SOLUTION INPUT REQUIRED FOR FASTRAN



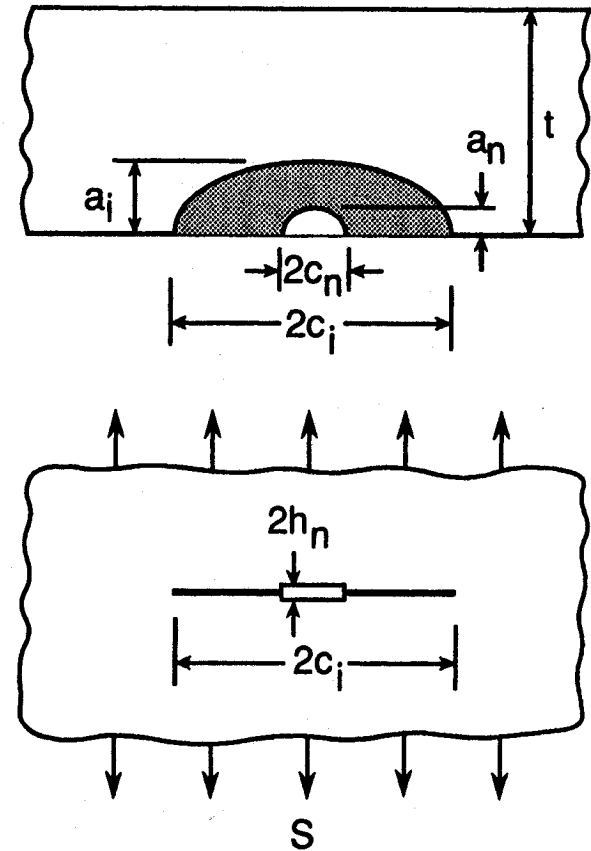
(a) Tension

NTYP = 1



(b) Bending

NTYP = 0; LTYP = 1

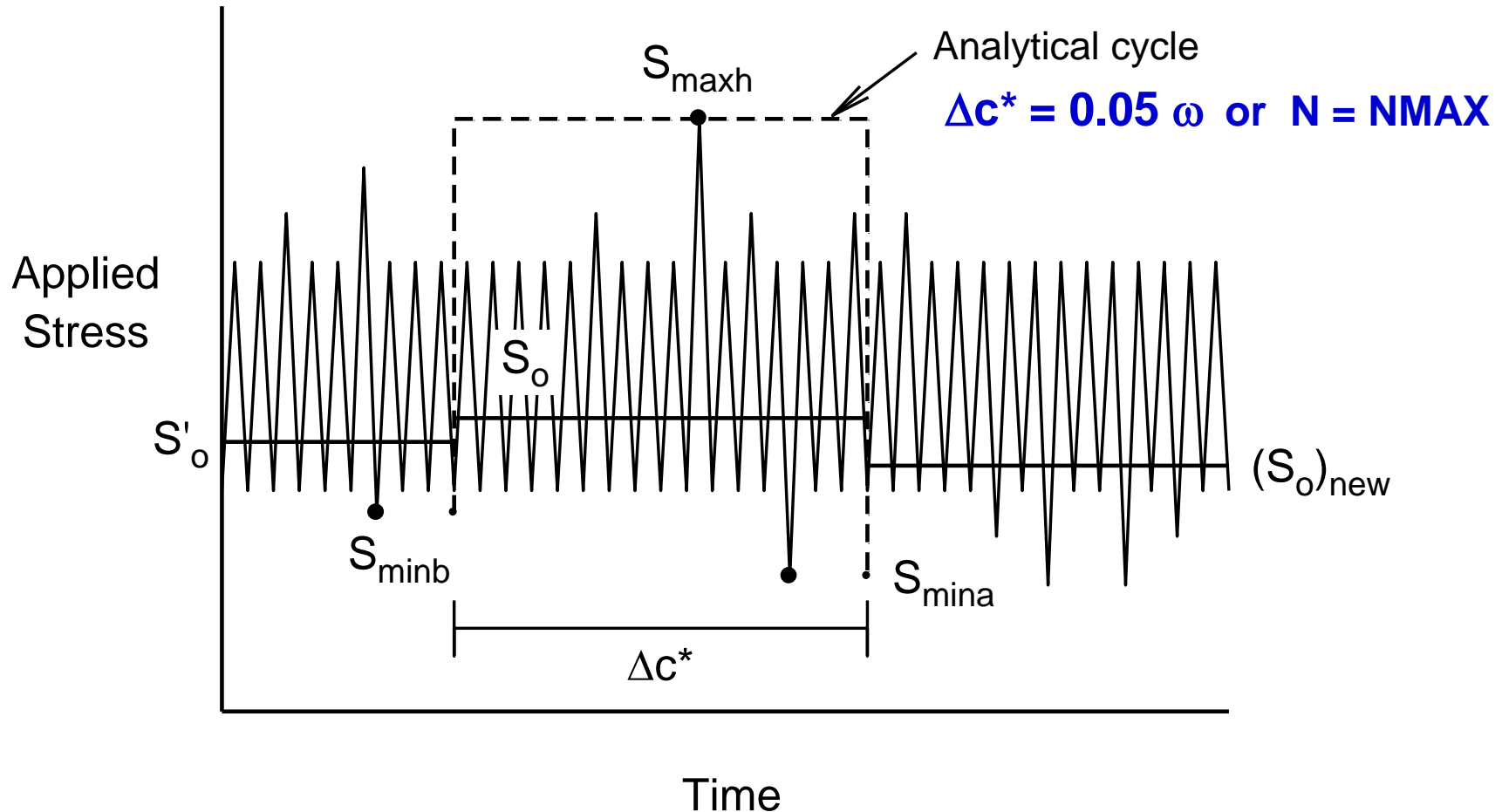


(c) Crack and starter notch

Pre-cracking option

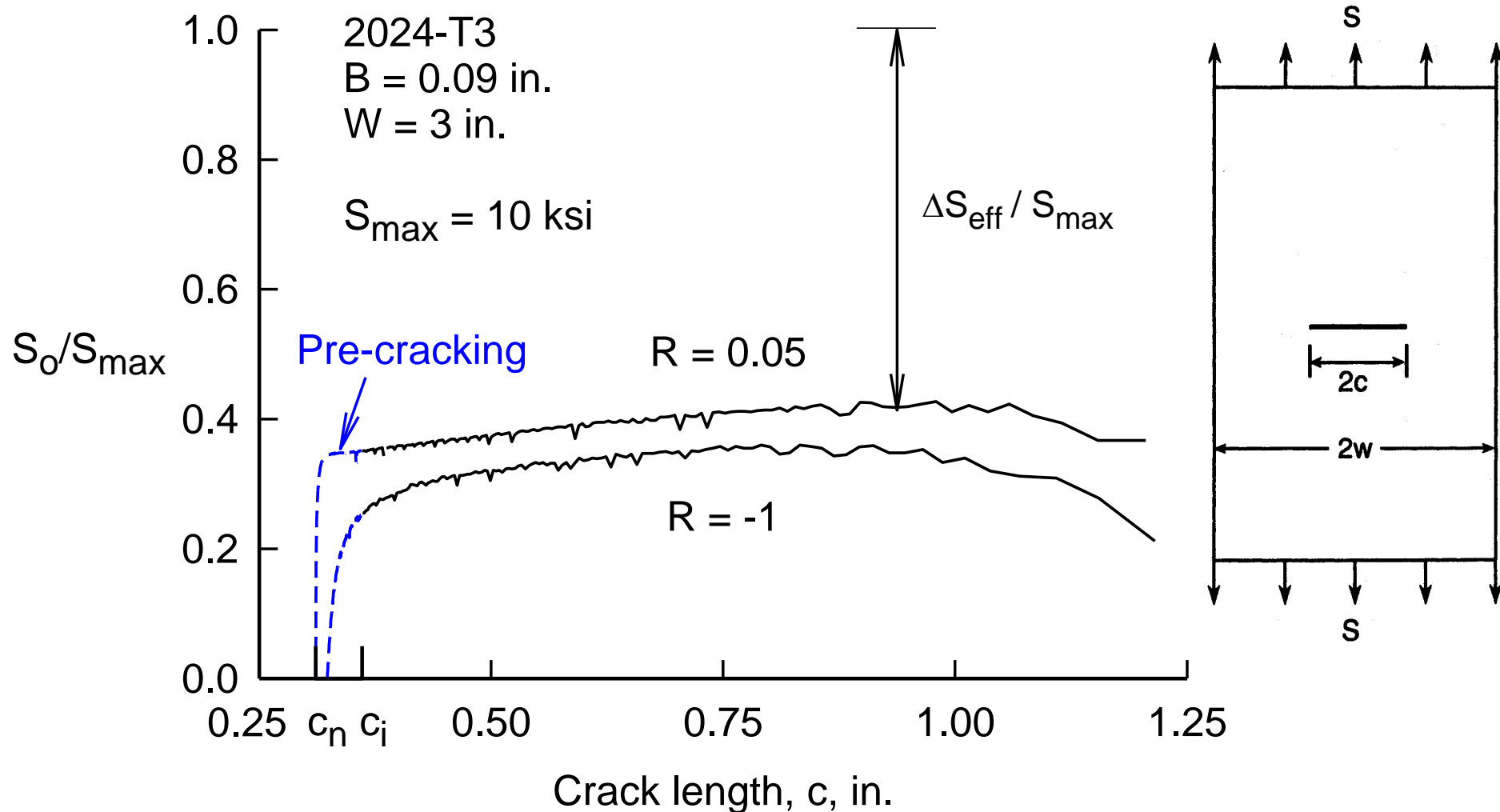
# MECHANICS OF THE ANALYTICAL CYCLE IN FASTRAN

**FASTRAN Version 5.4+ (recommend cycle-by-cycle, NMAX=1)**





# CALCULATED CRACK-OPENING STRESSES AT A LOW APPLIED STRESS LEVEL (MIDDLE-CRACK TENSION; NTYP = 1)



# CRACK-OPENING STRESSES UNDER CONSTANT-AMPLITUDE LOADING

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$$S_o/S_{\max} = f(R, S_{\max}/\sigma_o, \alpha, \Delta c/c)$$

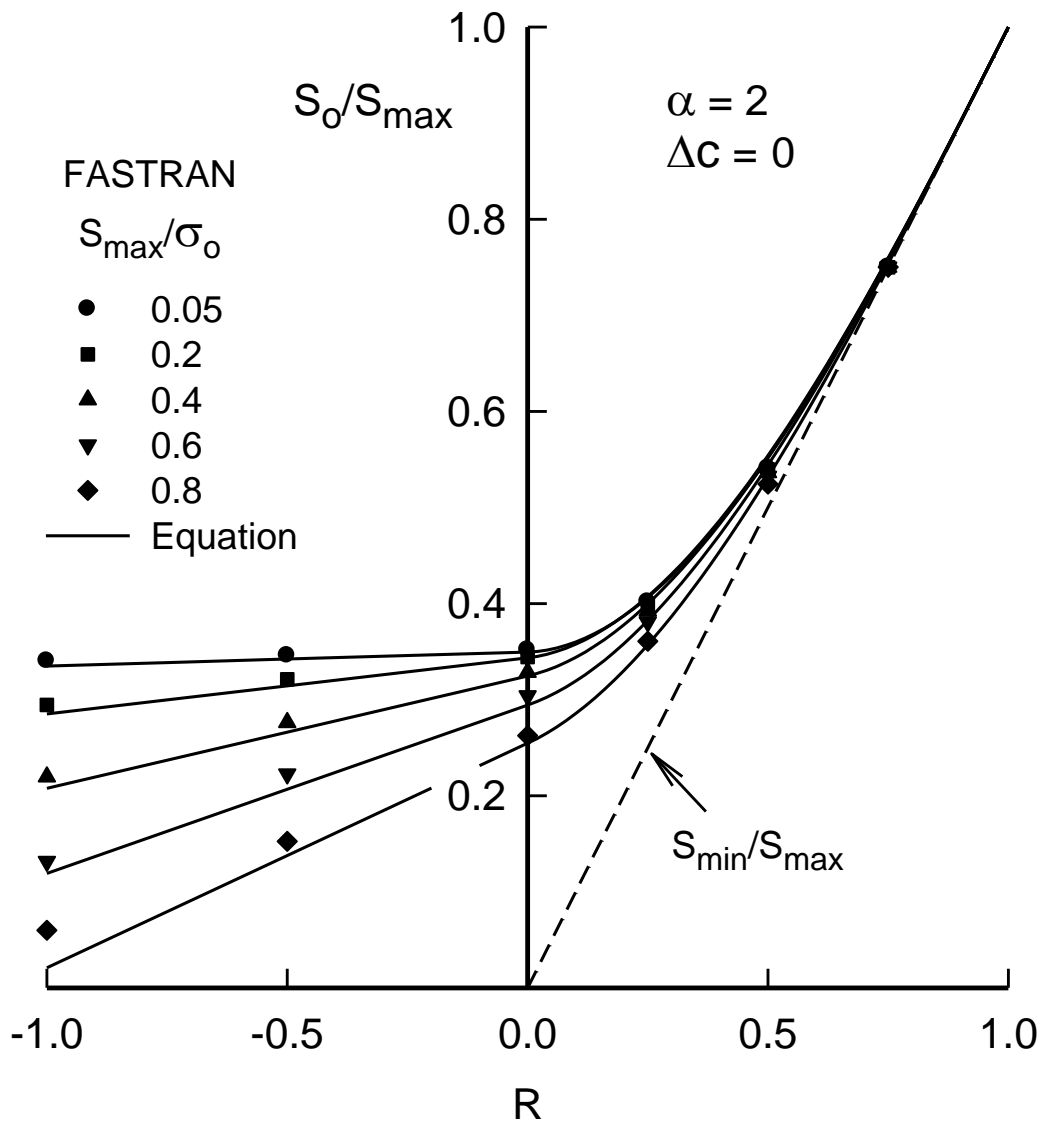
$$R = S_{\min}/S_{\max}$$

$$\sigma_o = (\sigma_{ys} + \sigma_{ult})/2$$

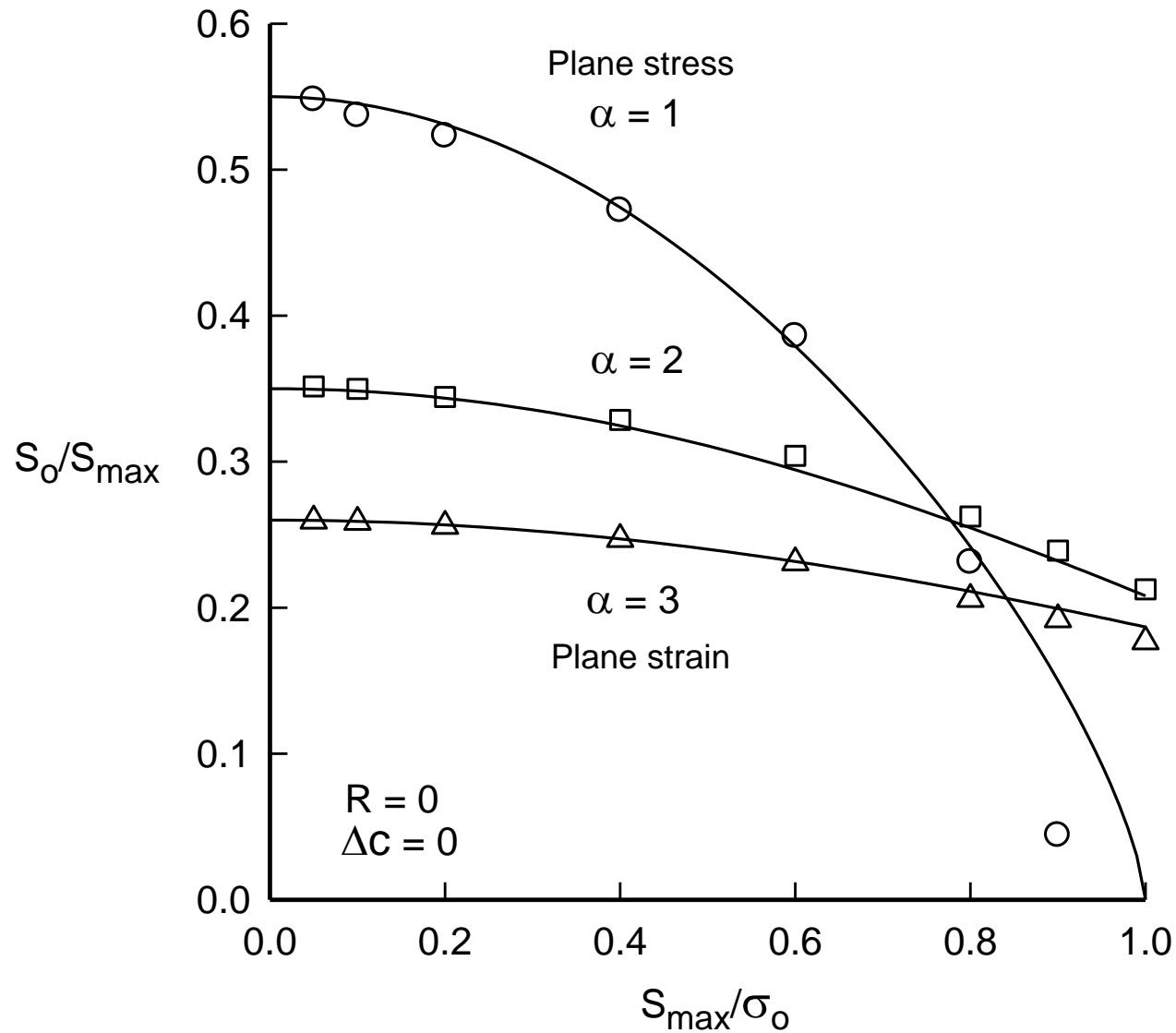
$\alpha = 1$  for plane-stress conditions

$\alpha = 3$  for plane-strain conditions

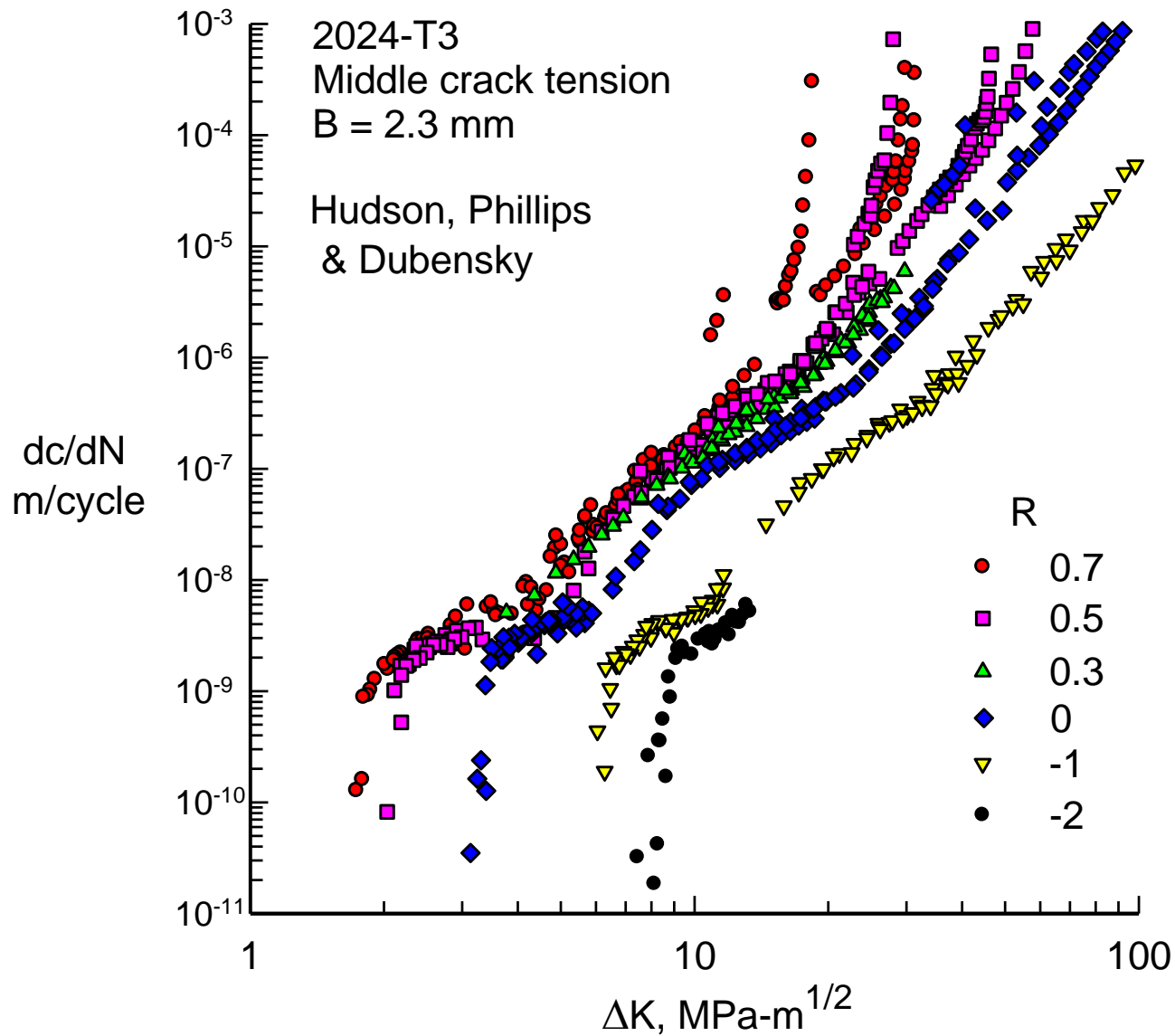
# CRACK-OPENING STRESSES AS A FUNCTION OF STRESS RATIO FOR A HIGH CONSTRAINT FACTOR



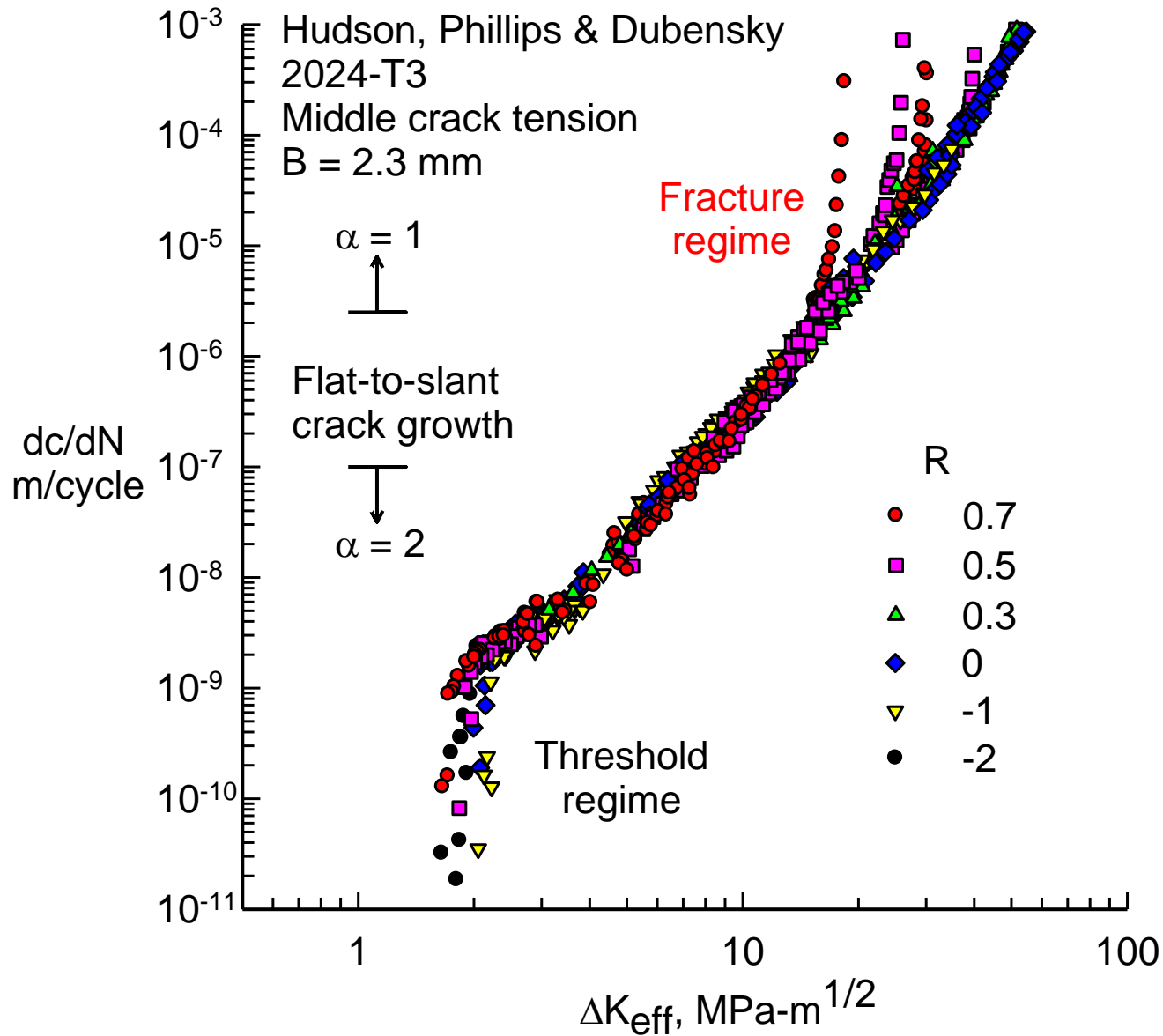
# CRACK-OPENING STRESSES AS A FUNCTION OF APPLIED STRESS FOR VARIOUS CONSTRAINT FACTORS



