EXPERIMENTAL VERIFICATION OF STRESS INTENSITY FACTOR SOLUTIONS FOR CORNER CRACKS AT A HOLE SUBJECT TO GENERAL LOADING

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Stress intensity factor ($K$) solutions for unsymmetric corner cracks at a straight shank hole subject to tension, bending, and bearing have been calculated. Fatigue cracks of this type are often found in service aircraft; therefore, the need for these $K$ solutions is paramount. All structurally significant crack shapes were considered; crack depth to crack length ratios ($a/c$) of 0.1 to 10.0, crack depth to sheet thickness ratios ($a/t$) of 0.1 to 0.99, and hole radius to sheet thickness ratios ($r/t$) of 0.1 to 10.0. Although the new $K$ solutions were calculated with error control (0.1% in $K$), an experimental program was completed to assess the accuracy of the new solutions in predicting fatigue life and crack shape development. Specimens were made from 7075-T651 bare aluminum plate with a centrally located hole and two diametrically opposed electro-discharge machined initial flaws at the hole edge. A marker load spectrum was used to aid post-test crack history reconstruction. Predictions of fatigue life and crack shape are conservative when compared to the experimental results. As a result, a new fatigue crack growth analysis procedure is suggested which considers propagating the crack at 32 points along the crack front.

INTRODUCTION

Mechanically fastened joints, riveted or bolted, offer many options to the designer and have been used for centuries to join structural members. For statically loaded structure, as a minimum a good joint design ensures the stress concentration at the fastener hole edge does not exceed the yield strength of the material. This is also the case for dynamically loaded structure; however, now the designer must also satisfy fatigue requirements. The fatigue problem can be attacked in two ways; one, use stress levels below the endurance limit of the material thus cracks never nucleate; two, design the structure such that the slow crack growth life of the component is greater than its design service goal plus some factor of safety. For primary aircraft structure, excluding landing gear, the second approach is preferred.

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and directed by civilian airworthiness authorities and military specifications. Assuming the simplest crack growth model, Paris Eqn. (1), the designer can meet the fatigue life requirements by choosing a material with good fatigue properties and designing to keep $\Delta K$ in an acceptable range.

$$
\frac{da}{dN} = C(\Delta K_i^{(a)}(a,c))^n
$$

$$
\frac{dc}{dN} = C(\Delta K_i^{(c)}(a,c))^n
$$

(1)

Where $da/dN$, $dc/dN$ are the crack growth rates, $\Delta K_i^{(a)}(a,c), \Delta K_i^{(c)}(a,c)$ are the stress intensity factors in the a- and c-direction; respectively, and $C$ and $n$ are material constants.

The focus of this research effort was to accurately calculate $K$ to add fidelity to the fatigue life predictions of mechanically fastened joints. Specifically, $K$ solutions were calculated for diametrically opposed unsymmetric corner cracks at a centrally located hole in a finite width sheet subject to tension, bending, and bearing as shown in Figure 1. Numerous combinations of $a/c$, $a/t$, and $r/t$ are analyzed at each side of the hole; specifically, crack depth to crack length ratios ($a/c$) of 0.1 - 10.0, crack depth to sheet thickness ratios ($a/t$) of 0.10 - 0.99, and hole radius to sheet thickness ratios ($r/t$) of 0.1 - 10.0. The 5,672,700 $K$ solutions were developed with control of the error. The relative error is generally much smaller than 0.1% along the entire crack front up to the vertices.