# FATIGUE-CRACK-GROWTH RATES USING LEFM ANALYSES

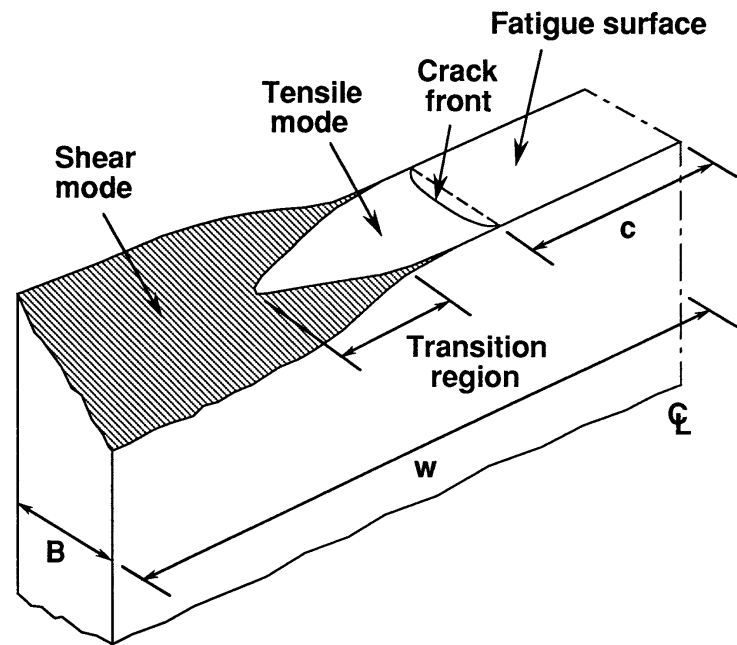


# FATIGUE-CRACK-GROWTH RATES CORRELATION USING CRACK-CLOSURE ANALYSES

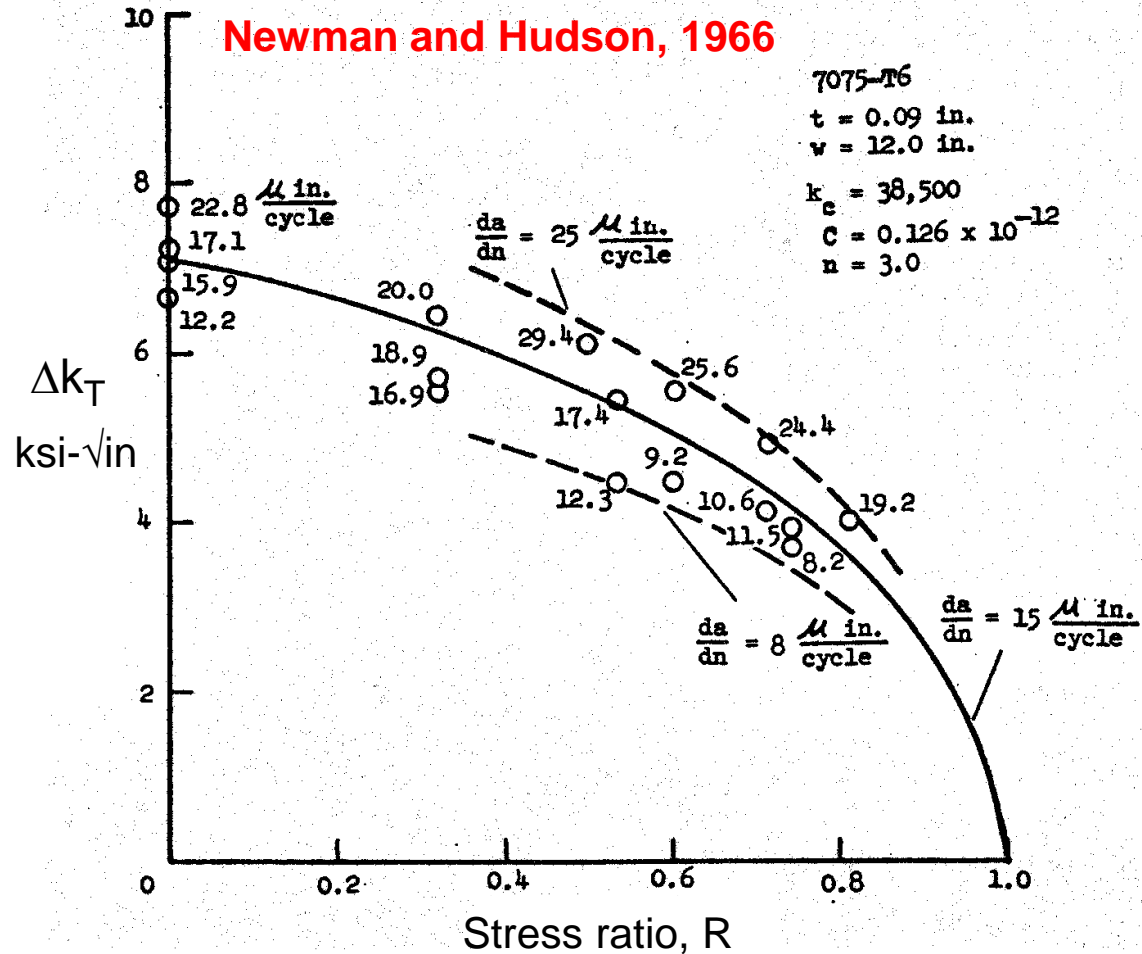


# FLAT-TO-SLANT FATIGUE-CRACK GROWTH

**Schijve (1966): Observed transition occurs at “constant rate”**

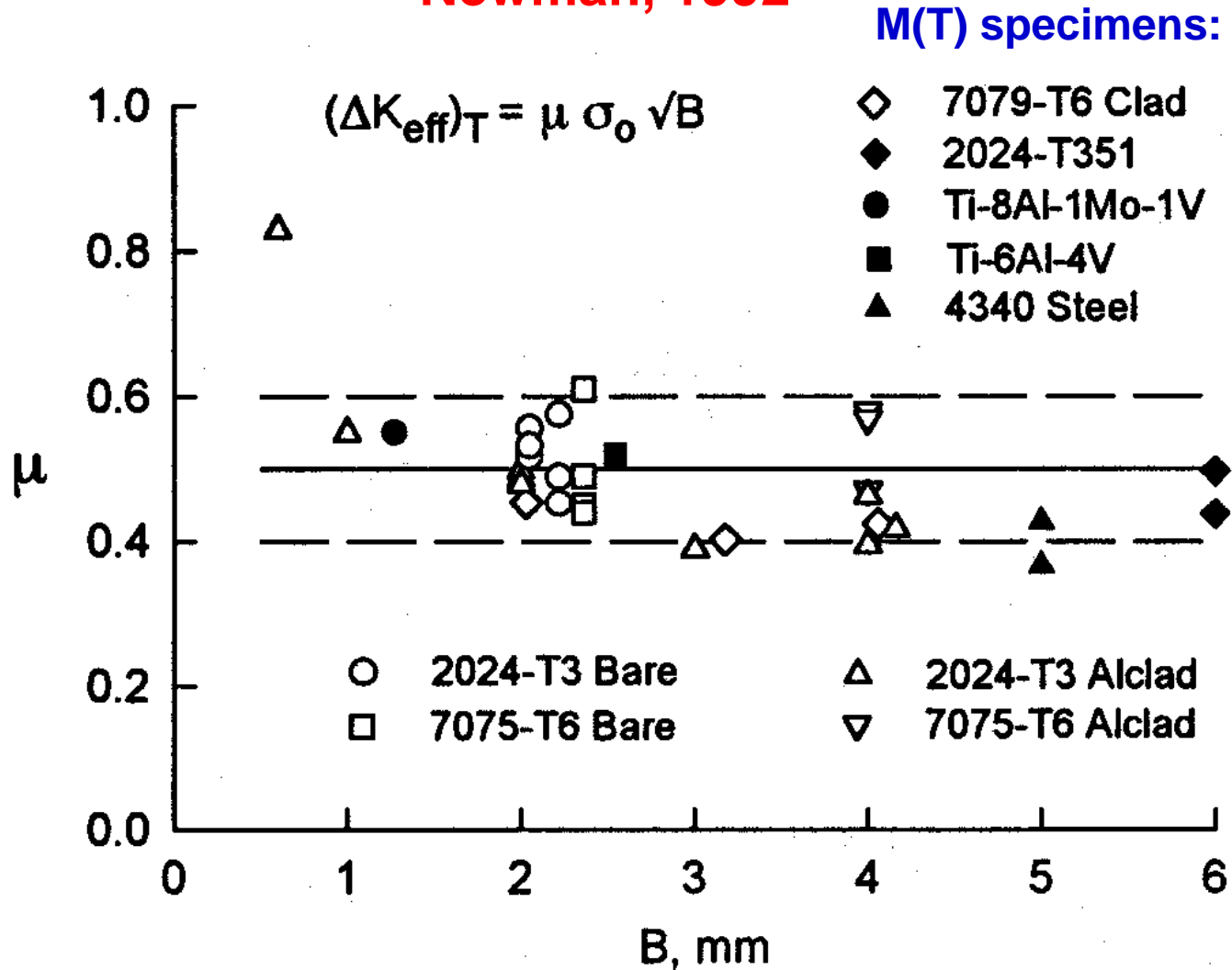


Constraint loss appears to occur on M(T) specimens, but **not** on deep-cracks in C(T) or bending specimens



# FLAT-TO-SLANT FATIGUE-CRACK GROWTH TRANSITION

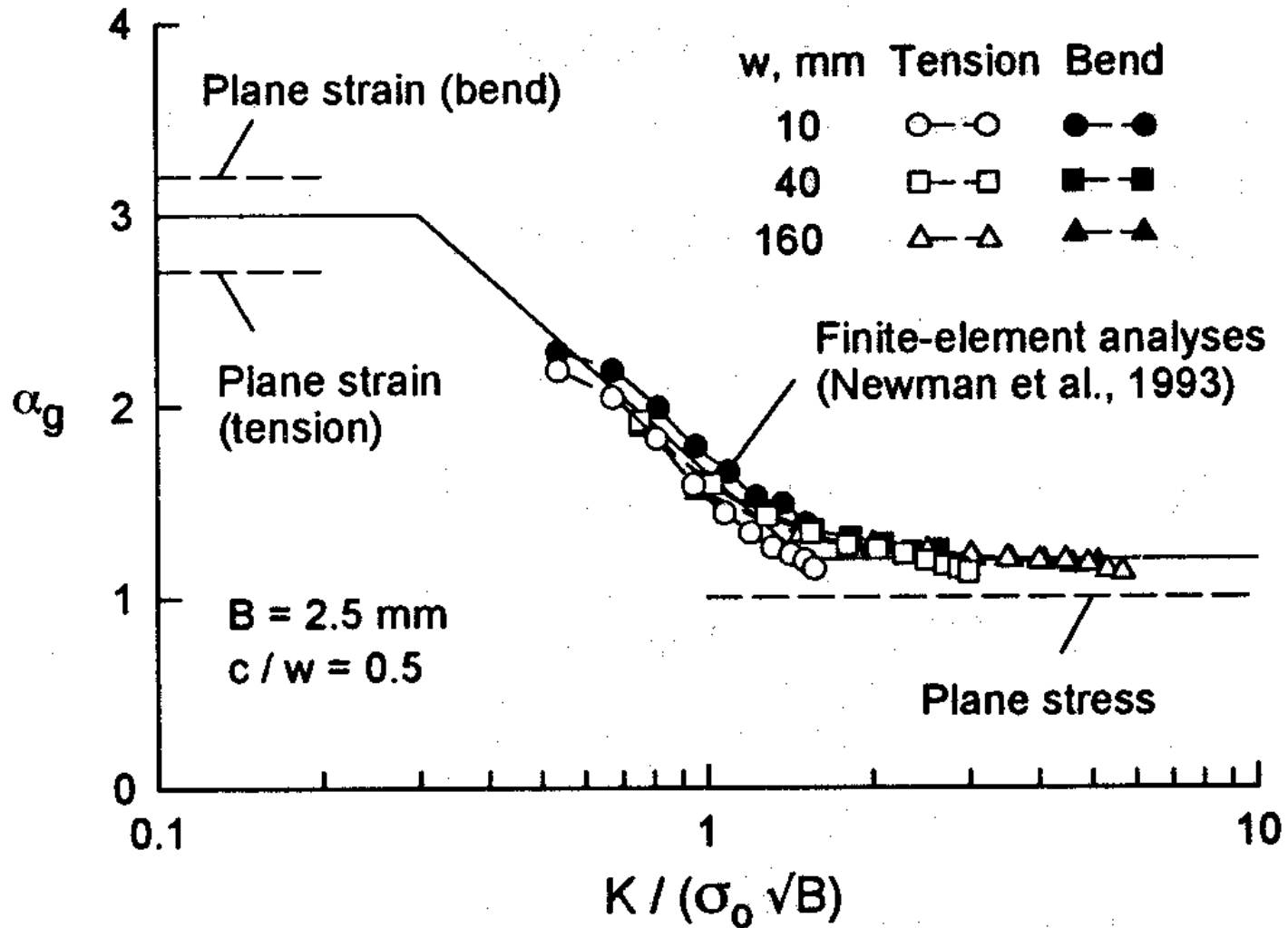
Newman, 1992





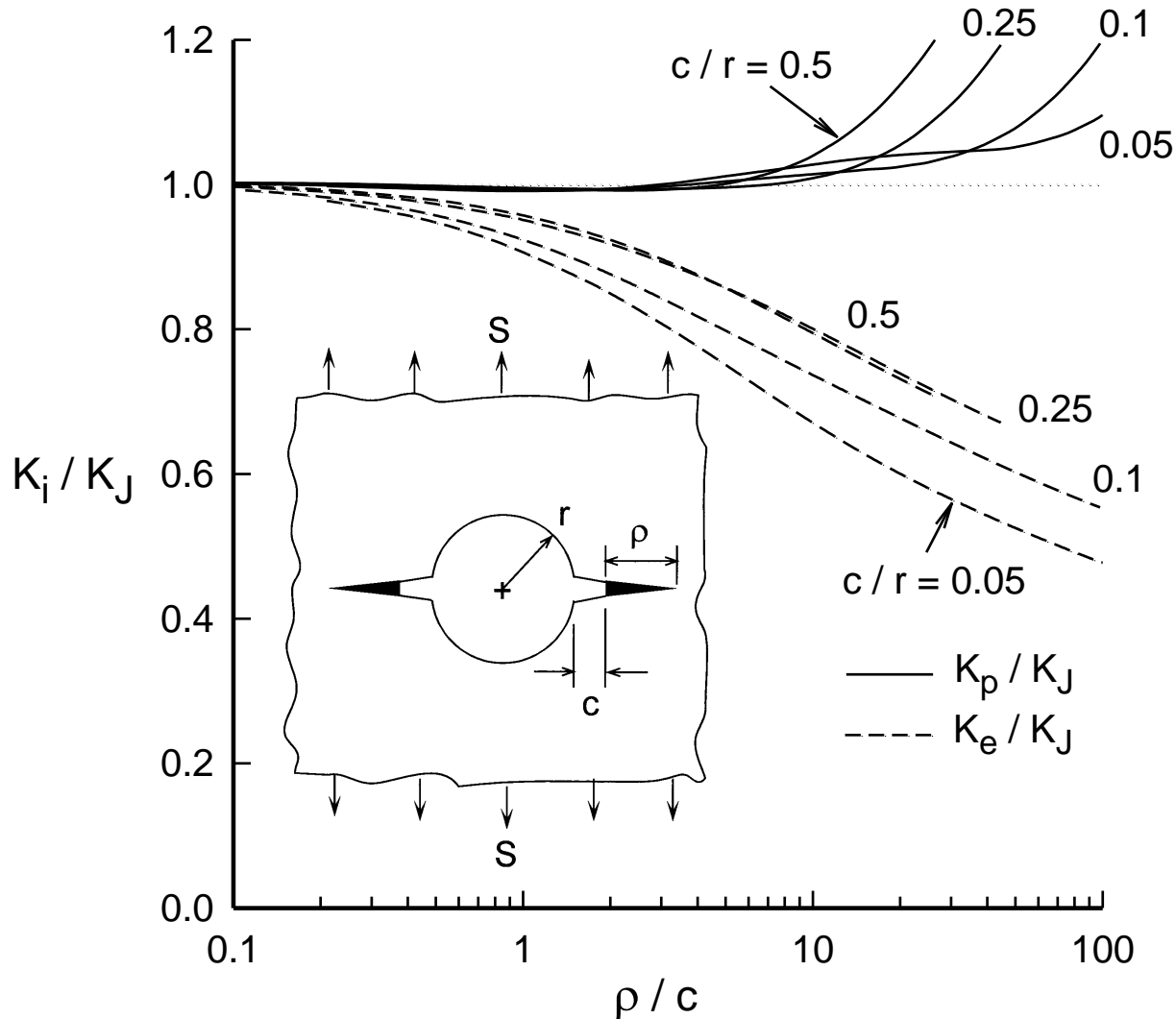
# CONSTRAINT EFFECTS IN THREE-DIMENSIONAL CRACKED BODIES

Newman, Bigelow & Shivakumar, 1993



# ELASTIC-PLASTIC STRESS-INTENSITY FACTORS

Newman, 1992



Crack Parameters:

$$K_i = S (\pi d)^{1/2} F(d/r)$$

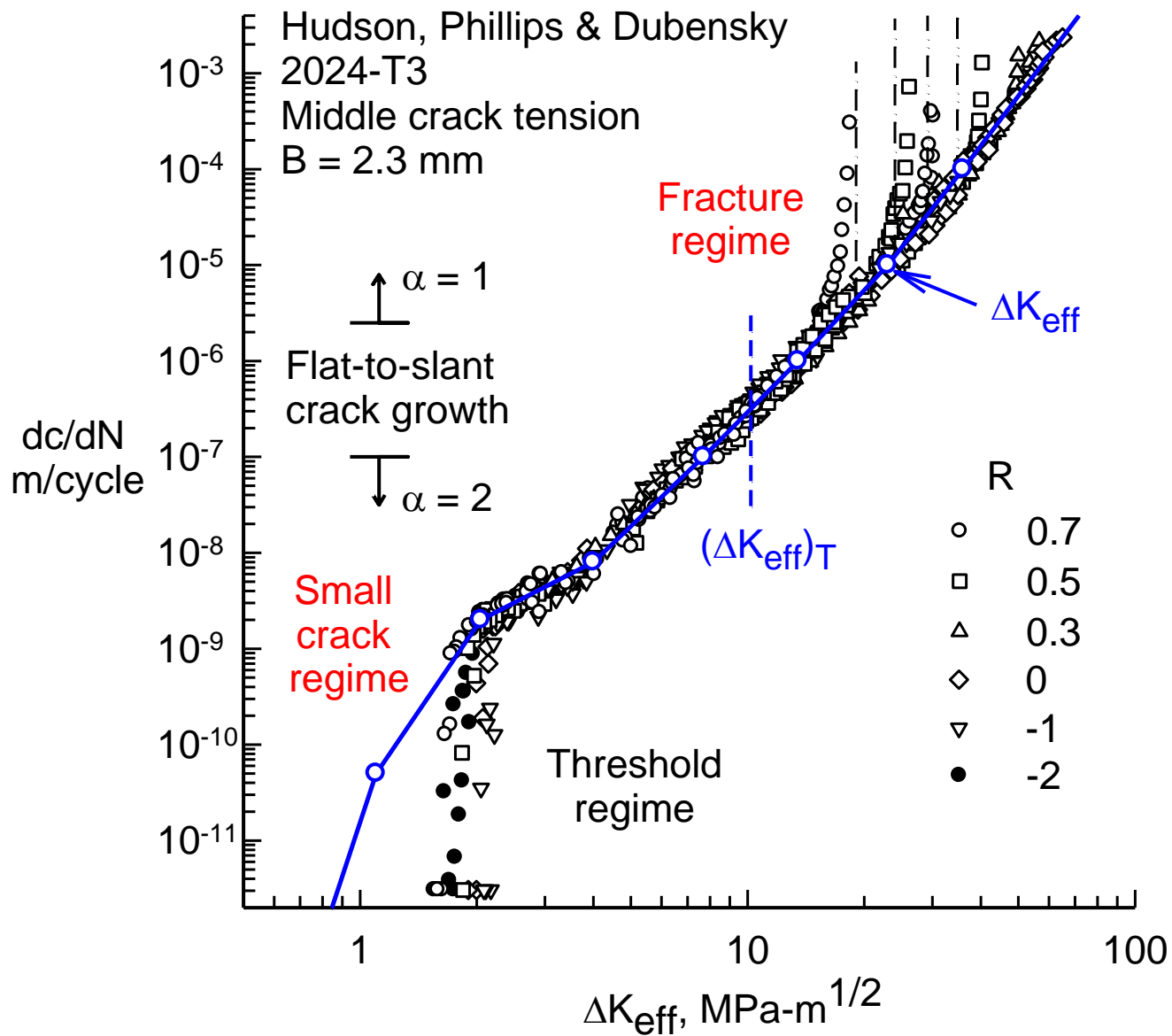
where  $d = c + \gamma \rho$

$\gamma = 0$  elastic

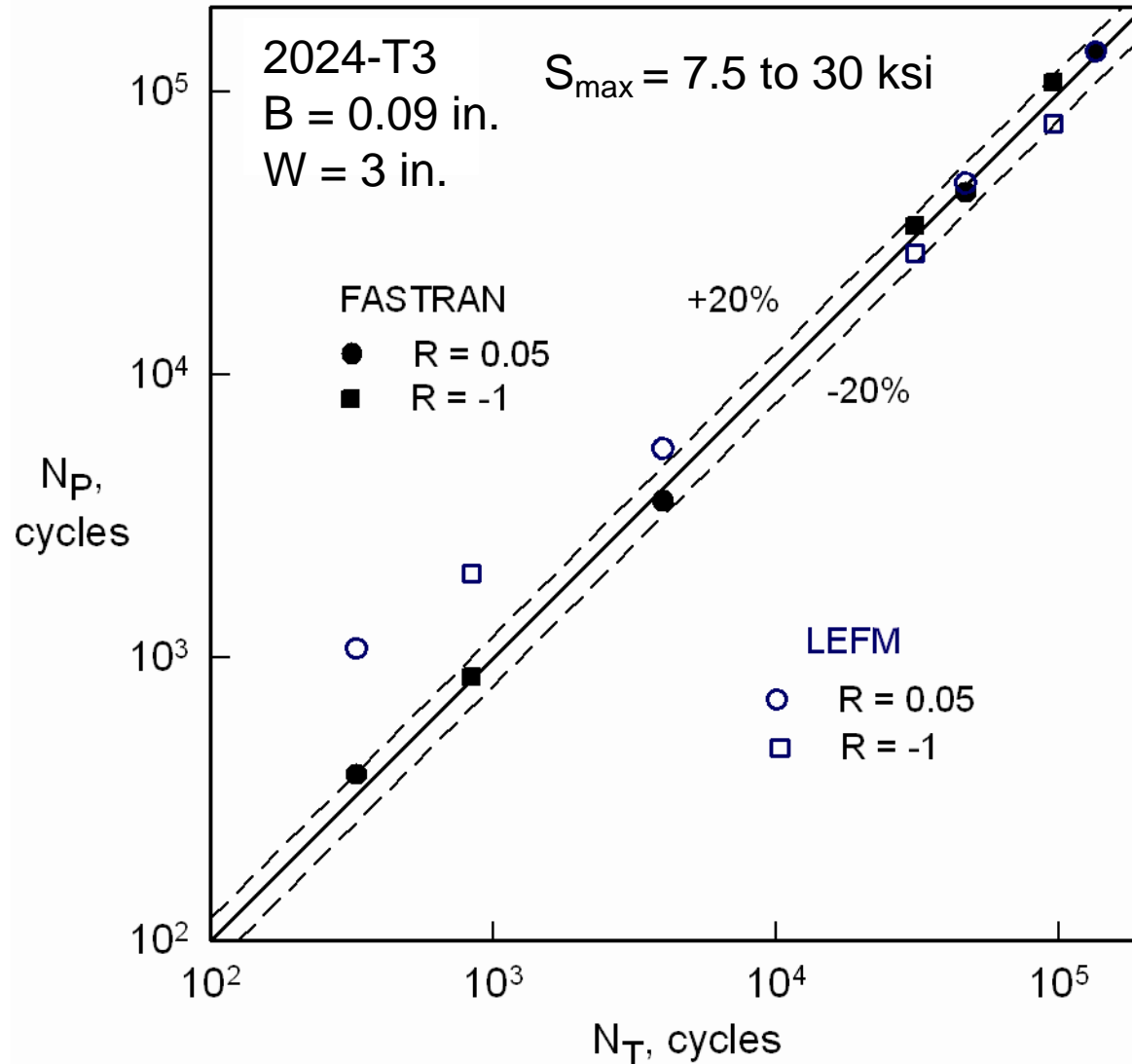
$\gamma = 1/4$  elastic-plastic

$$J = K_p^2 / E$$

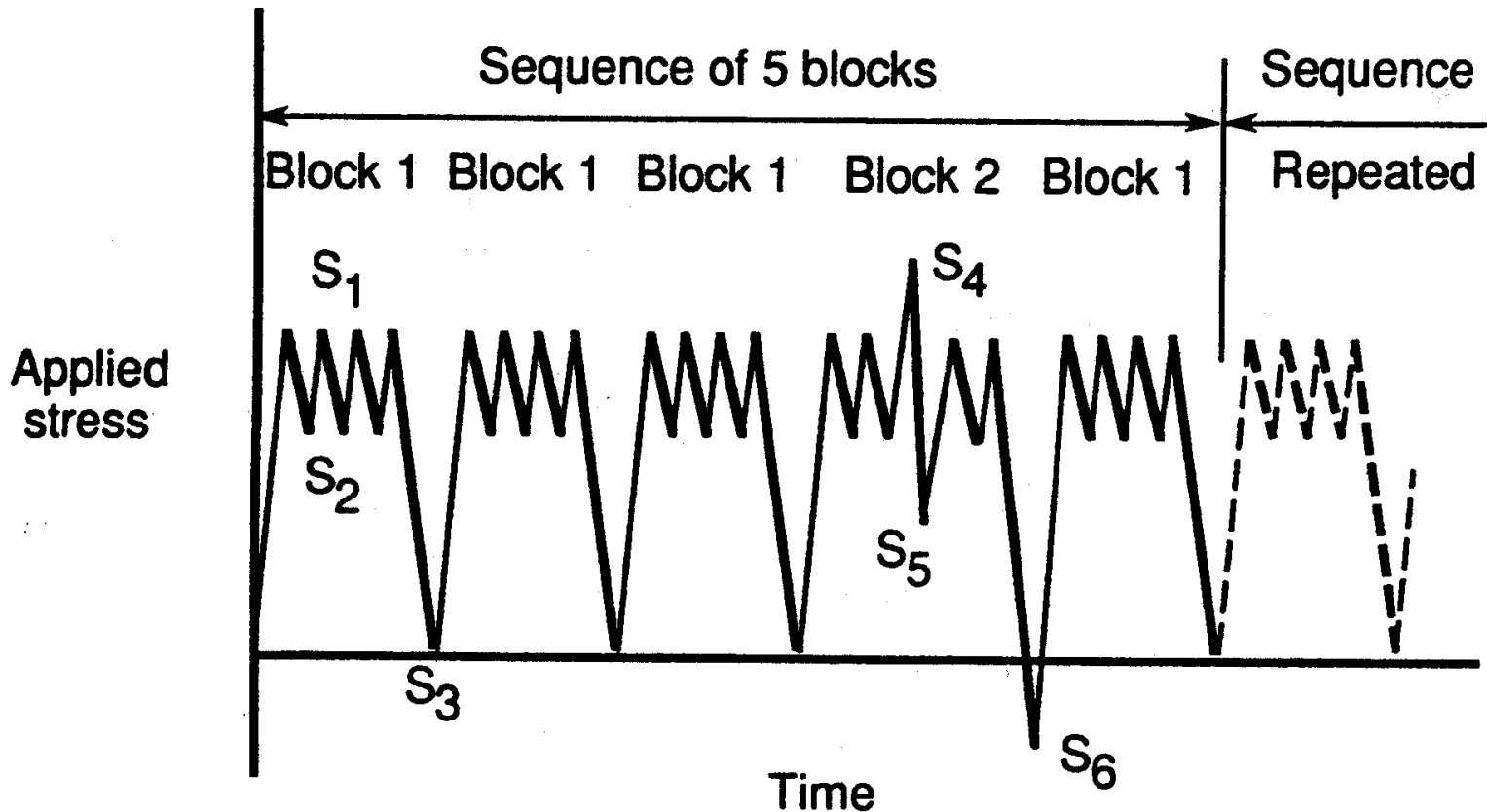
# CRACK-CLOSURE ANALYSES OF 2024-T3 ALUMINUM ALLOY



# COMPARISON OF MEASURED AND PREDICTED CRACK GROWTH USING LEFM AND FASTRAN



# VARIABLE-AMPLITUDE LOADING OPTION (NFOPT = 1)



# SPECTRUM LOADING OPTIONS IN FASTRAN

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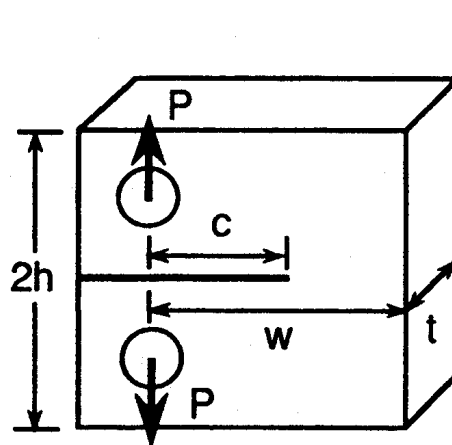
- **TWIST or MINI-TWIST - Transport Spectra (NFOPT = 2 or 3)**
- **FALSTAFF - Fighter Spectra (NFOPT = 4)**
- **SPACE SHUTTLE Load Spectra (NFOPT = 5)**
- **Gaussian (R ~ -1) Load Sequence (NFOPT = 6)**
- **Felix & Helix Helicopter Flight-Load Sequence (NFOPT = 7)**
- **Spectrum Read from List of Stress Points (NFOPT = 8)**
- **Spectrum Read from Flight-by-Flight Loading (NFOPT = 9)**
- **Spectrum Read from Flight Schedule (NFOPT = 10)**

# CRACK CONFIGURATION OPTIONS IN FASTRAN

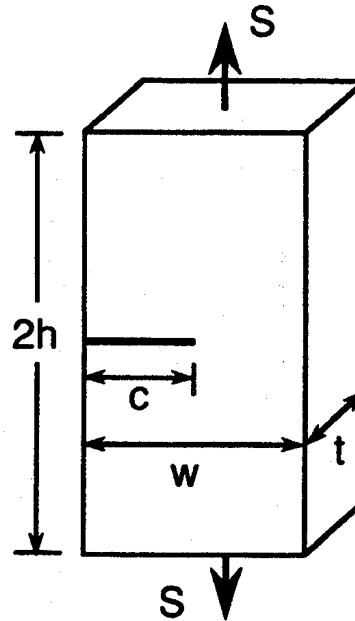
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- **Two-dimensional crack configurations (15)**
  - Middle-crack tension
  - Compact and bend type specimens
  - Crack(s) from an open hole
  - Crack in a pressurized cylinder
  - Periodic array of cracks at holes
  - User defined crack configuration
- **Three-dimensional crack configurations (11)**
  - Surface crack (tension or bending loads)
  - Surface or corner crack(s) at an open hole
  - AGARD small-crack specimen
  - Periodic array of surface or corner cracks at pin-loaded holes

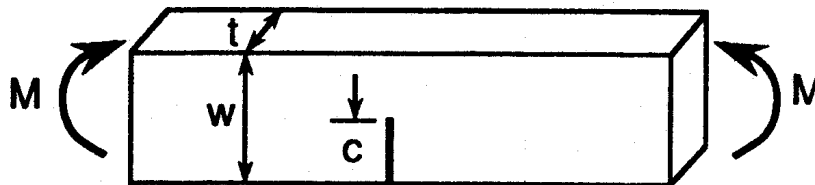
# LABORATORY SPECIMENS



NTYP = 2  
(h/w = 0.6)



NTYP = 3  
(h/w = 2)



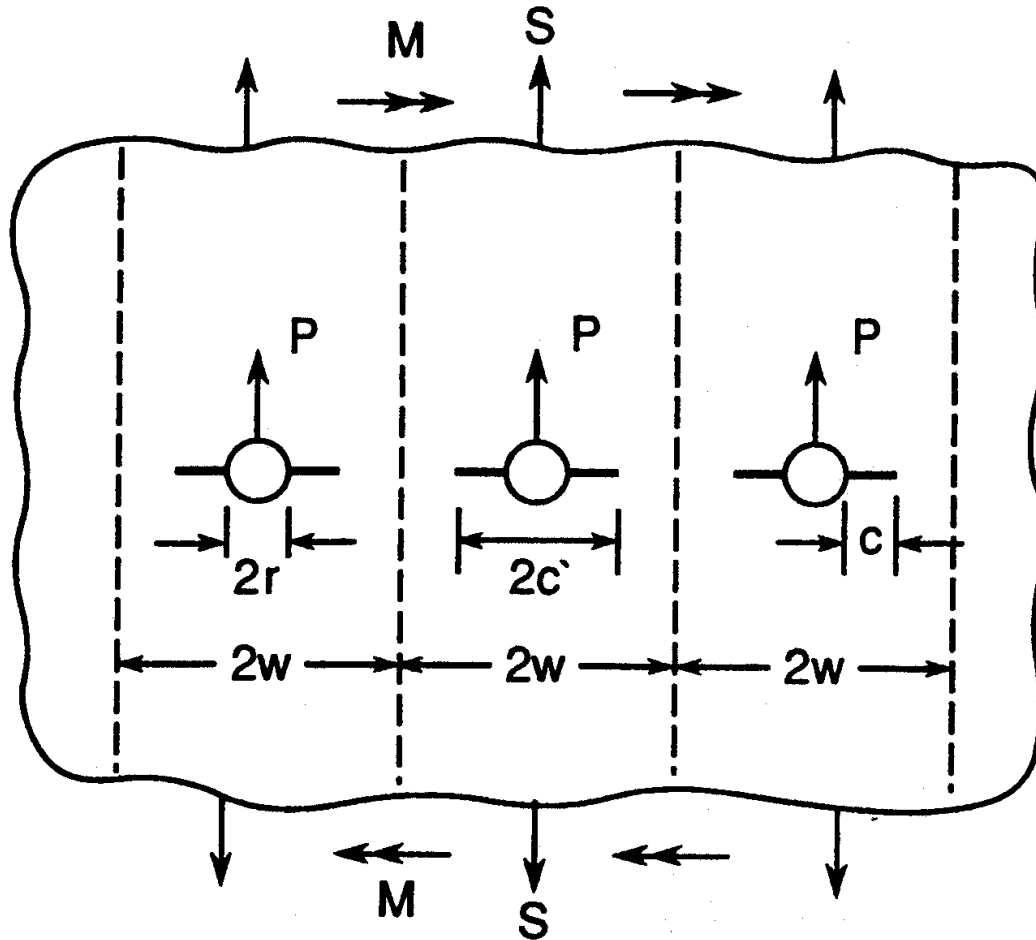
NTYP = 99

$$S_b = \frac{6M}{tw^2}$$

Example of user defined crack configuration  
(NTYP = -99 Crack(s) from hole)



# RIVETED AIRCRAFT JOINT CRACK CONFIGURATION

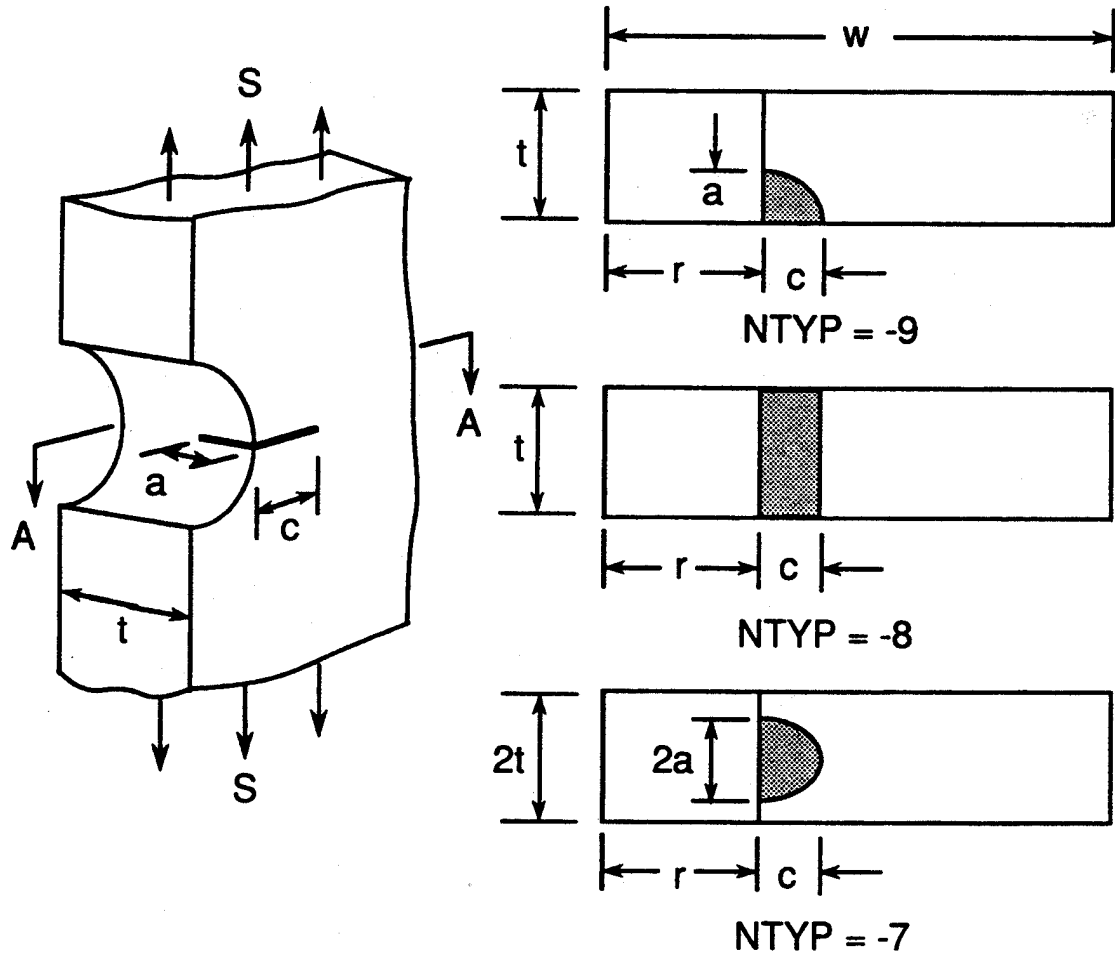


$$\text{NTYP} = -11$$
$$(P = M = 0)$$

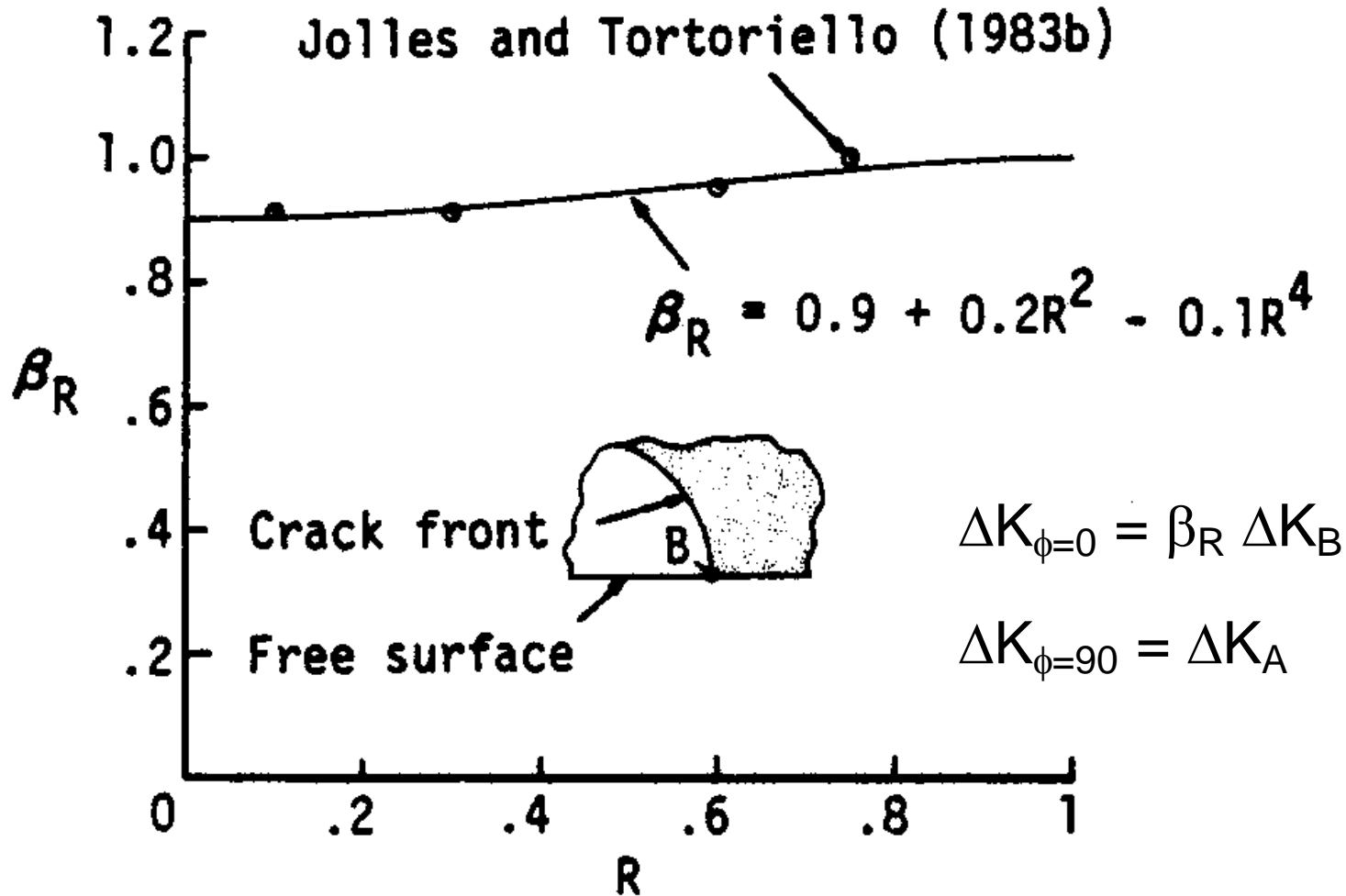
$$\text{NTYP} = -10$$
$$(S_b = \gamma S)$$

$$c' = r + c$$

# AGARD SMALL-CRACK SPECIMEN



# CRACK-CLOSURE CORRECTION FOR FREE SURFACE



# FATIGUE-CRACK GROWTH RATE OPTIONS

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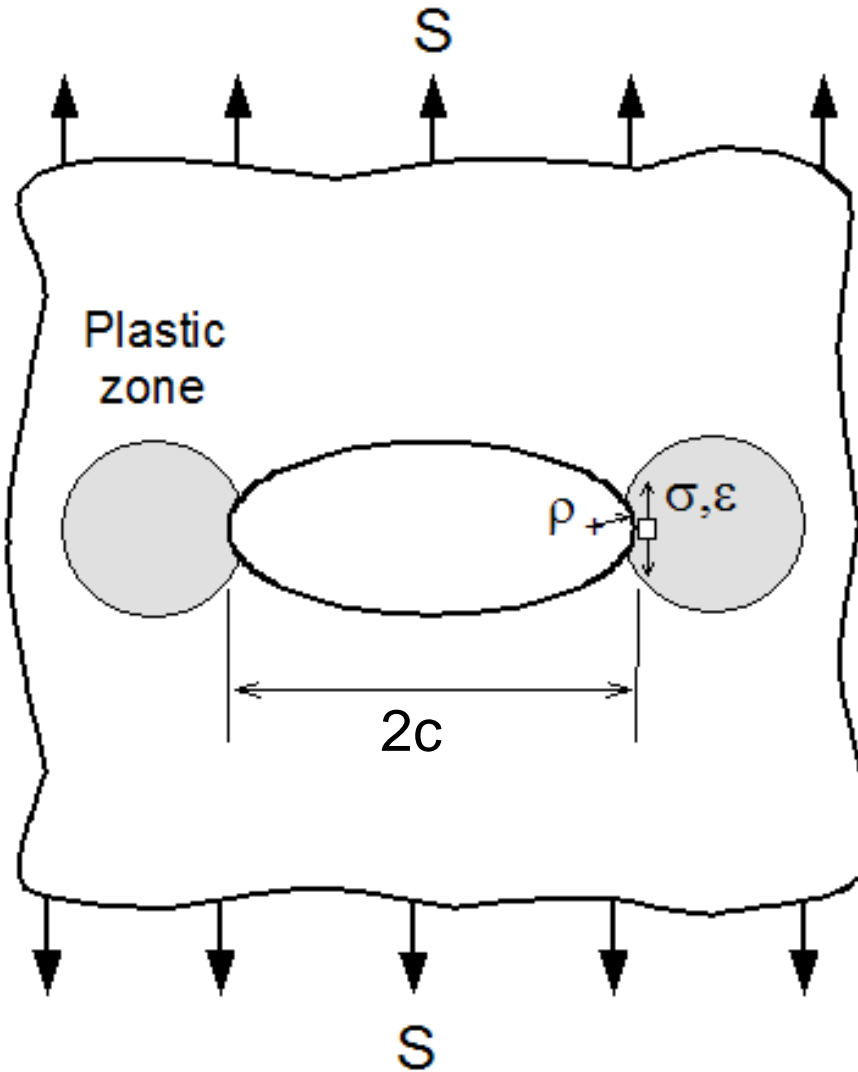
- **Equation:**  $dc/dN = C_1 \Delta K_{\text{eff}}^{C_2} f(\Delta K_{\text{th}}) / g(K_c)$ 
  - $f(\Delta K_{\text{th}}) = 1 - (\Delta K_o / \Delta K_{\text{eff}})^p$   
 $\Delta K_o = C_3 (1 + C_4 R)$  or  $\Delta K_o = C_3 (1 - R)^{C_4}$
  - $g(K_c) = 1 - (K_{\text{max}} / C_5)^q$
- **Table Look-up:**  $dc/dN = f(\Delta K_{\text{eff}})$  (Max 35 points)
  - $f(\Delta K_{\text{eff}}) = C_{1i} \Delta K_{\text{eff}}^{C_{2i}}$  (i = 1 to 34)
  - $f(\Delta K_{\text{eff}}) = C_{1i} \Delta K_{\text{eff}}^{C_{2i}} f(\Delta K_{\text{th}}) / g(K_c)$
- **Crack growth** ( $da/dN = dc/dN$  or  $da/dN \neq dc/dN$ )

# FRACTURE CRITERIA

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- **Two-Parameter Fracture Criterion –  $K_F$  and  $m$** 
  - $m = 0$  LEFM ( $K_c = K_F$  for  $S_n < \sigma_{ys}$ )
  - $m = 1$  Plastic-collapse criteria ( $K_F$  large)
- **Cyclic fracture toughness exceeded ( $K_{max} > C_5$ )**
- **Plastic-zone size exceeds net-section region**

# Elastic-Plastic Stress- and Strain-Concentration Factors using Neuber's Equation



Neuber (1961):

$$K_{\sigma} K_{\epsilon} = K_T^2$$

$$\sigma \epsilon E = \sigma_e^2$$

Hutchinson, Rice (1968) showed that the stress-strain field for a **crack** in a non-linear elastic material verified Neuber's equation

Crews (1974) experimentally validated Neuber's equation for **elliptical hole** in finite plate under remote uniform stress