**BACKGROUND**

In-service [1], full-scale fatigue test [2], and laboratory component/coupon [3] fatigue test evidence indicates cracks in riveted joints commonly nucleate as corner cracks at the intersection of the faying surface and rivet hole bore. A closed form solution for this stress intensity factor is not known; thus, all investigators have used engineering estimates or numerical methods. Engineering estimates for stress intensity factors for some corner-crack configurations have been made by Hall and Finger [4], Liu [5], and Newman [6]. These investigators did not consider the variation of stress intensity factor along the crack front. Rather, their estimates gave a single value of the stress intensity factor for each crack configuration; therefore, their estimates might be considered only as an average stress intensity factor value along the crack front. Shah [7] used the alternating method, along with an engineering estimate, to calculate the stress intensity factor variation along the crack front for the corner-crack configuration subjected to either remote tension or pin loading in the hole. The pursuit of a numerical estimate for $K$ began over twenty years ago with Smith and Kullgren [8] using the finite element alternating method.
while Kathiresan [9] and Heckmer and Bloom [10] as well as Raju and Newman [11] using the finite element method. Nishioka and Atluri [12, 13] analyzed corner cracks from a circular hole and corner cracks in a lug by the finite-element-alternating method. Stress intensity factors by all of the previously mentioned numerical methods agreed well with one another, except in the region where the crack intersected the hole boundary. Here the stress intensity factors from [11] showed a precipitous "drop-off." The stress intensity factors calculated by other investigators [8, 12] did not show the large drop-off near the hole-crack intersection. In 1990, Tan et al. [14] re-evaluated the surface crack stress intensity factors from the finite-element meshes used by Raju and Newman. They found some ill-shaped (large aspect ratios) elements located at the hole-crack intersection. These ill-shaped elements were also found in the corner crack finite element meshes.

Newman and Raju [15, 16] had previously developed stress intensity factor equations for the corner crack at a hole configuration that covered a wide range in crack size \((a/t)\) and crack shape \((a/c)\) for two hole radius to thickness ratios \((r/t = 0.5 \text{ and } 1.0)\) using the results from [11]. These equations were developed for one corner crack or two-symmetric corner cracks at a circular hole in a plate subjected to either remote tension or bending loads. The single corner crack equations used the Shah [7] one to two crack conversion factor. However, the equations were not fitted to the results showing the rapid drop-off at the hole-crack intersection. In this vertex region, engineering judgement was used. The equations have been found to be relatively accurate over the range of \(a/t\) and \(a/c\) ratios analyzed with the finite-element method. Engineering estimates were used to extrapolate the equations to large \(a/t\) (> 0.8) ratios and for intermediate \(a/c\) ratios of 0.2 to 2 for \(r/t\) ratios from 0.5 to 1.0.

An extensive summary of the numerical simulations to estimate \(K\) solutions for corner cracked holes has been compiled by Bakuckas [17]. The report documents \(K\) solutions calculated by various methods; conventional finite element method (FEM), finite element alternating method (FEAM), boundary element method (BEM), and three-dimensional weight function method (WFM). The \(K\)'s are extracted from the analysis results using indirect or direct methods; the former derives \(K\) from energy, usually the energy release rate and the latter derives \(K\) directly from the forces (force method) or displacements (crack opening displacement method) solution. A more complete discussion on indirect and direct methods for calculating \(K\) can be found in references [18] and [19].

Recently, Fawaz and Andersson [20] using the \(p\)-version finite-element method have generated accurate stress intensity factors for corner crack(s) at the edge of a circular hole subjected to remote tension, bending and simulated pin loading for an \(r/t\) ratio of 1.0. The solution space of these latest results have been extended and experimentally verified in the current effort.
The \textit{p-version} finite element code, STRIPE [21], developed by the Aeronautical Research Institute of Sweden (now known as the Aeronautics Division of the Swedish Defense Research Agency) is used in the current investigation. Recall, in \textit{p-version} FEA the number of elements is kept fixed and convergence is obtained by increasing the order of approximation, \( p \), of the shape functions within each element; whereas, in the \textit{h-version}, convergence is obtained by increasing the number of elements. In STRIPE, a combination of the \textit{h-} and \textit{p-version} FEA is used to obtain exponential convergence of the stress intensity factor solution by simultaneously decreasing the element size (increasing the total number of elements in the model) and increasing the \( p \)-level. At present, the maximum \( p \)-level available in STRIPE is 15. For a uniform order of approximation, the total number of degrees of freedom per element is [22]

\[
\text{DoF} = \frac{(p+1)(p+2)(p+3)}{6} + 3(p+1)
\]  

(2)

where \( \text{DoF} \) is the total degrees of freedom in the model.

For cracks in bodies of finite dimension, the crack front usually intersects a geometric boundary. The point of intersection is called the \textit{vertex}. The \( K \)-variation is very complicated near a vertex for crack fronts intersecting perpendicular to a stress free surface (as in our case). In fact, the gradient along the crack front in \( K \) is infinitely large at the vertex and hence difficult to capture in a numerical analysis. This problem in our case is completely resolved by employing a strongly graded mesh not only in the neighborhood of the crack front, but also along the crack front towards the two vertices.