# Original Two-Parameter Fracture Criterion

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- Inglis' stress-concentration equation for elliptical hole,

$$K_T = 1 + 2 \sqrt{c/\rho}$$

- Neuber's equation:  $K_\sigma K_\varepsilon = K_T^2$

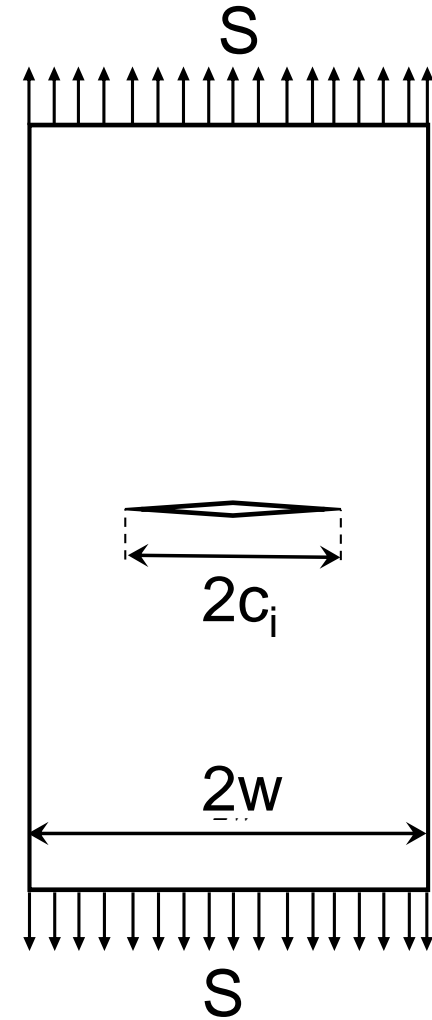
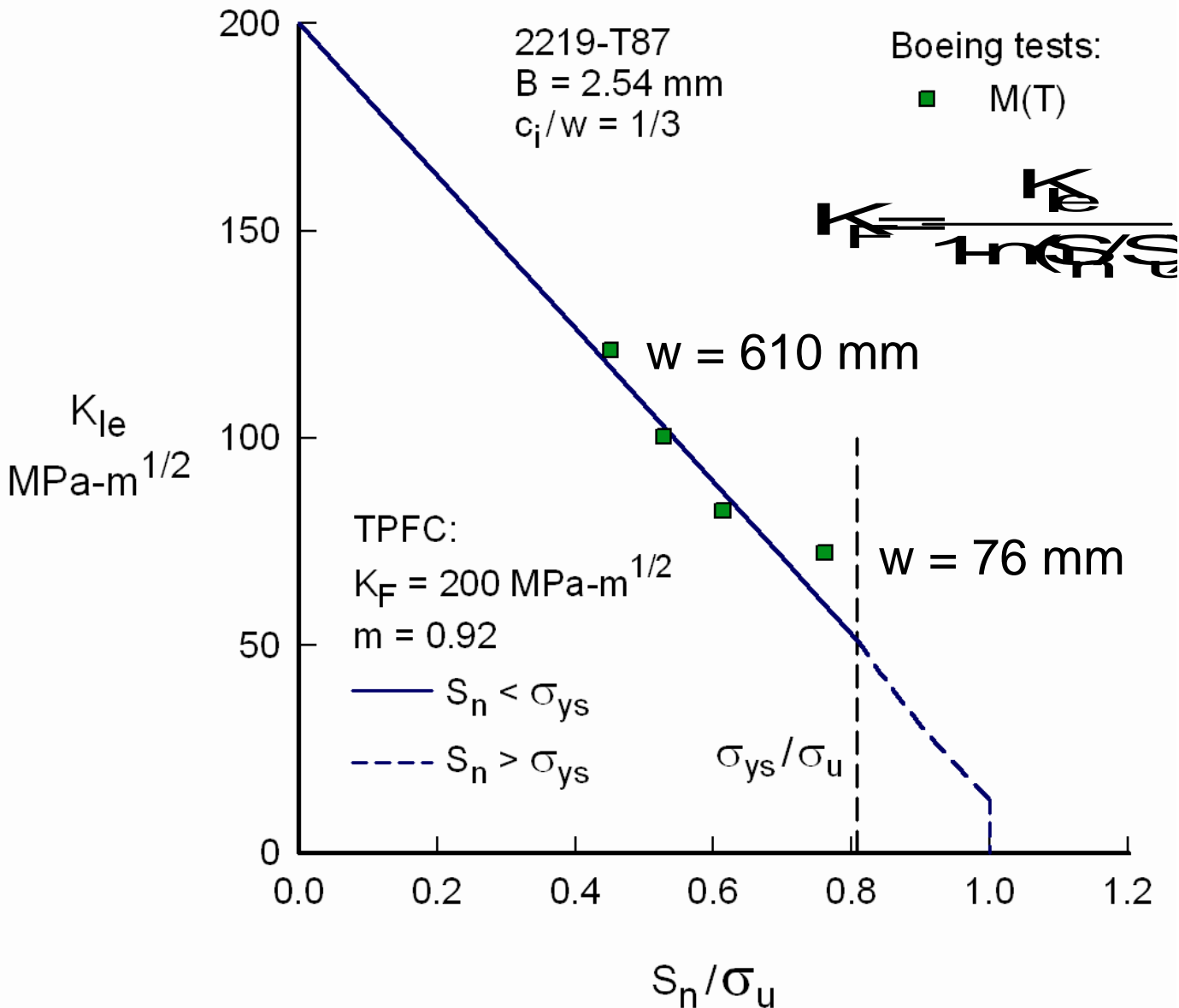
$$K_F = K_{le} / \Phi$$

$$\Phi = 1 - m (S_n / \sigma_u) \quad \text{for } S_n < \sigma_{ys}$$

$$\Phi \approx (\sigma_{ys} / S_n) [1 - m (S_n / \sigma_u)] \quad \text{for } S_n \geq \sigma_{ys}$$

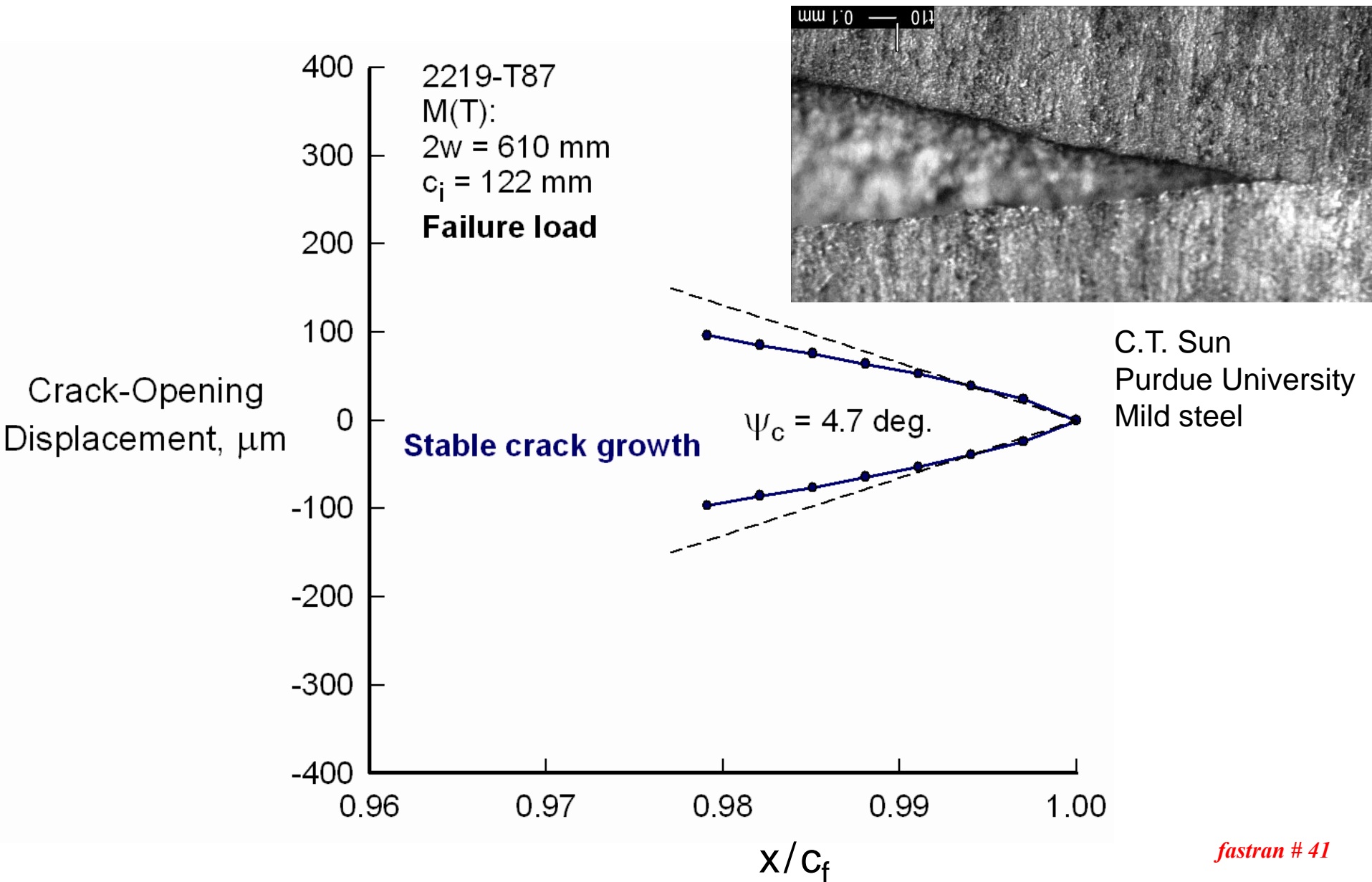
**Constraint effects on net-section *NOT* considered !**

# Two-Parameter Fracture Criterion Analysis on 2219-T87 Aluminum Alloy M(T) Specimens

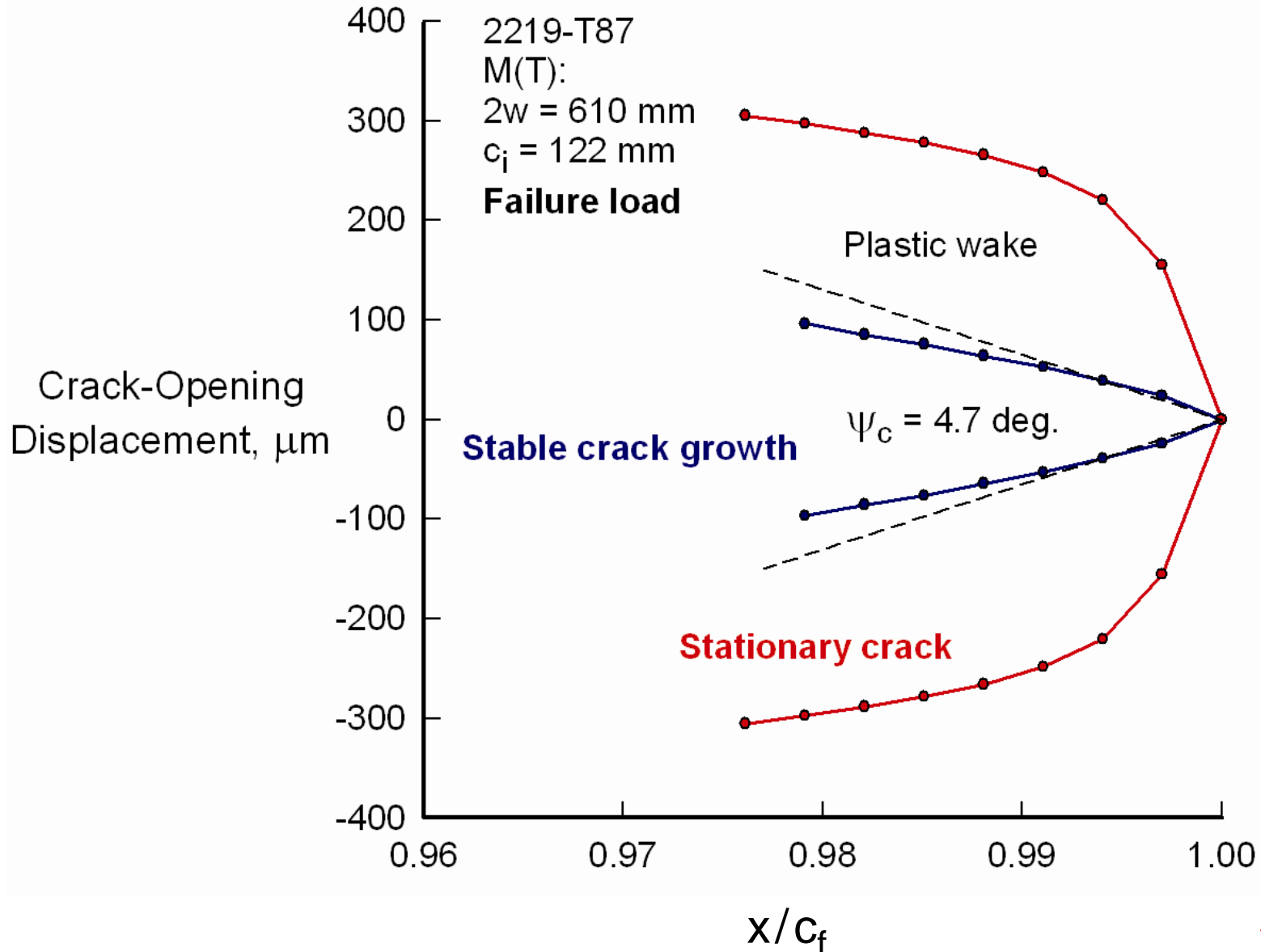




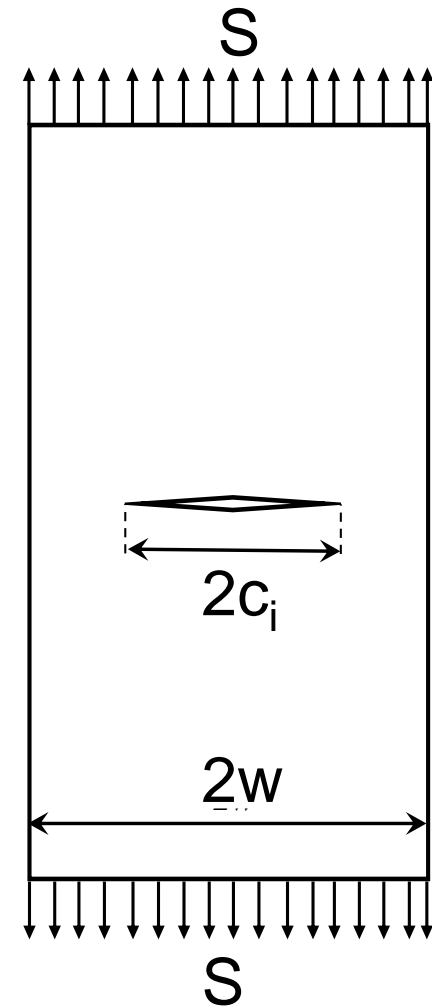
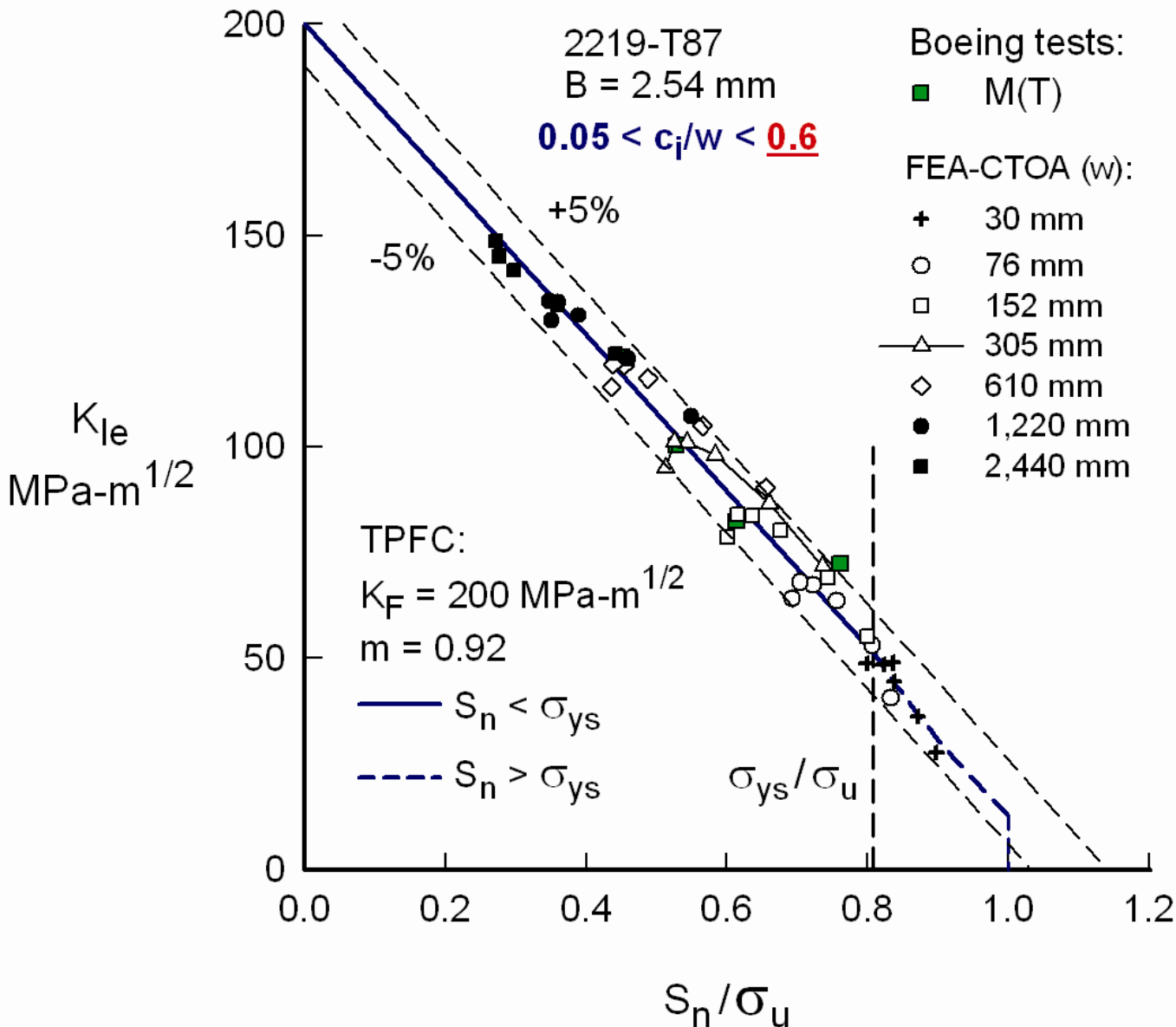
# Crack-Opening Displacements for Stably Tearing Crack using Critical CTOA and Finite-Element Analyses



# Crack-Opening Displacements for Stationary and Stably Tearing Crack using Critical CTOA-FEA Analyses



# Elastic Stress-Intensity Factor at Failure for Wide Range of Middle-Crack Tension Specimens



# OUTLINE OF PRESENTATION

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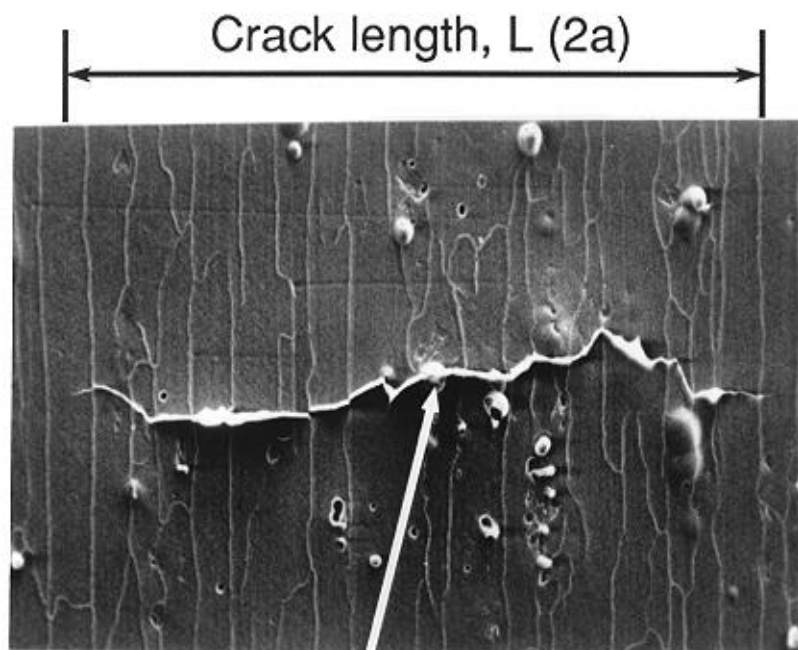
- Brief History on Fatigue-Crack Growth
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- **Crack Initiation and Small-Crack Behavior**
- **Fatigue-Crack Growth and Fracture**
- **Concluding Remarks**

# CRACK INITIATION AND SMALL-CRACK BEHAVIOR

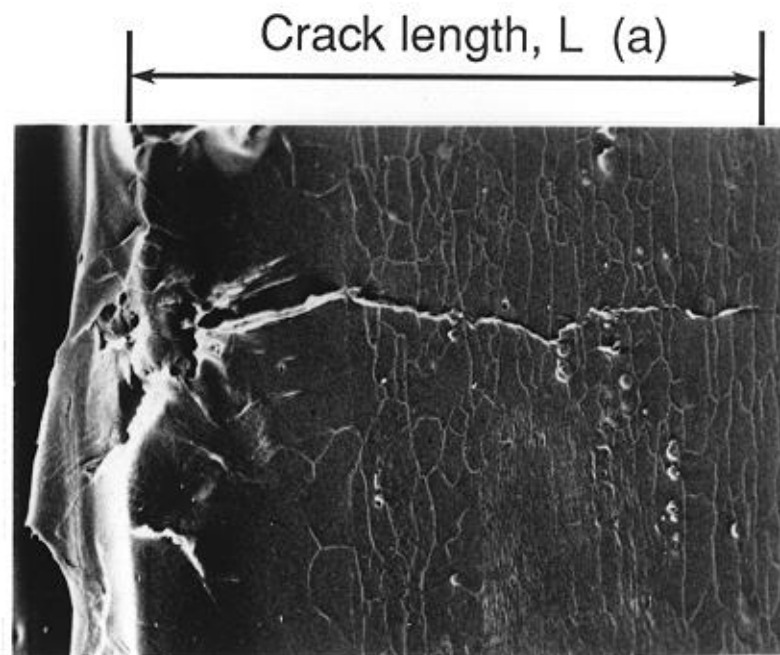
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- **AGARD Structures and Materials Panel (1984-91) and NASA/CAE (1987-1994) Small-Crack Test and Analysis Programs**
- **Small- and Large-Crack Growth Rates**
- **DARPA SIPS Program (2003-2008)**

# SMALL-CRACK MEASUREMENTS IN ALUMINUM ALLOYS

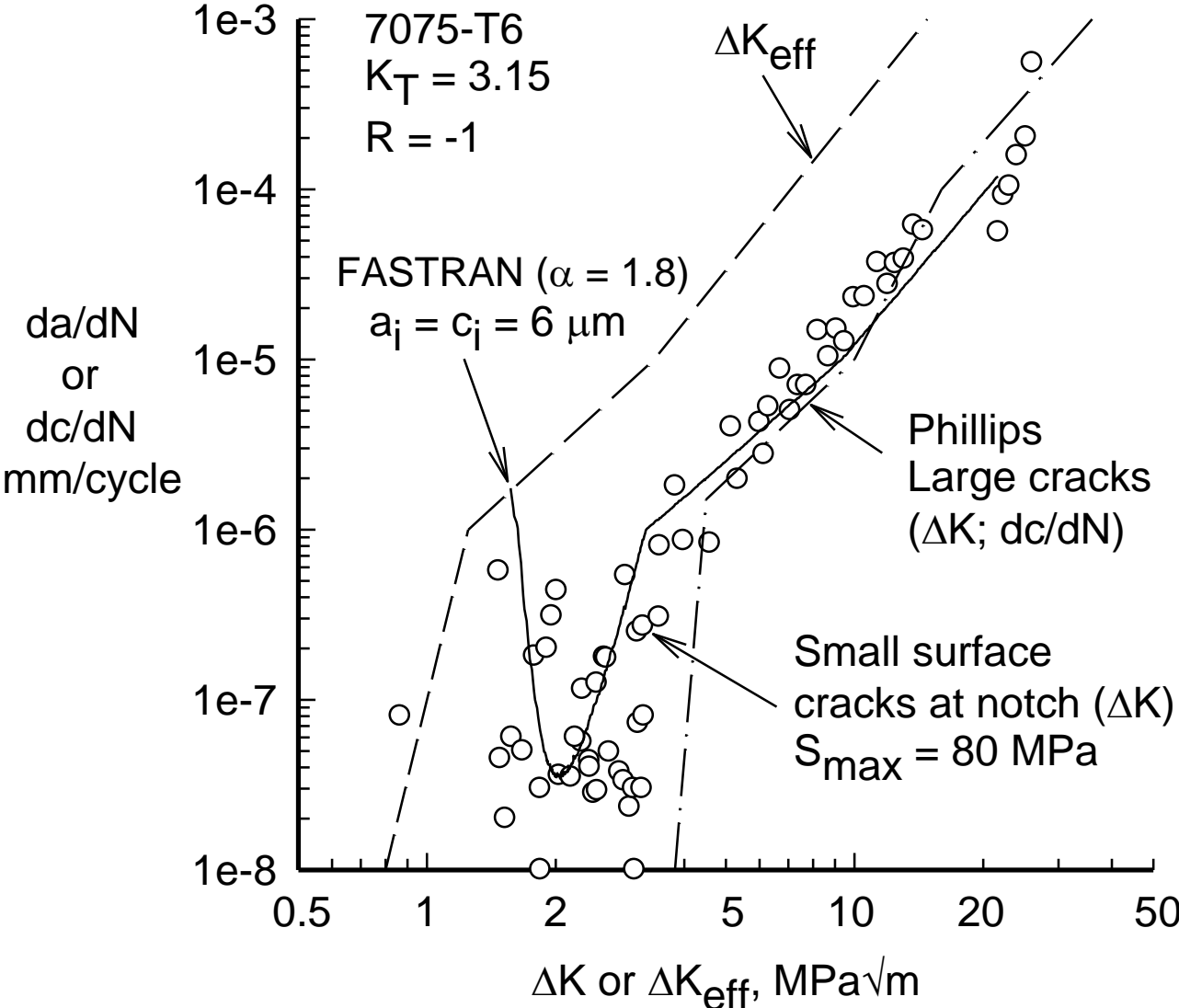


(a) 7075-T6

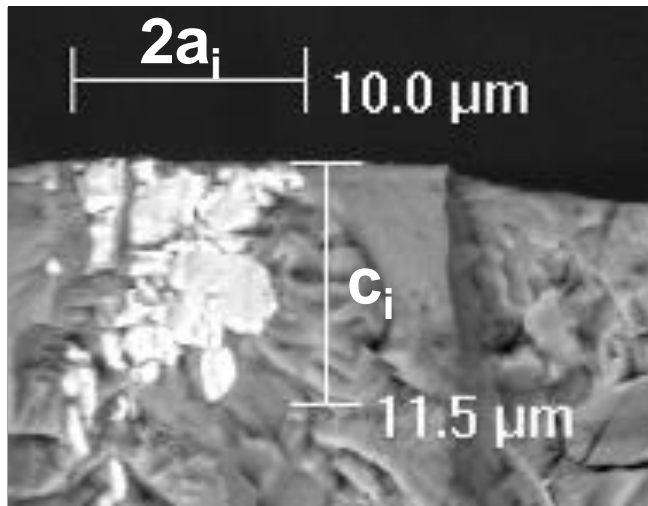
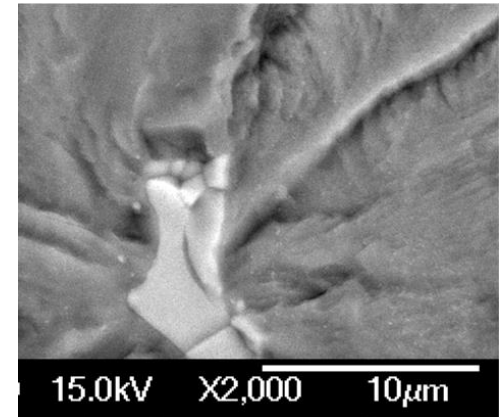
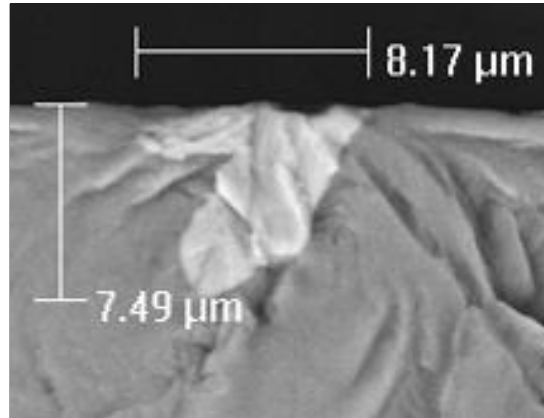
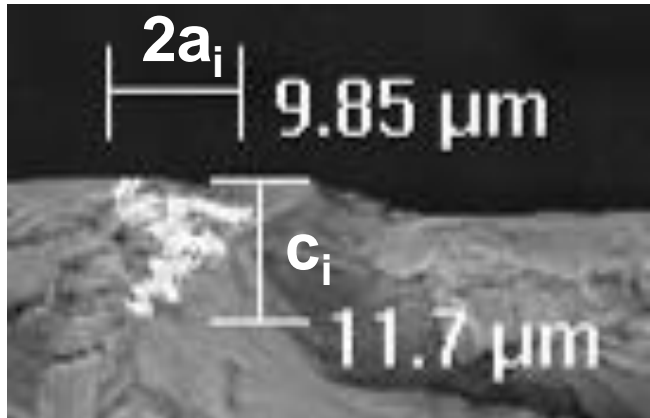


(b) Lc9CS (7075-T6 clad)

# SMALL- AND LARGE-CRACK GROWTH RATES IN 7075-T6



# TYPICAL INITIATION SITES IN SIPS 7075-T651



## Average flaw size:

$a_i = 4.3 \mu\text{m}$  (along bore)

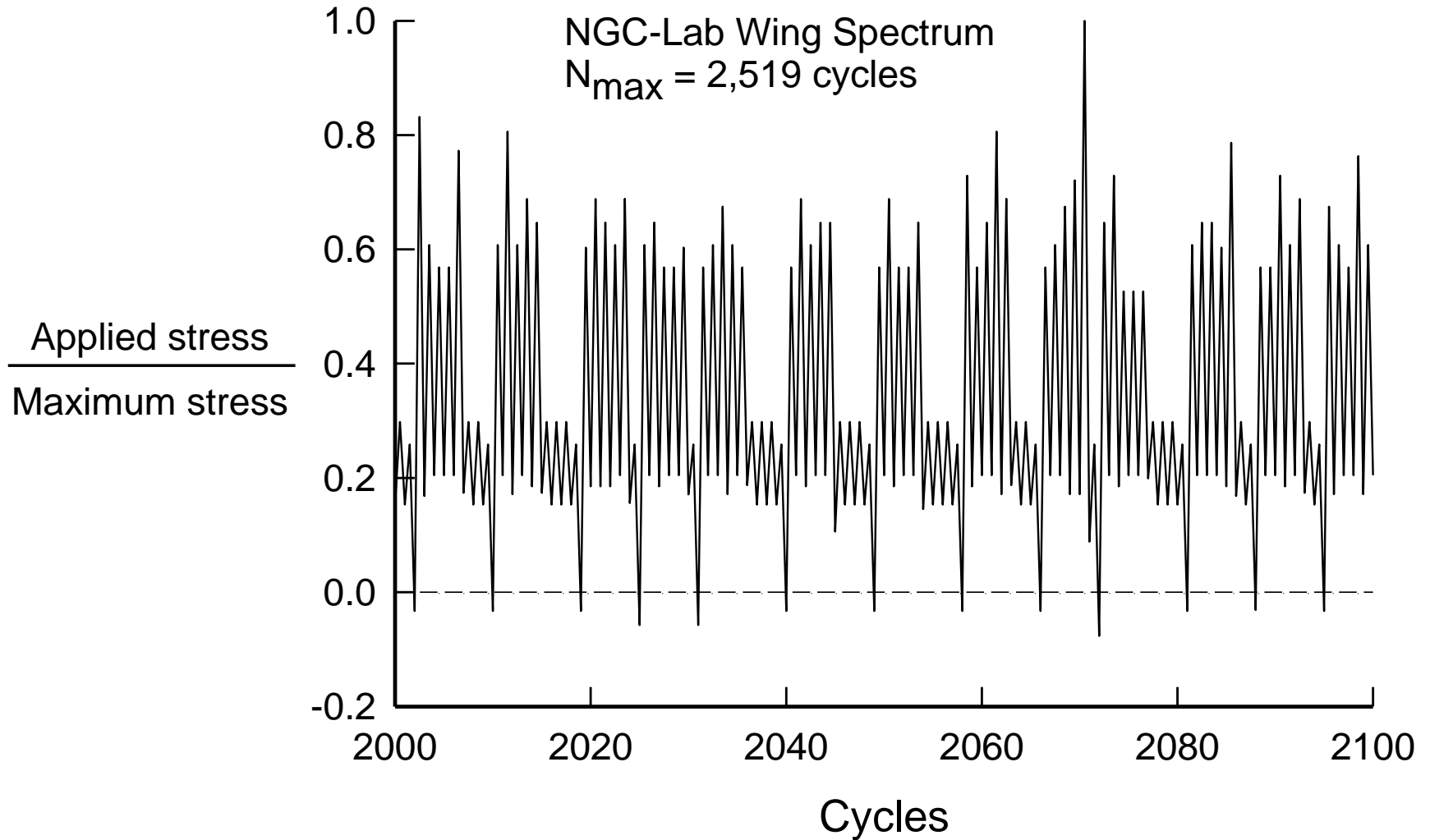
$c_i = 9.9 \mu\text{m}$  (depth from hole)

Semi-circular:  **$6.2 \mu\text{m}$**   
(equal area)

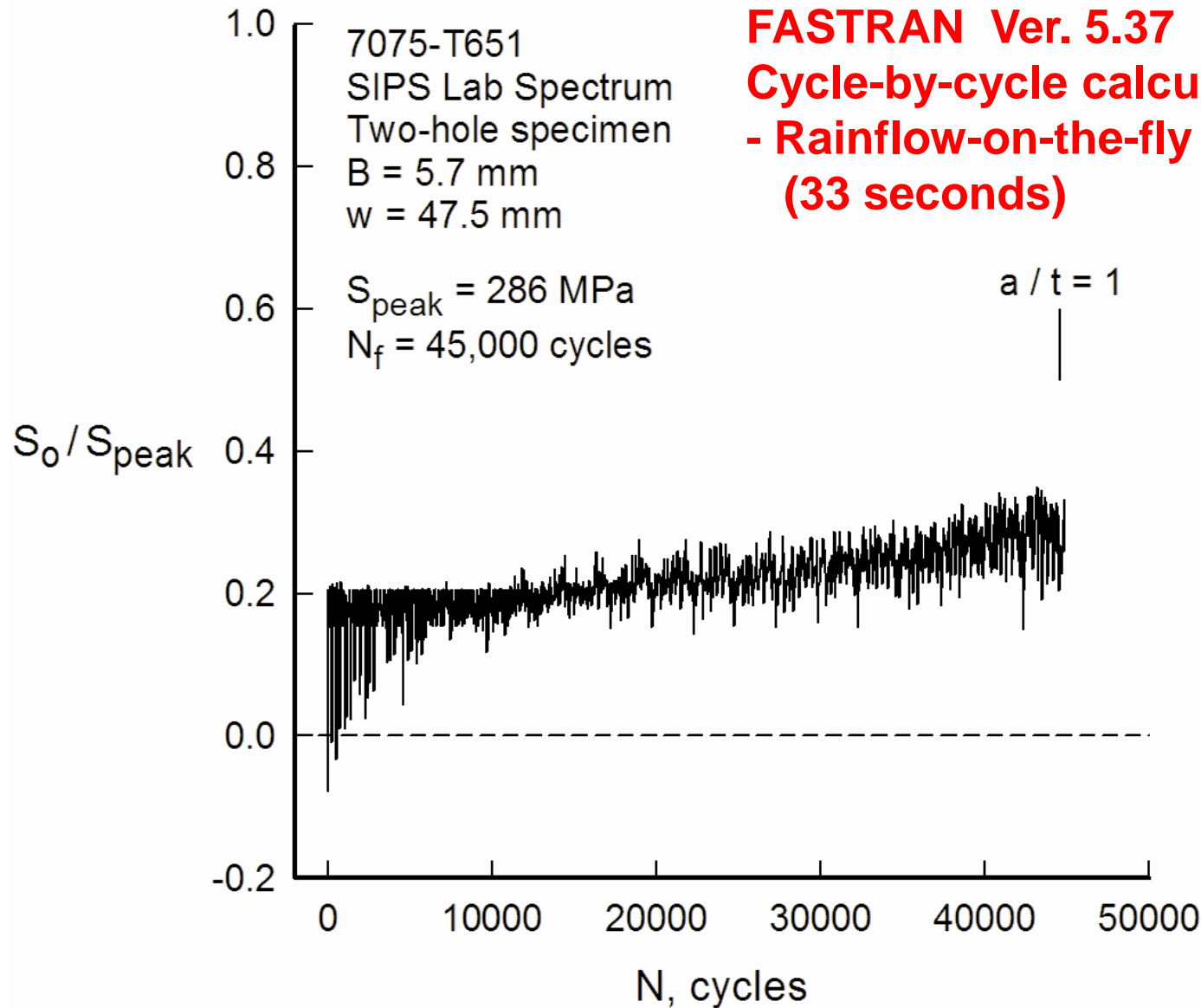
Not an EIFS, but RIFS – Real Initial Flaw Size



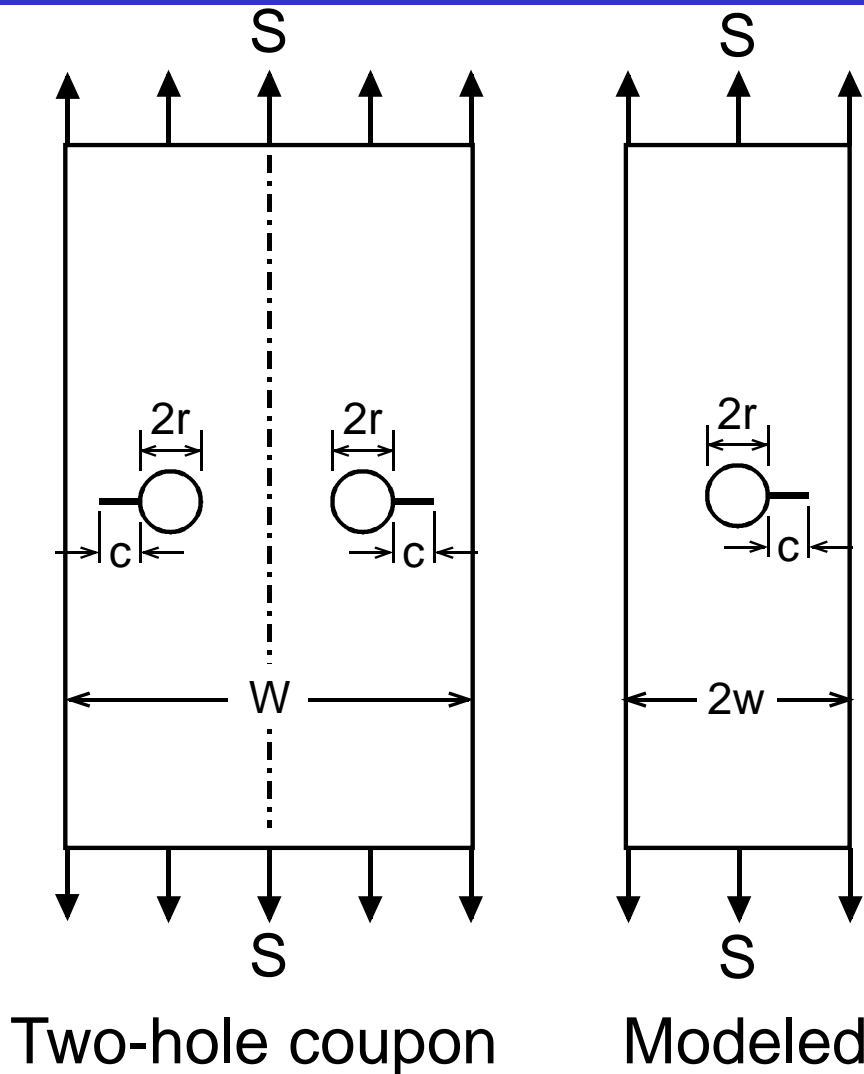
# SIPS LABORATORY WING SPECTRUM



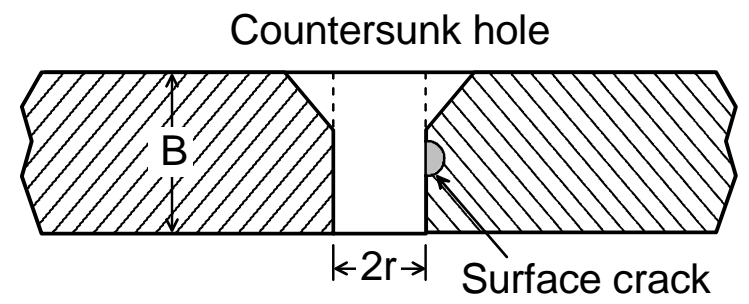
# CALCULATED CRACK-OPENING STRESSES UNDER SIPS LABORATORY WING SPECTRUM LOADING



# METHOD USED TO ANALYZE TWO-HOLE COUPONS

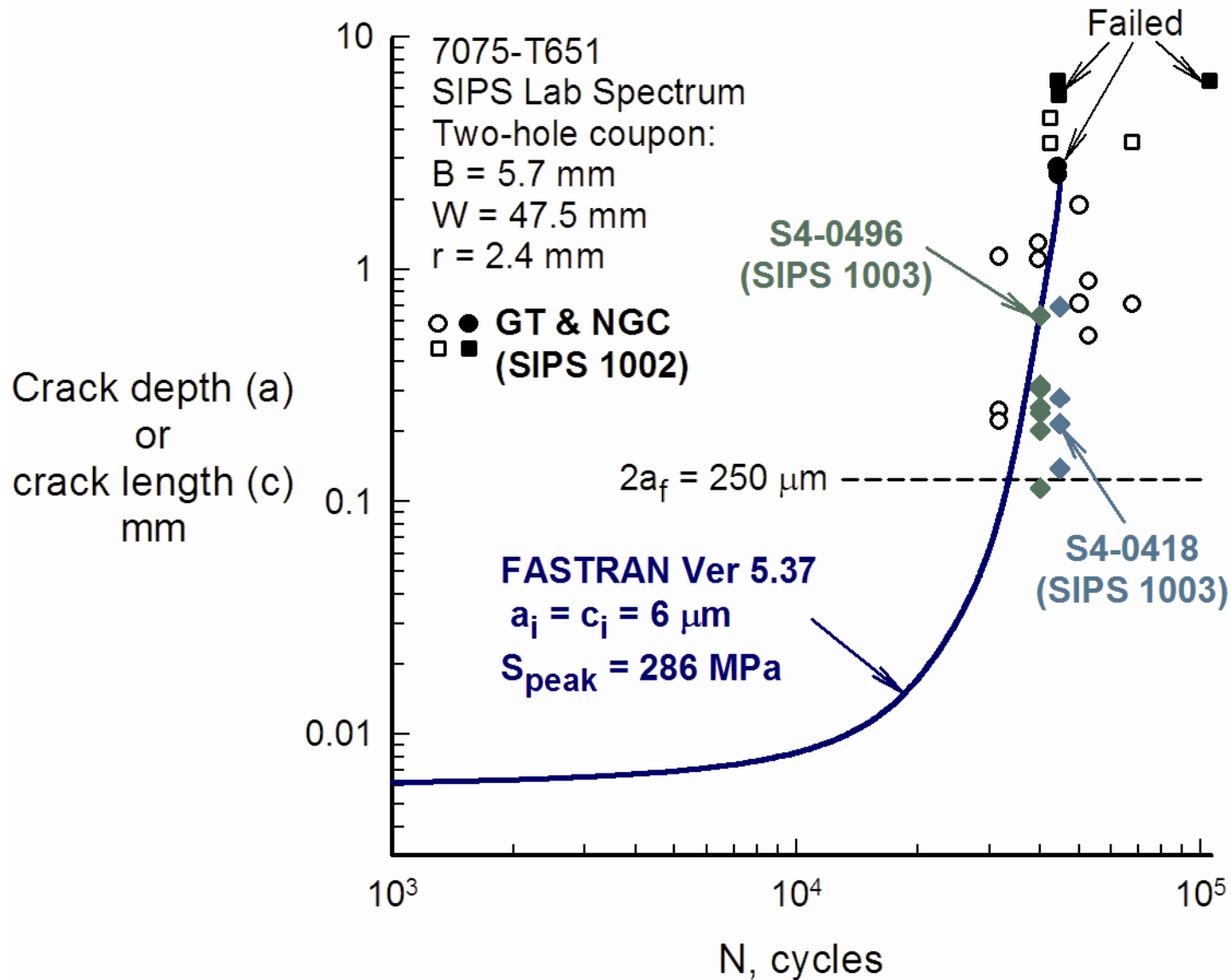


Countersunk or straight-shank holes

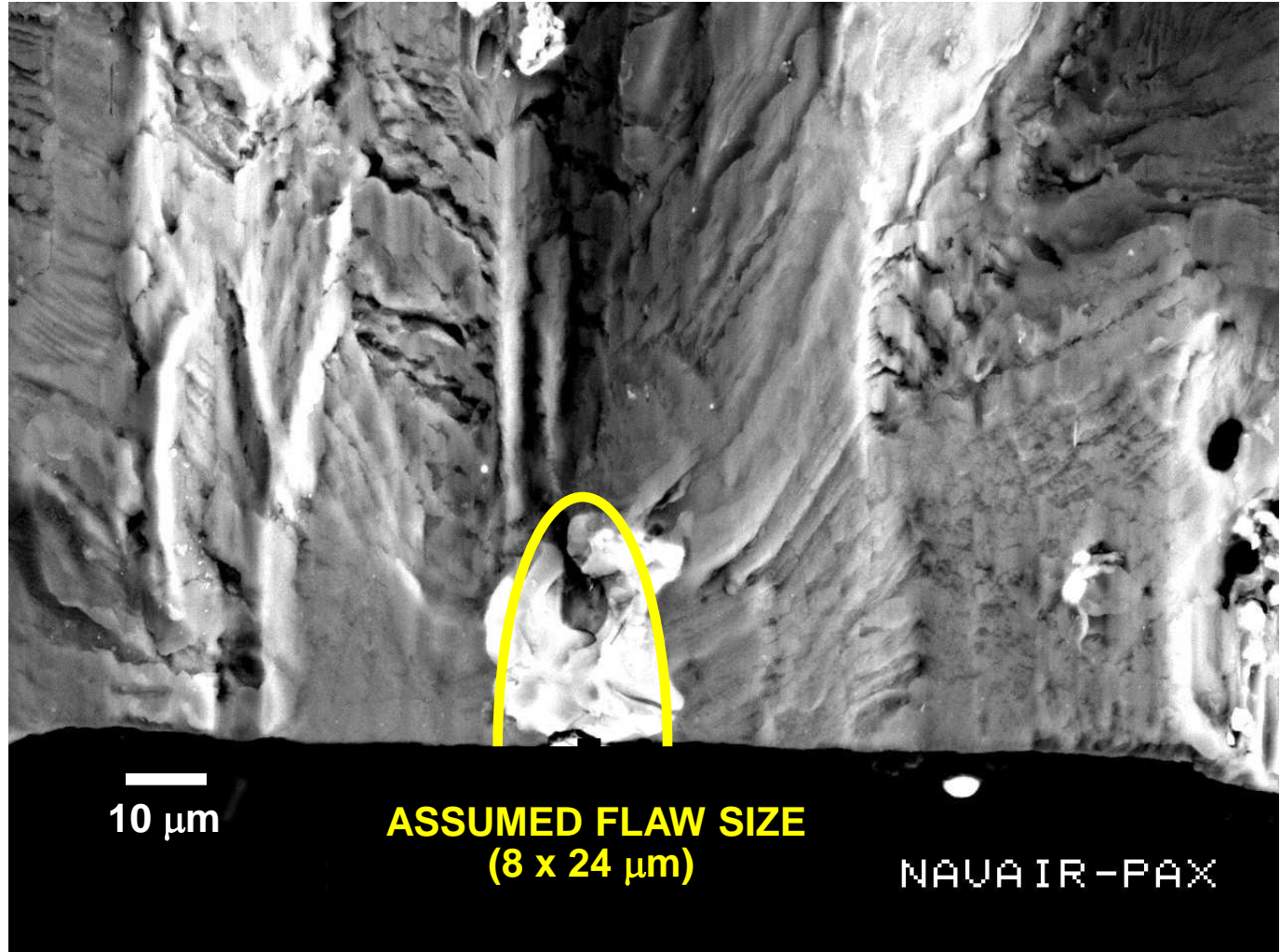
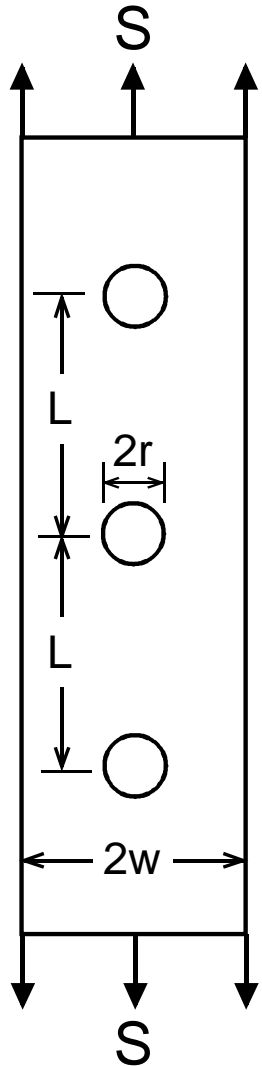


Initial flaw assumed to be a surface crack along hole bore and neglecting countersunk

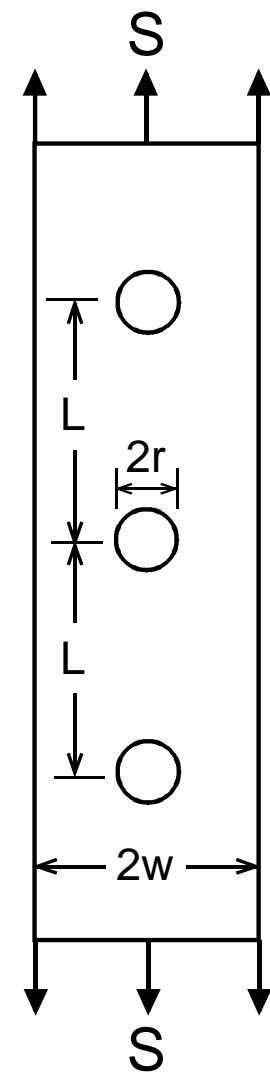
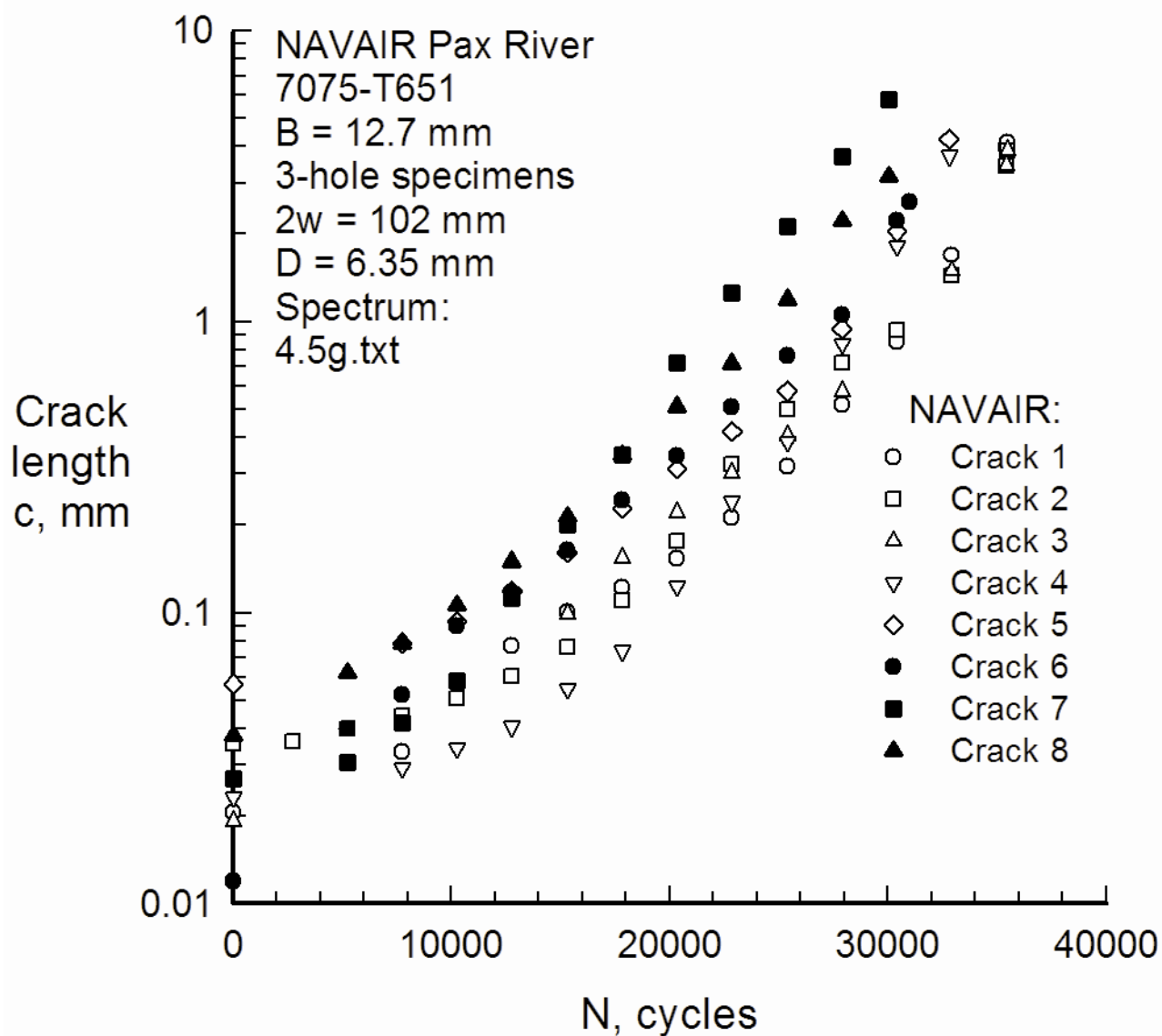
# MEASURED AND PREDICTED CRACK GROWTH UNDER SPECTRUM LOADING