\textbf{MODELING THEORY AND IMPLEMENTATION}

\textbf{K-EXTRACTION FROM FEA RESULTS}

We briefly review our method for reliable extraction of stress intensity factors for general smooth edges in 3D domains. Consider a smooth edge, \( \gamma \), in Figure 2. The angle \( \omega \) is assumed to be constant. In the present paper, we are interested in the special case when \( \omega = 2\pi \) and \( \gamma \) has a part-elliptical shape. The distance (short) from the edge to a point is denoted by \( r \), and \( \theta \) \((-\omega \leq \theta \leq \omega /2)\) is the polar angle. The displacements, \( \mathbf{u} \) can be written in the form (excluding exceptional angles \( \omega \)),

\[
\mathbf{u}(r,\theta,x_3) = \sum_{a=I,II,III} K_a(x_3) r^{\omega} \Psi_a(\theta) + \text{smoother terms}
\]  

(3)

where \( K_I(x_3) \), \( K_{II}(x_3) \), and \( K_{III}(x_3) \), are the mode I, mode II, and mode III stress intensity functions, where for example \( K_I \) is defined by,
\[ K_f(x_3) = \lim_{r \to 0} \sqrt{2\pi r^{(1-\lambda)}} \sigma_3(r,0,x_3) \]  

(4)

where \( \sigma_3(r, \theta, x_3) \) is the normal stress in the local (1,2,3) system. The exponents \( \lambda_\alpha \) \( (\alpha = I, II, \text{ or } III \text{ in the following}) \) depend only on the local geometry and local boundary conditions (faces are traction free in our case). The functions \( \Psi_\alpha \) in Eqn. (3) for isotropic materials are,

\[
\Psi_\alpha(\theta, \lambda_\alpha) = \begin{bmatrix}
\Psi_{a1}(\theta, \lambda_\alpha) \\
\Psi_{a2}(\theta, \lambda_\alpha) \\
\Psi_{a3}(\theta, \lambda_\alpha)
\end{bmatrix}
\]

(5)
or

\[
\Psi_I(\theta, \lambda_I) = \beta_I \begin{bmatrix}
(\kappa - Q_I(\lambda_I + 1)) \cos(\lambda_I \theta) - \lambda_I \cos((\lambda_I - 2)\theta) \\
0 \\
(\kappa + Q_I(\lambda_I + 1)) \sin(\lambda_I \theta) + \lambda_I \sin((\lambda_I - 2)\theta)
\end{bmatrix}
\]

(6)

\[
\Psi_{II}(\theta, \lambda_{II}) = \beta_{II} \begin{bmatrix}
(\kappa - Q_{II}(\lambda_{II} + 1)) \sin(\lambda_{II} \theta) - \lambda_{II} \sin((\lambda_{II} - 2)\theta) \\
0 \\
-((\kappa + Q_{II}(\lambda_{II} + 1)) \cos(\lambda_{II} \theta) + \lambda_{II} \cos((\lambda_{II} - 2)\theta)
\end{bmatrix}
\]

(7)

\[
\Psi_{III}(\theta, \lambda_{III}) = \beta_{III} \begin{bmatrix}
0 \\
\sin(\lambda_{III} \theta) \\
0
\end{bmatrix}
\]

(8)

where \( \kappa = 3 - 4 \) is Poisson’s ratio, \( G \) the shear modulus, and

\[ Q_I = -\left( \frac{\lambda_I - 1}{\lambda_I + 1} \right) \frac{\sin \left( \frac{\omega(\lambda_I - 1)}{2} \right)}{\sin \left( \frac{\omega(\lambda_I + 1)}{2} \right)} \]

\[ Q_{II} = -\frac{\sin \left( \frac{\omega(\lambda_{II} - 1)}{2} \right)}{\sin \left( \frac{\omega(\lambda_{II} + 1)}{2} \right)} \]

\[ \beta_I = \frac{2G}{\sqrt{2\pi \lambda_I (\lambda_I + 1)(1 + Q_I)}} \]

(9)
\[ \beta_{II} = -\frac{2G}{\sqrt{2\pi \lambda_{II}}((\lambda_{II} - 1) + Q_{II}(\lambda_{II} + 1))} \]

\[ \beta_{III} = \frac{4G}{\sqrt{2\pi \lambda_{III}}} \]