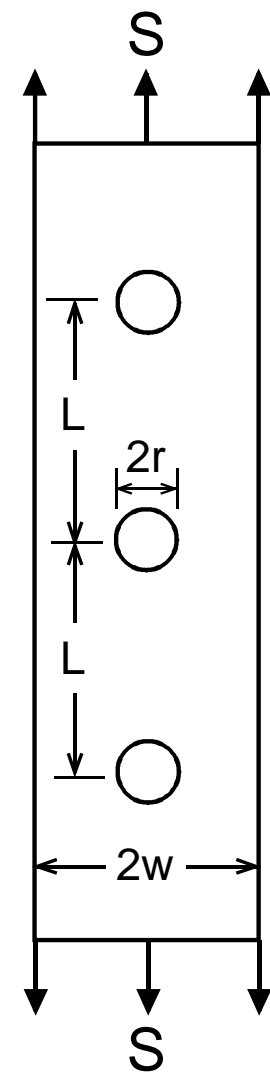
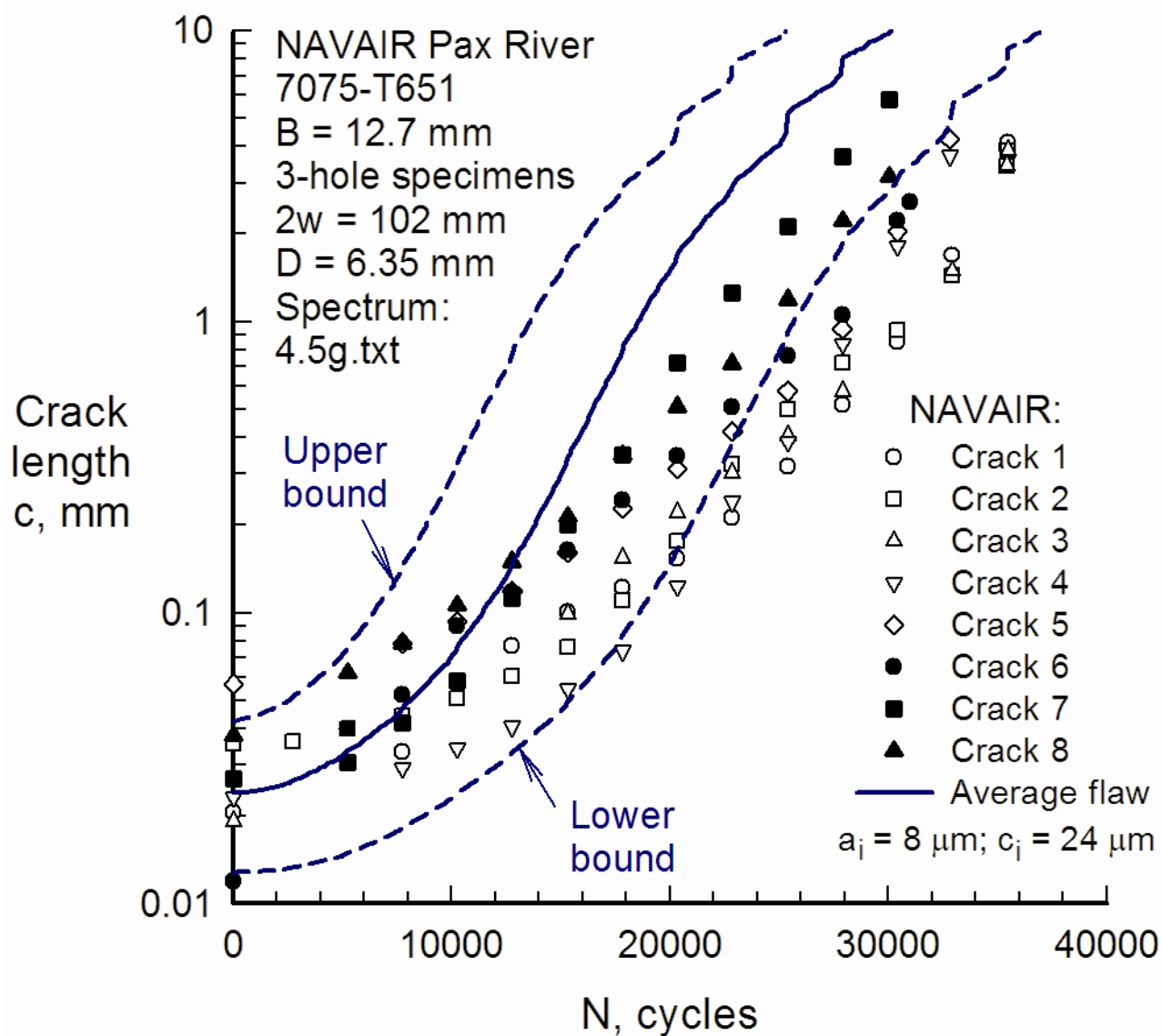
# INITIATION SITE IN NAVAIR THREE-HOLE COUPON



# MEASURED CRACK GROWTH UNDER SPECTRUM LOADING ON NAVAIR TESTS



# MEASURED AND PREDICTED CRACK GROWTH UNDER SPECTRUM LOADING ON NAVAIR TESTS



# OUTLINE OF PRESENTATION

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- Brief History on Fatigue-Crack Growth
- Plasticity-Induced Crack-Closure Model
- Crack Initiation and Small-Crack Behavior
- **Fatigue-Crack Growth and Fracture**
- **Concluding Remarks**

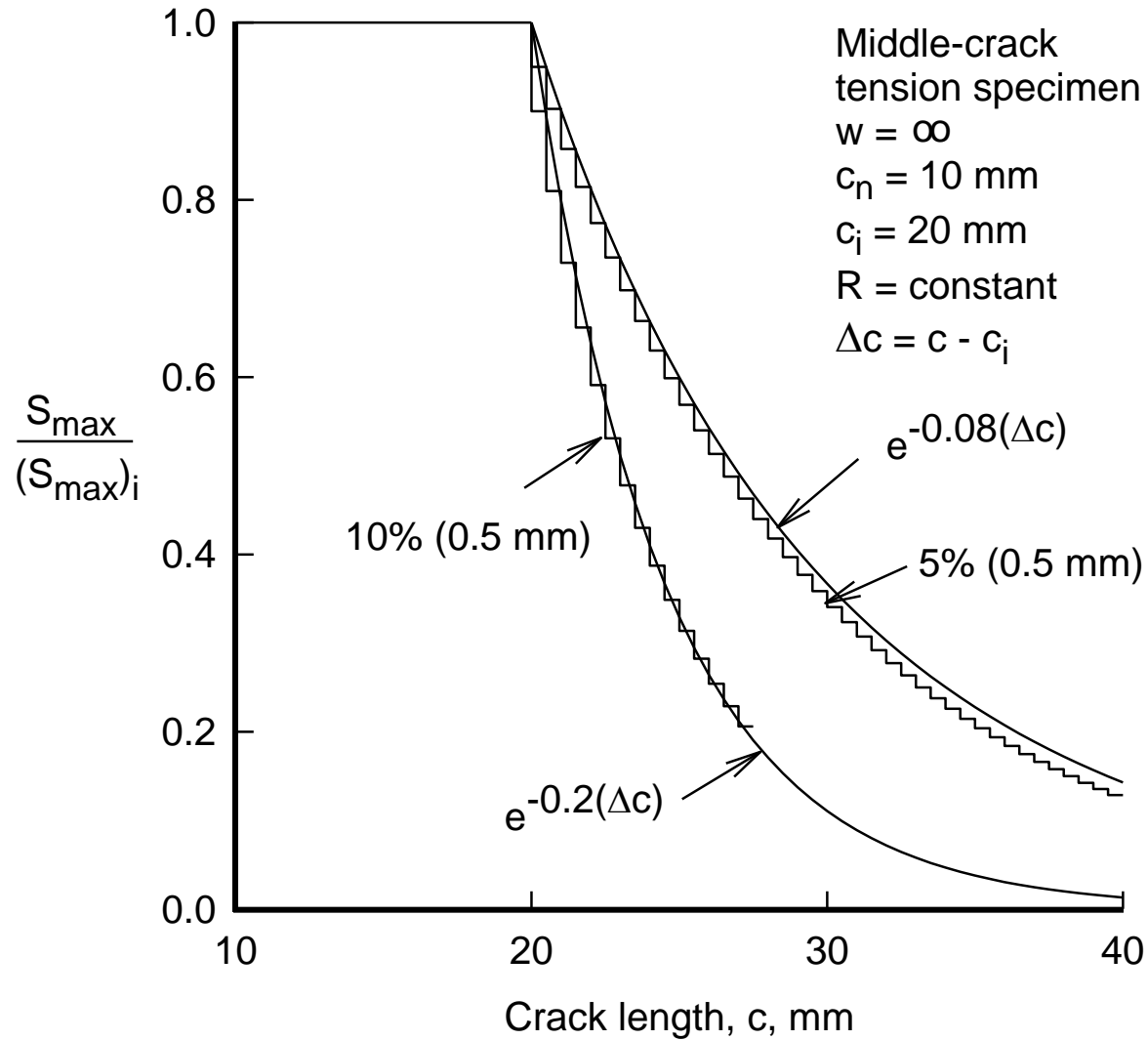


# FATIGUE CRACK GROWTH

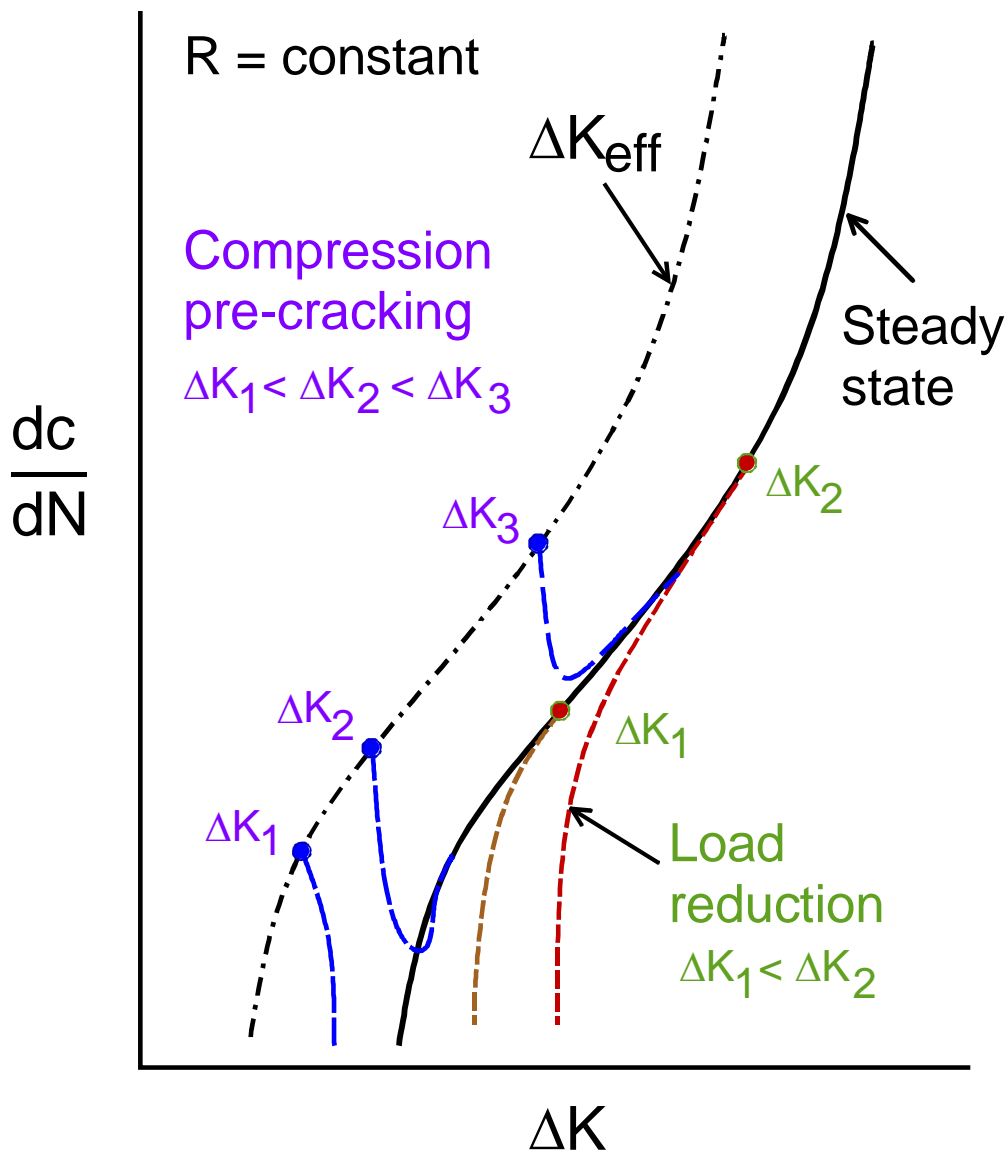
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- **Thresholds for large cracks**
- **Cold-worked hole effects**
- **Spectrum loading effects**

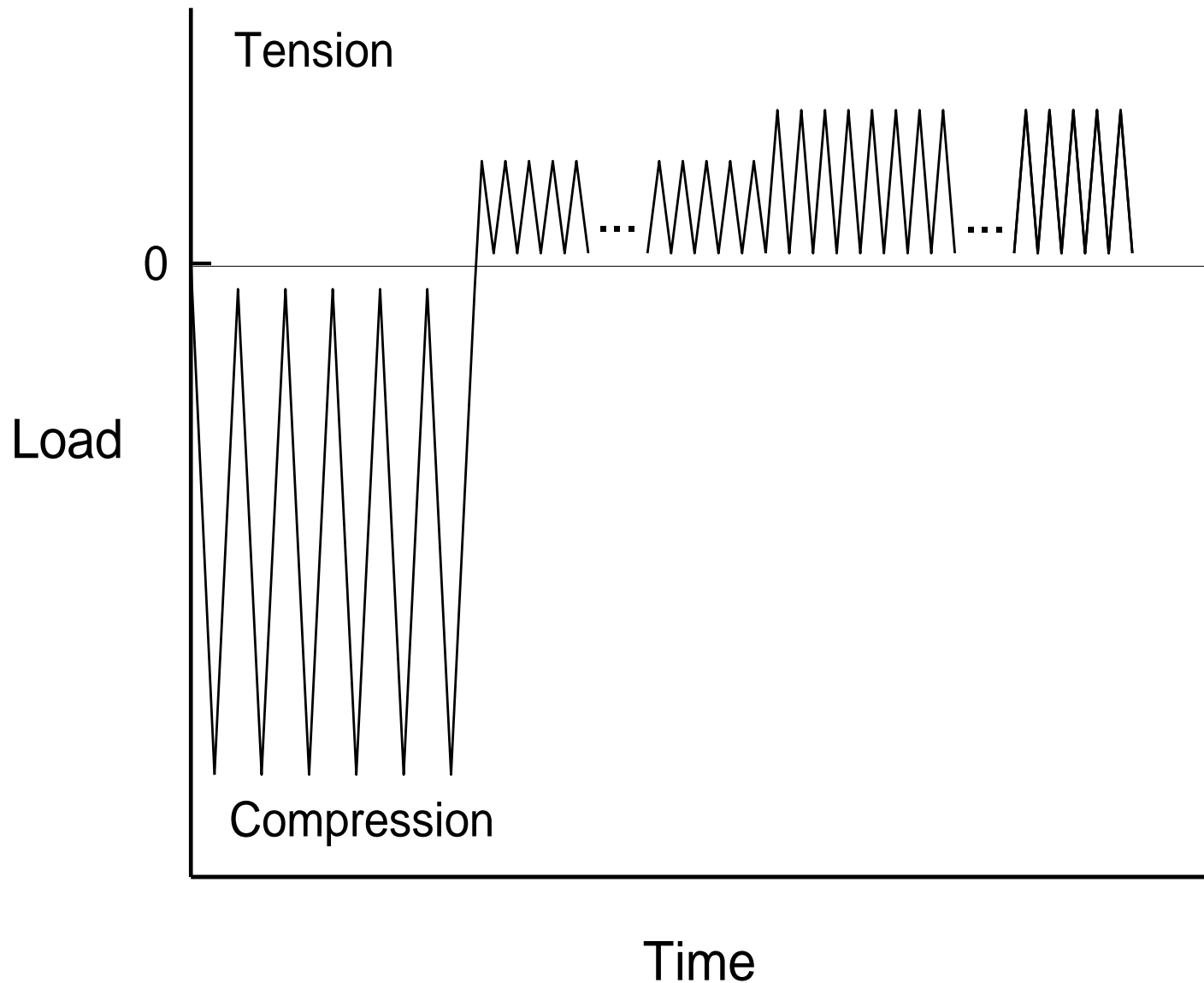
# ASTM LOAD-REDUCTION PROCEDURES



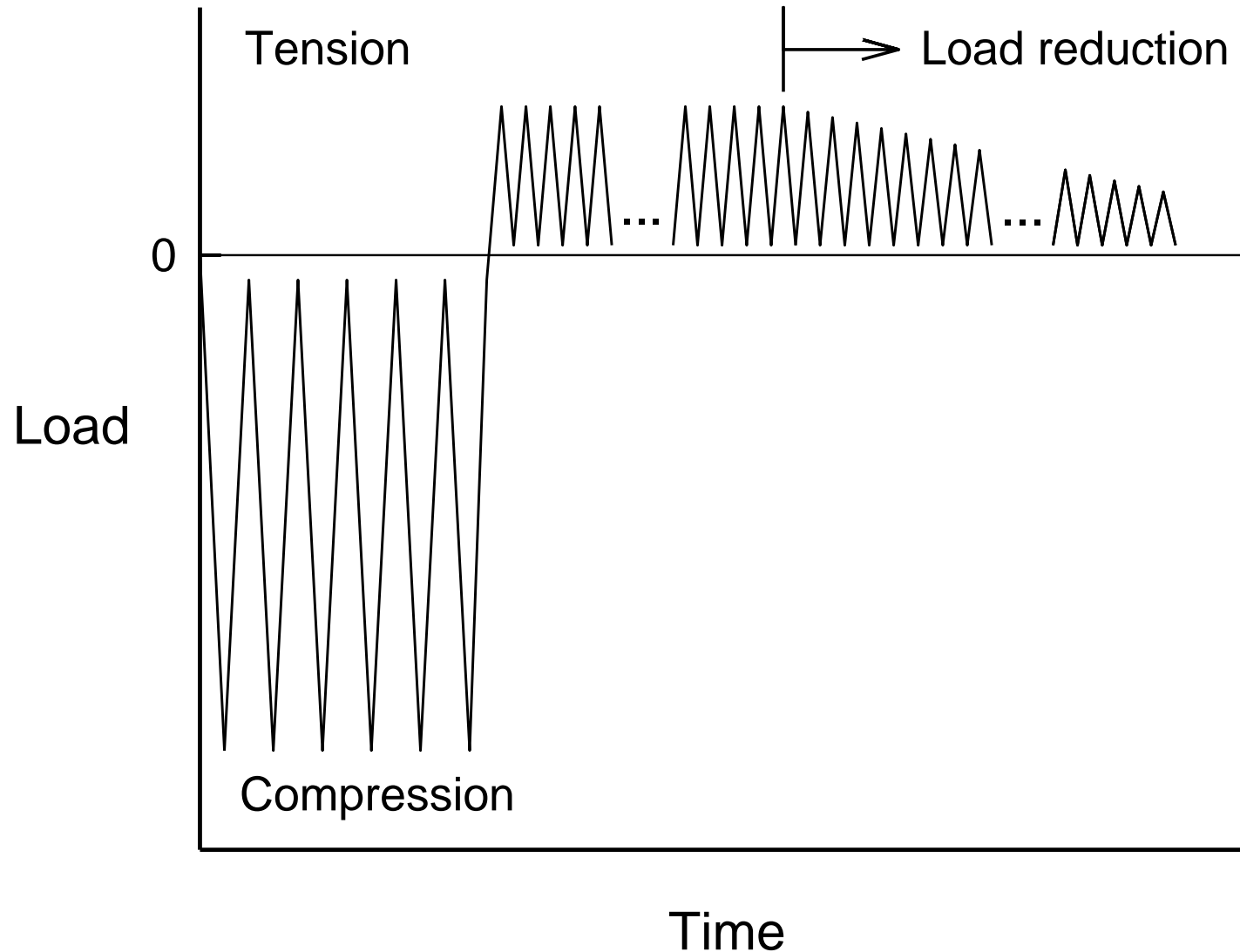
# TYPICAL BEHAVIOR FOR LOAD-REDUCTION AND COMPRESSION PRE-CRACKING PRE-CRACKING THRESHOLD TESTING



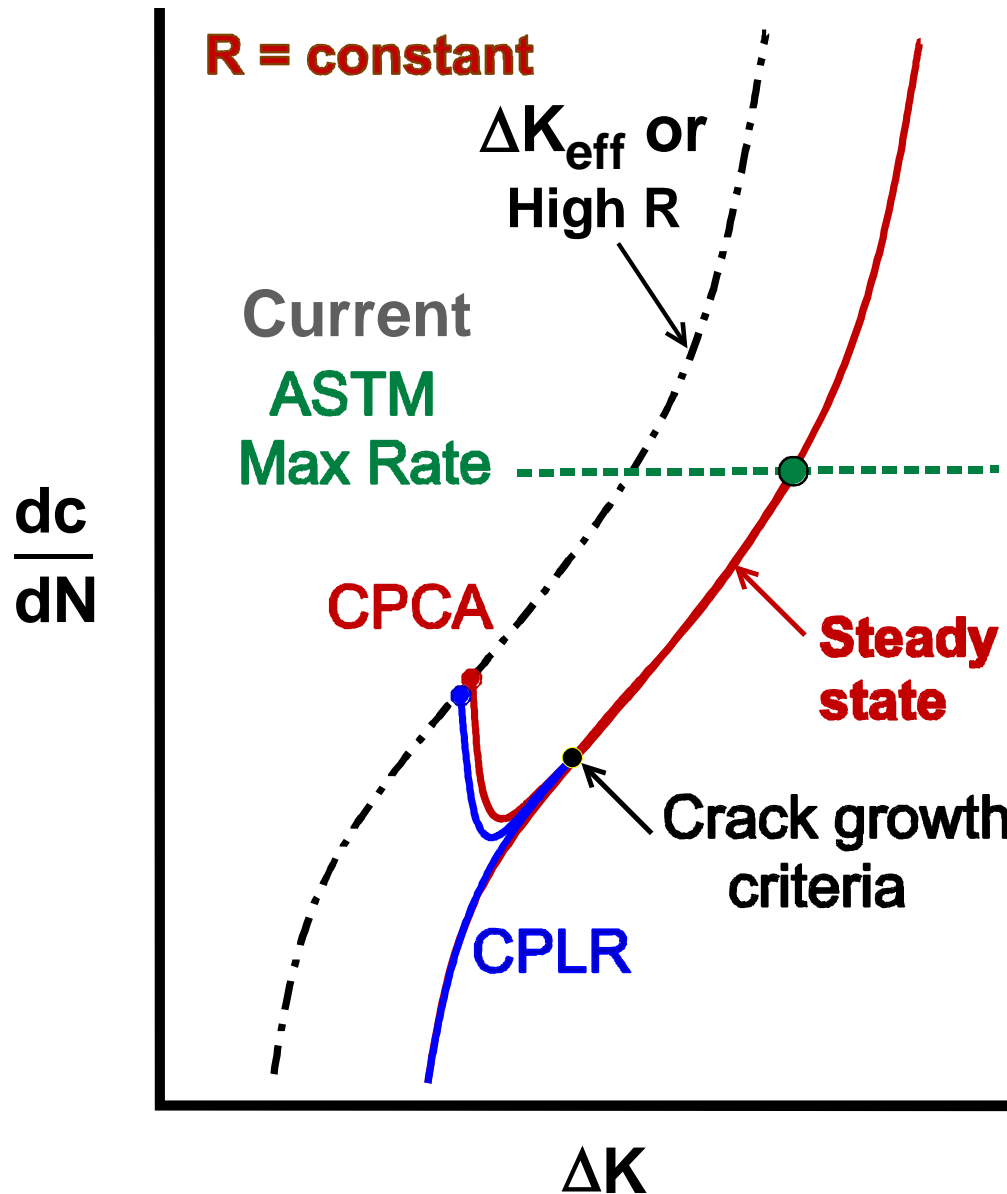
# COMPRESSION-COMPRESSION PRECRACKING AND CONSTANT-AMPLITUDE (CPCA) LOAD TESTING



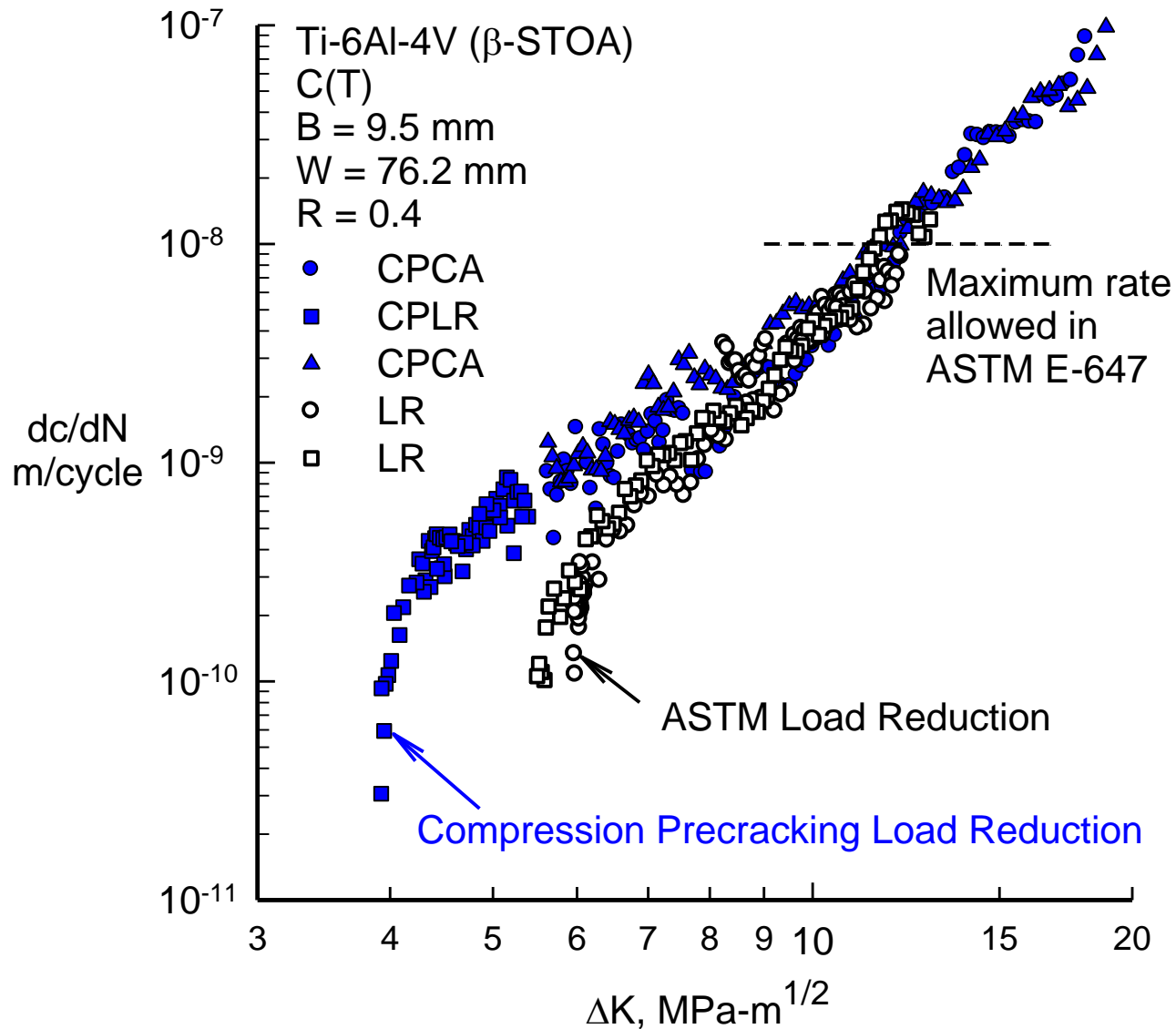
# COMPRESSION-COMPRESION PRECRACKING AND LOAD-REDUCTION (CPLR) THRESHOLD TESTING



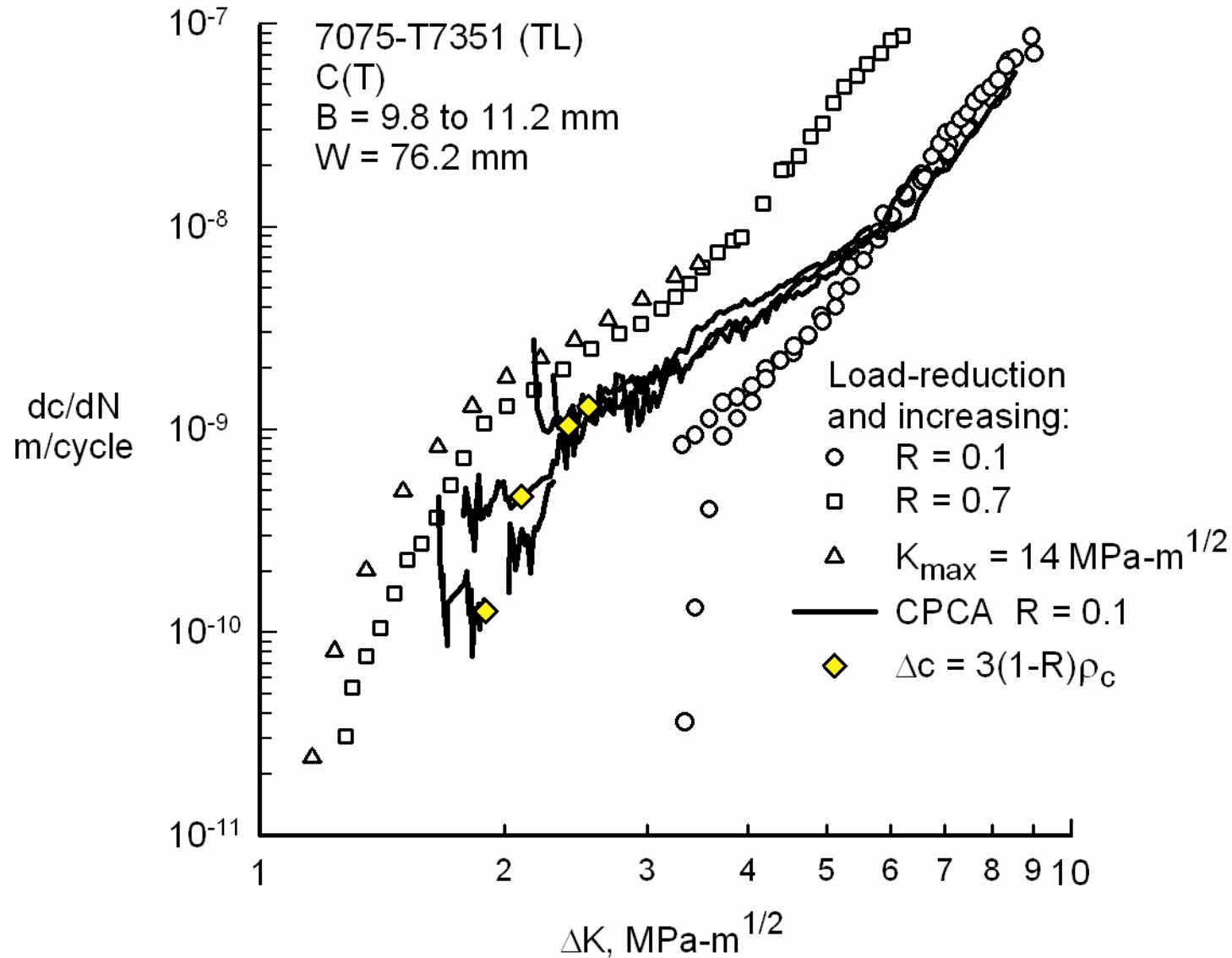
# METHODS TO GENERATE STEADY-STATE DATA



# CPCA AND LOAD-REDUCTION TESTING AT MEDIUM STRESS RATIO ON TITANIUM $\beta$ -STOA ALLOY

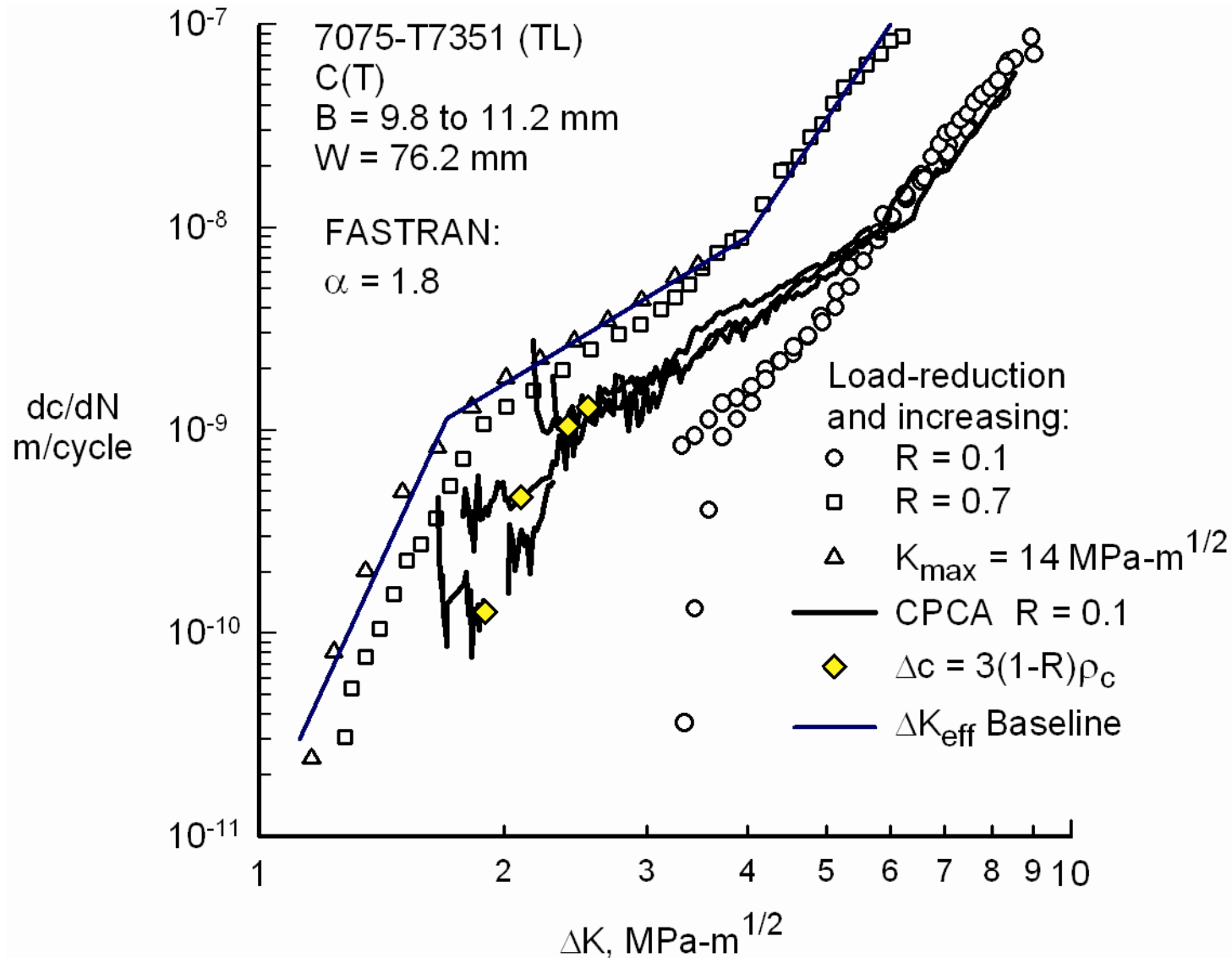


# CPCA AND LOAD-REDUCTION THRESHOLD TESTING ON 7075-T7351 AT R = 0.1 CONDITIONS

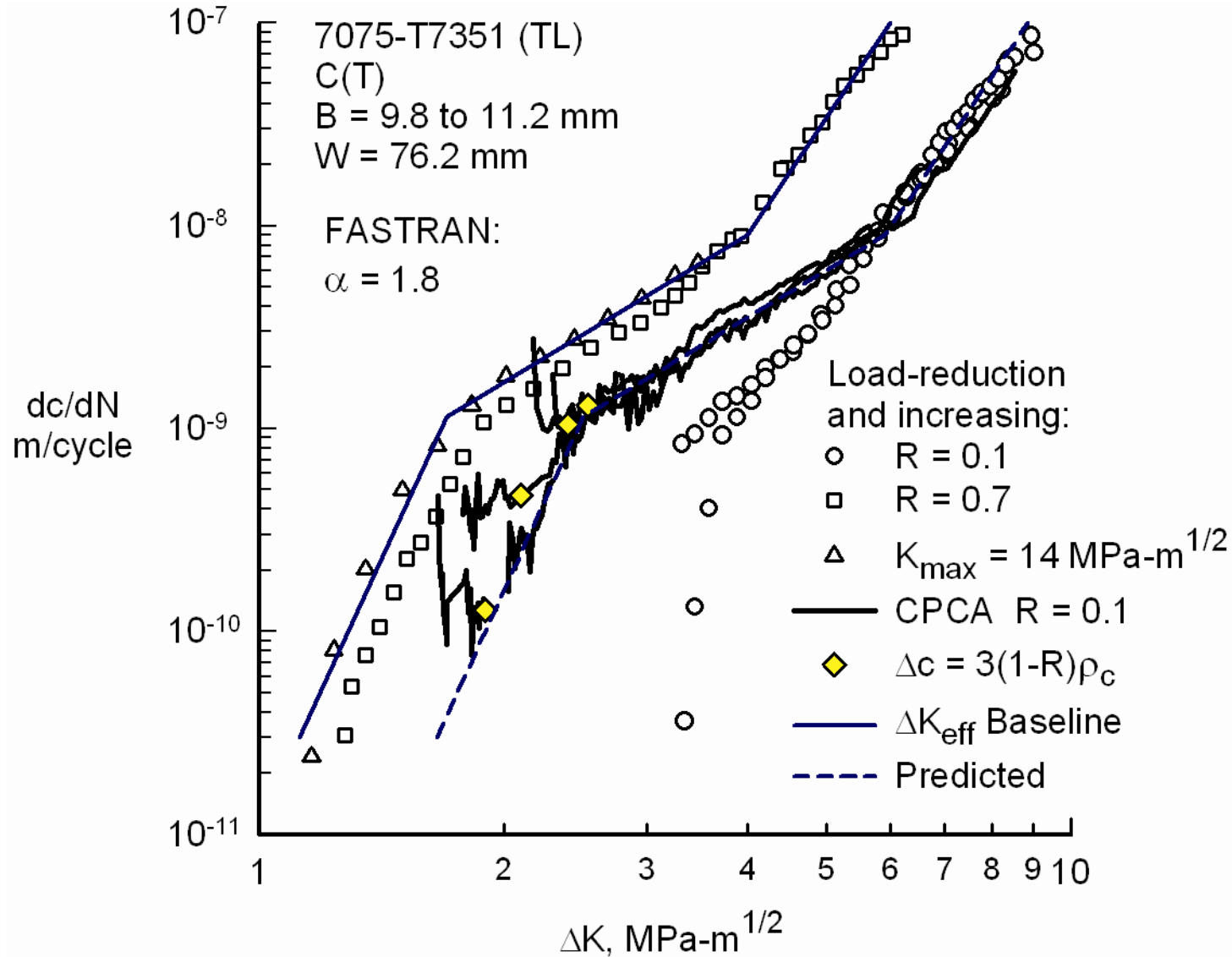




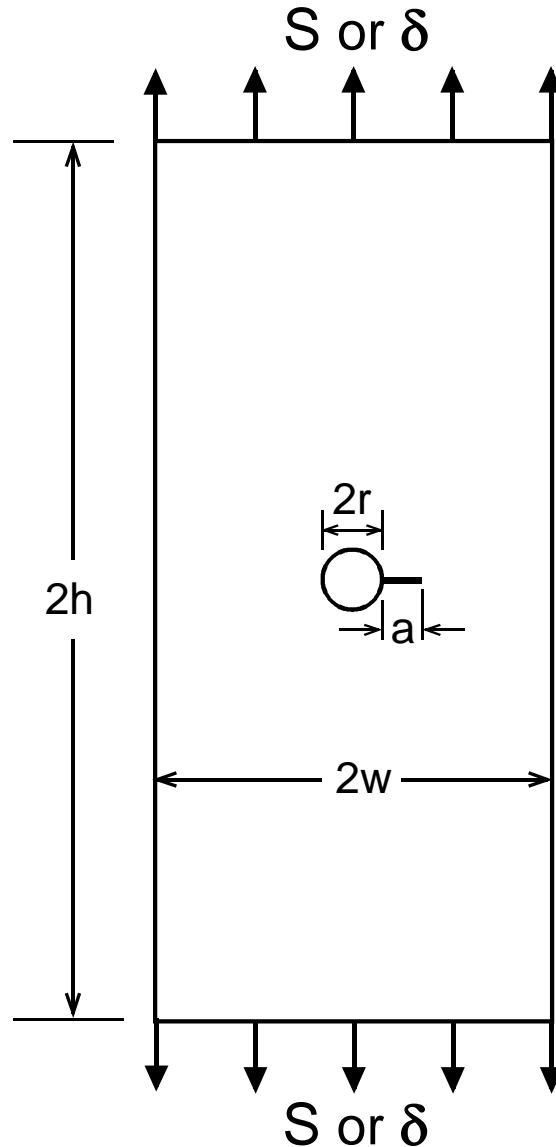
# CPCA AND LOAD-REDUCTION THRESHOLD TESTING ON 7075-T7351 AT R = 0.1 CONDITIONS



# CPCA AND LOAD-REDUCTION THRESHOLD TESTING ON 7075-T7351 AT R = 0.4 CONDITIONS



# COLD-WORKED HOLE TEST SPECIMEN



Test procedure:

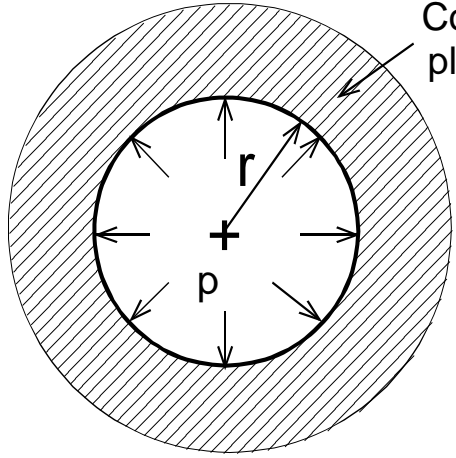
Sikorsky:

- (1) Holes drilled
- (2) Cold-worked (4.5%)
- (3) Reamed
- (4) EDM Notched

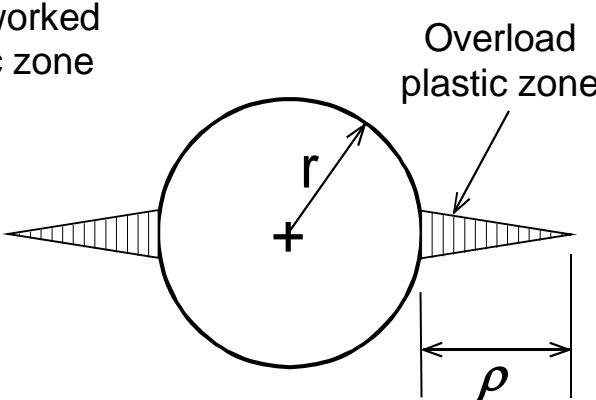
Tested at MSU:

- (5) Constant-amplitude loading

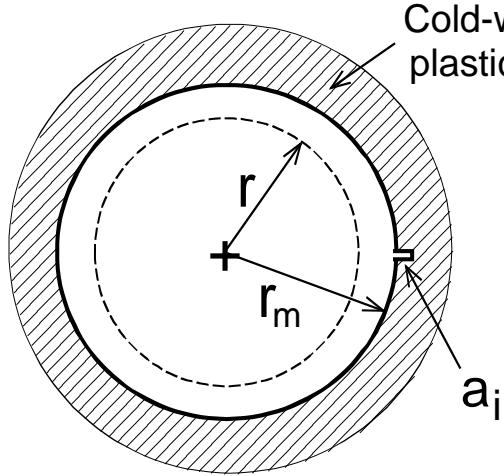
# SIMULATION OF COLD-WORKING AND NOTCHING



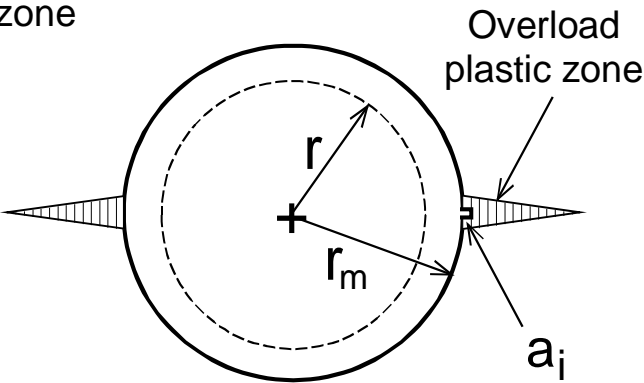
(a) Cold-worked hole



(b) Simulated cold-worked hole

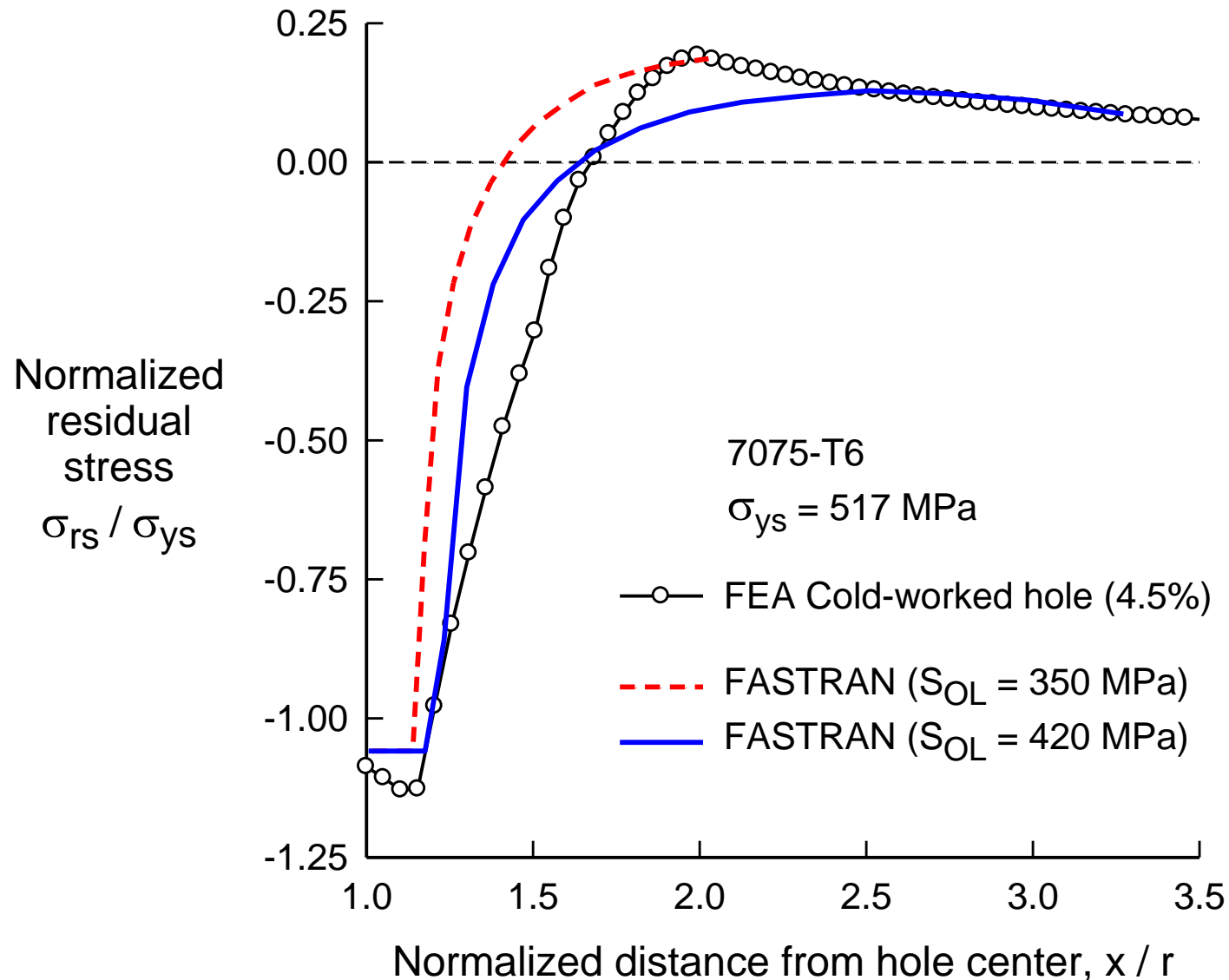


(c) Cold-worked hole after reaming and cutting notch

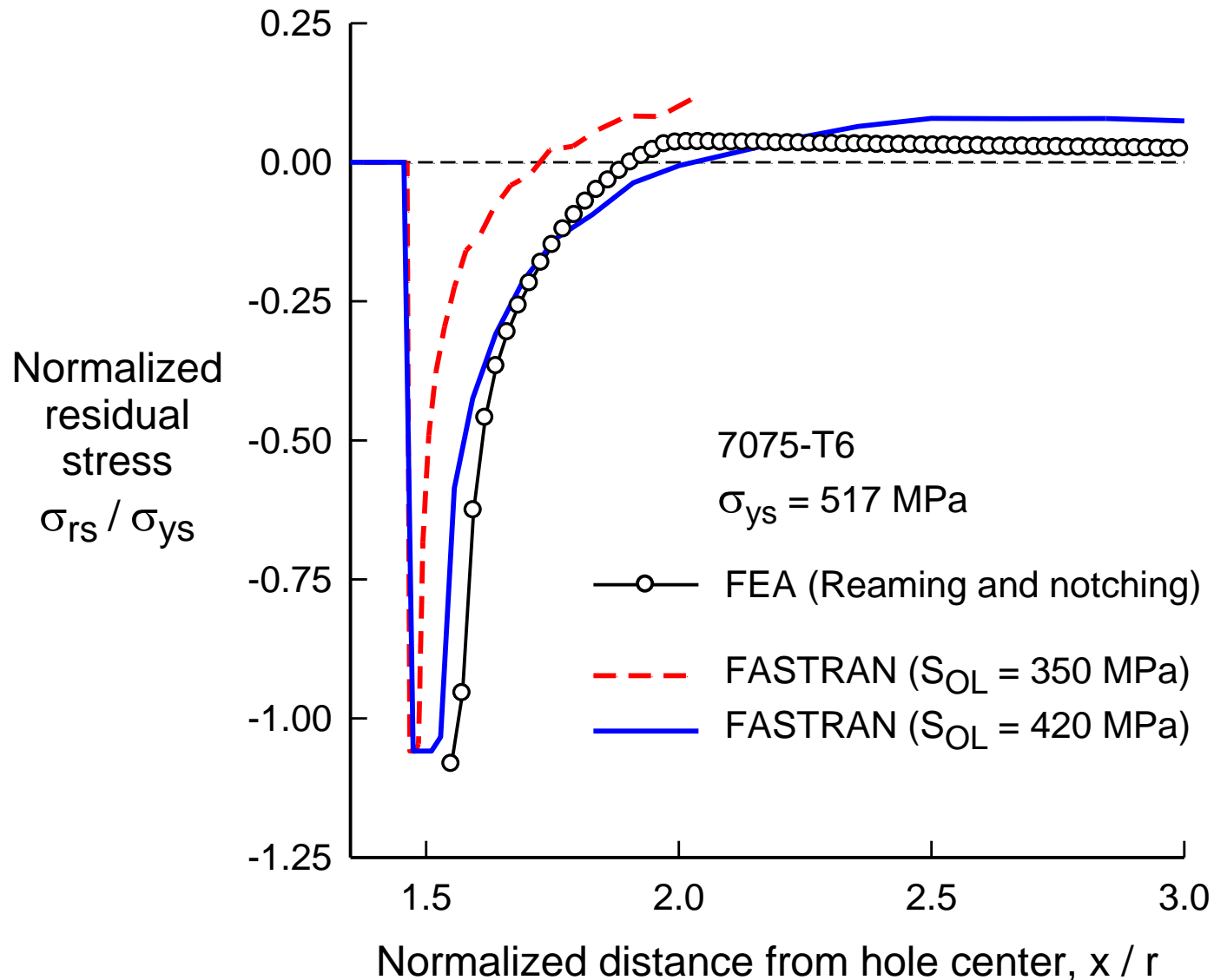


(d) Simulated cold-worked hole after reaming and cutting notch

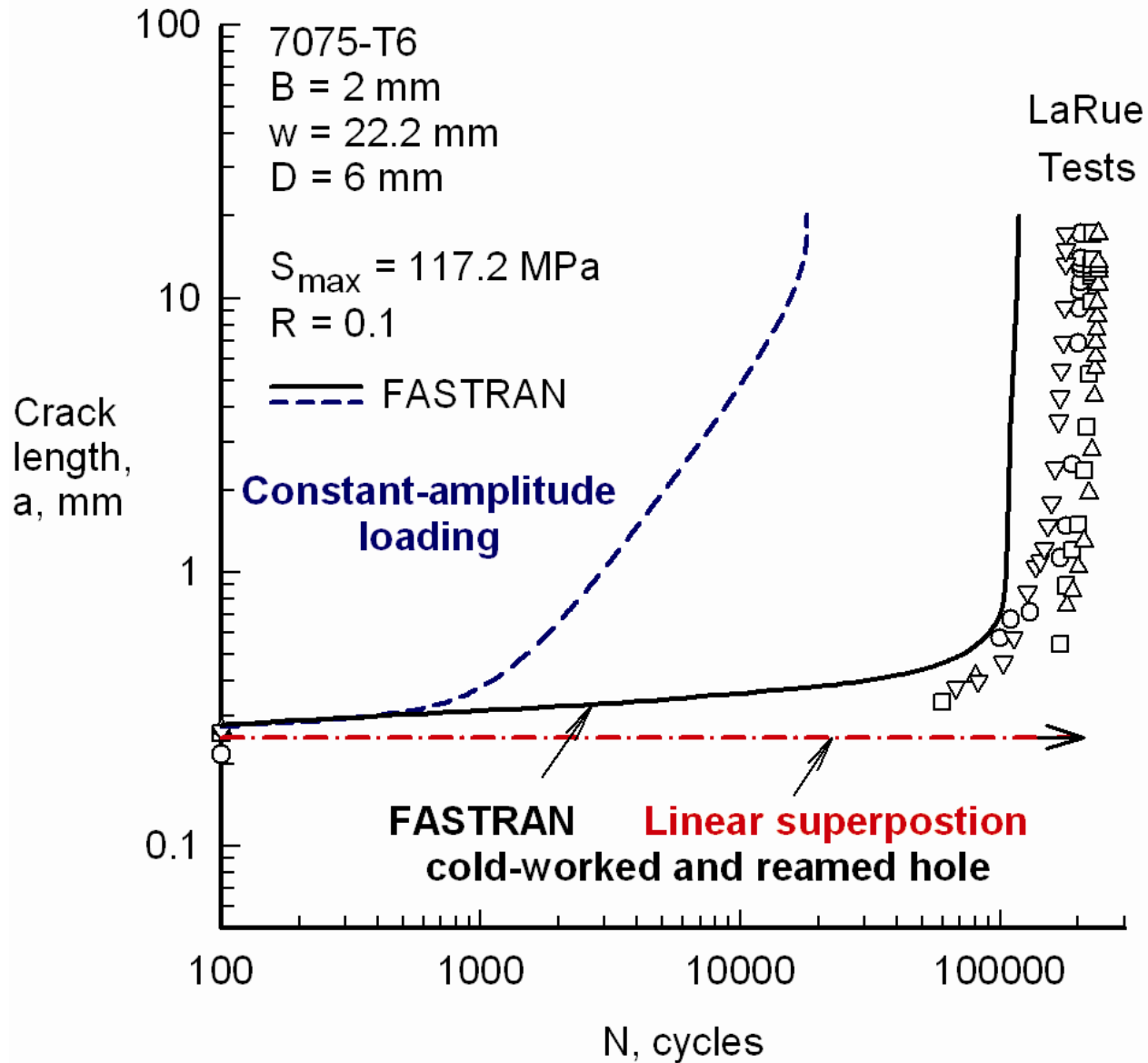
# RESIDUAL STRESSES AFTER COLD-WORKING



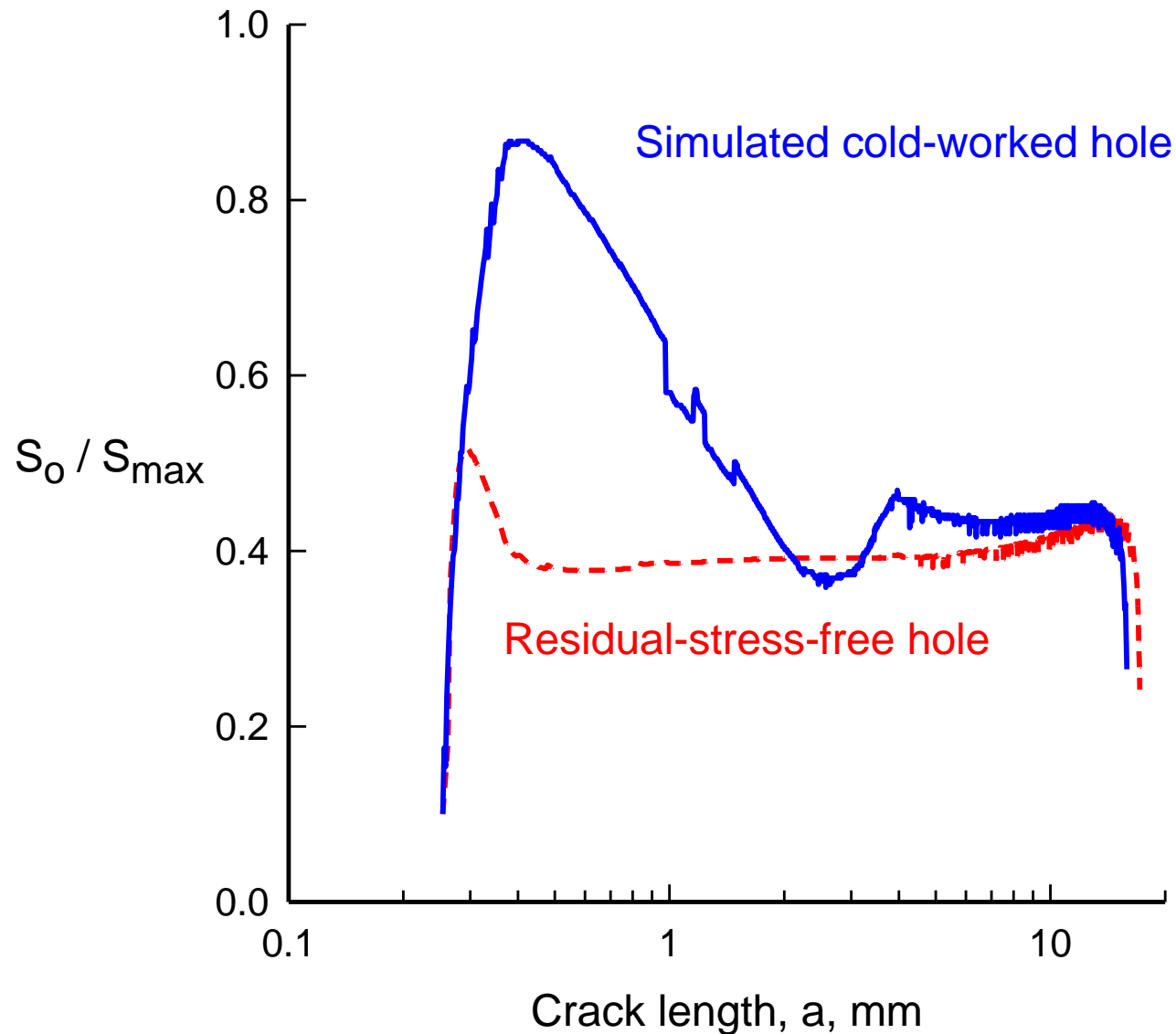
# RESIDUAL STRESSES AFTER COLD-WORKING, REAMING AND NOTCHING



# MEASURED AND PREDICTED CRACKING BEHAVIOR ON COLD-WORKING HOLE SPECIMENS

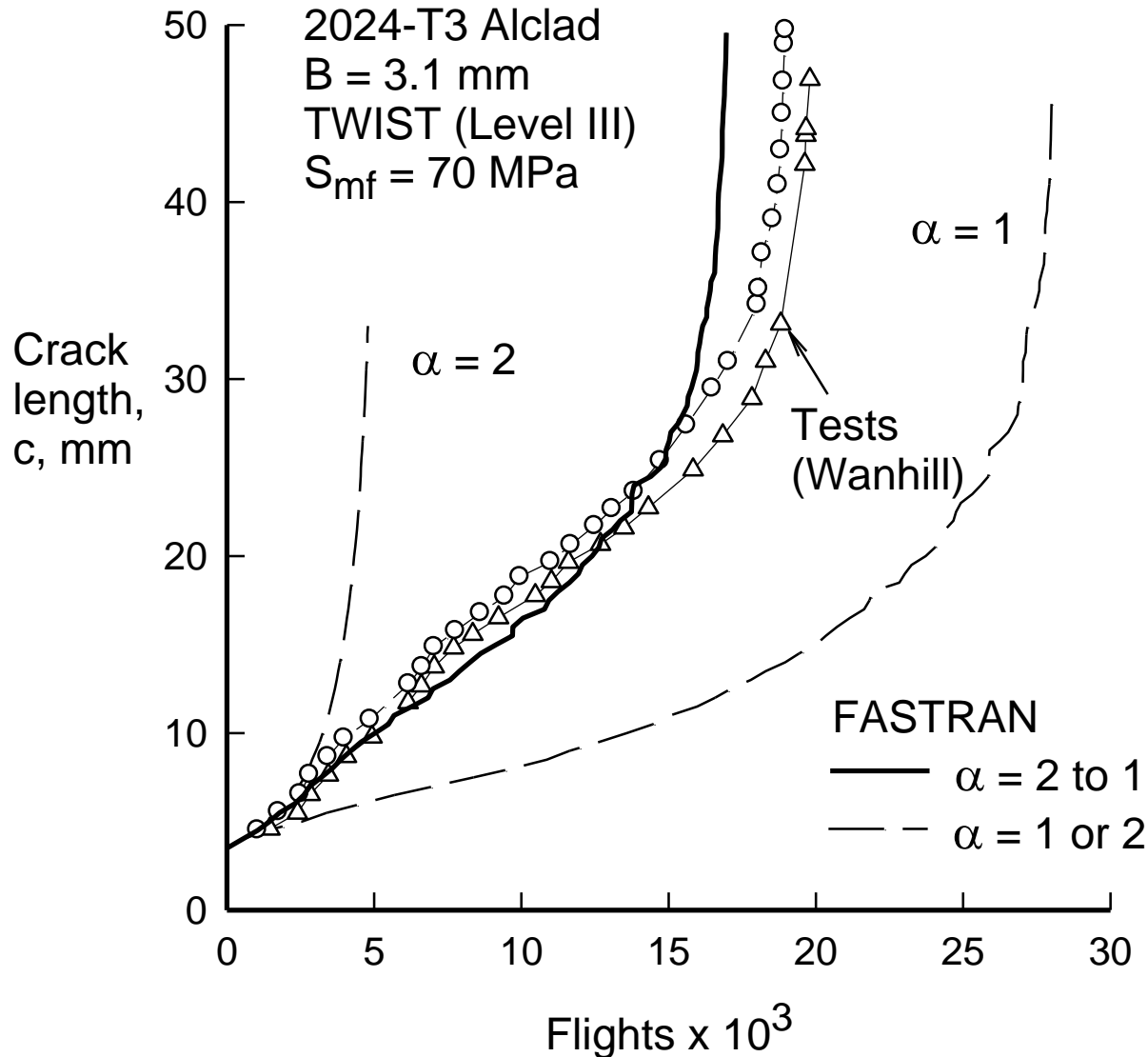


# CALCULATED CRACK-OPENING STRESSES FOR CRACK GROWTH WITH OR WITHOUT COLD-WORKED HOLE

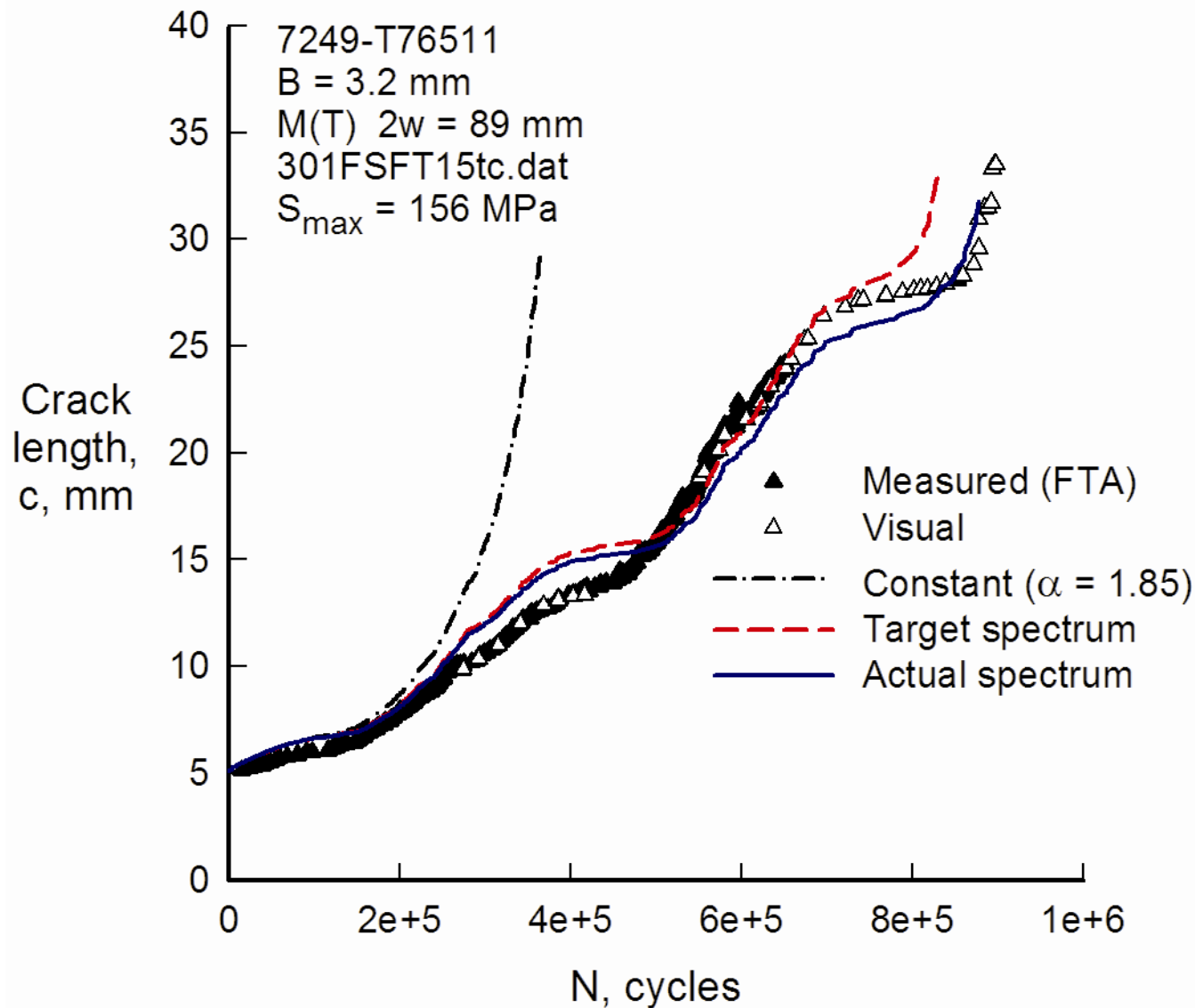




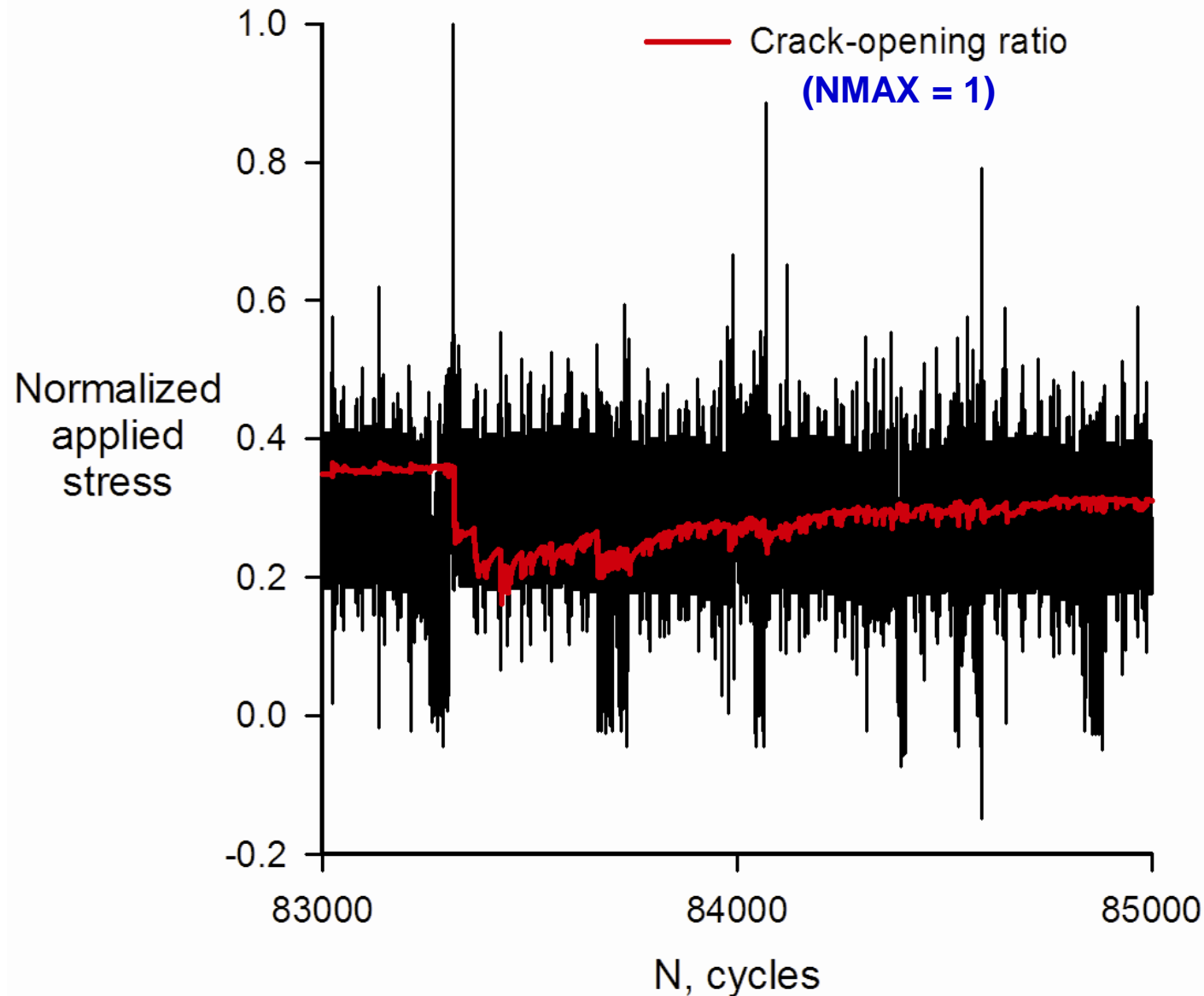
# MEASURED AND PREDICTED CRACK-GROWTH UNDER TWIST SPECTRUM LOADING



# Measured and Calculated Crack-Length-Against-Cycles for Modified FSFT Spectrum Loading



# Typical Crack-Opening Stress Calculations for P-3C Modified FSFT Spectrum Loading



# CONCLUDING REMARKS

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- FASTRAN is an advanced life-prediction code, which accounts for the effects of plasticity on fatigue-crack growth behavior in metallic materials for a variety of crack configurations and loading conditions.
- “Fatigue” is crack propagation from micro-structural features for many engineering materials and fatigue lives can be predicted with small-crack theory.
- Fatigue-crack growth can be predicted reasonably well under aircraft spectrum loading with the plasticity-induced crack-closure concept.
- Constraint effects and non-linear fracture mechanics parameters are keys to improving life-prediction models.
- Fatigue-crack growth-rate data in the near threshold regime should be obtained with no load-history effects.

# Future FASTRAN Modifications

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- Incorporate **T-stress** in the evolution of plastic deformation around crack fronts (bending crack configurations have +T stresses, while tension-loaded configurations have –T stresses)
- Include **roughness- and debris-induced** crack-closure behavior in the model
- Development of **plasticity, creep, and relaxation behavior** in time-dependent crack growth during load-time-temperature cyclic histories (creep brittle and creep ductile materials)