Table 1 shows \( \lambda_{\alpha} \)-values for two technically important \( \omega \)-values.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \omega = 3\pi/2 )</th>
<th>( \omega = 2\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.54448374</td>
<td>0.50000000</td>
</tr>
<tr>
<td>II</td>
<td>0.90852919</td>
<td>0.50000000</td>
</tr>
<tr>
<td>III</td>
<td>0.66666667</td>
<td>0.50000000</td>
</tr>
</tbody>
</table>

For smooth edges, the edge intensity functions \( K_{\alpha}(x) \) are analytic on open intervals \( s_k \leq x_3 \leq s_{k+1} \). Hence, we approximate the edge intensity functions \( K_{\alpha} \) with the polynomials

\[ K_{\alpha}(x_3) = \sum_{n=0}^{p} \bar{k}_{\alpha n} P_n(s), \quad s = \frac{2(x_3 - s_k)}{s_{k+1} - s_k} - 1 \]  

where \( \bar{k}_{\alpha n} \) are unknown coefficients, \( p \) is the polynomial order of the finite element trial functions, and \( P_n \) the Legendre polynomials.

Figure 3 illustrates a domain \( \Omega^e \) used for extraction of the coefficients \( \bar{k}_{\alpha n} \). The extraction domain has three cylindrical surfaces with circular cross sections \( \rho_2 > \rho_1 > \varepsilon \) perpendicular to the edge considered. We denote by \( \Omega^e \), the cylinder with inner radius \( \varepsilon \) and outer radius \( \rho_2 \). The surface of \( \Omega^e \) is denoted \( \Gamma^e \). The two outer cylindrical surfaces are exactly modeled in the finite element analysis.

The coefficients \( \bar{k}_{\alpha n} \), we calculate by applying the Maxwell-Betti reciprocity theorem, Eqn. (11), \( p+1 \) times for each value of \( \alpha \)

\[ \int_{\Gamma^*} \left( u_i^{(e)} T_j^{(e)} - U_i^{(e)} t_j^{(e)} \right) d\Gamma = \int_{\Omega^*} \left( u_i^{(e)} X_j^{(e)} - U_i^{(e)} x_j^{(e)} \right) d\Omega - \int_{\Gamma^*} \left( u_i^{(e)} T_j^{(e)} - U_i^{(e)} t_j^{(e)} \right) d\Gamma \]  

In Eqn. (11), \( u_i^{(e)}, t_j^{(e)}, x_j^{(e)} \) are the displacements, tractions, and volume force density, respectively, for a loading system having identical edge stress.
intensity functions $K_\alpha$ as the original load system. The fields $U_i^{(n)}, T_i^{(n)}, X_i^{(n)},\quad 0 \leq n \leq p,$ are auxiliary solutions used to calculate the coefficients $\bar{k}_e$.

As auxiliary displacement solutions, we use

$$U_i^{(n)}(s) = f(r) r^{-\lambda_\alpha} \Psi_\alpha(\theta, -\hat{\lambda}_\alpha) P_n^\alpha(s)$$  \hspace{1cm} (12)

with $\Psi_\alpha$ from Eqn. (6) to (8) and $f(r)$ being the cut-off function

$$f(r) = \begin{cases} 
1-3 \left( \frac{r-\rho_1}{\rho_2-\rho_1} \right)^2 + 2 \left( \frac{r-\rho_1}{\rho_2-\rho_1} \right)^3 & \text{for } r \leq \rho_1 \\
2 & \text{for } \rho_1 < r < \rho_2 \\
0 & \text{for } r \geq \rho_2 
\end{cases}$$  \hspace{1cm} (13)

where $f(s_1) = 1, f(s_2) = 0, \frac{df}{dr}(s_1) = \frac{df}{dr}(s_2) = 0$.

The set of functions $U_i^{(n)}(s)$, which have a strong singularity at the edge, have desirable orthogonality properties with respect to $u_i^{(n)}$ and $u_i$ correspond to identical edge stress intensity functions $K_\alpha$.

Substituting $u_i$ from Eqn. (3) and $U_i^{(n)}$ from Eqn. (12) into the left hand side of Eqn. (11), and shrinking $\varepsilon \to 0$, the unknown coefficient $\bar{k}_e$ is obtained as

$$\bar{k}_e = C_n \left[ \int_{\Gamma} \left( u_i^{(n)} x_i^{(n)} - U_i^{(n)} x_i^{(n)} \right) d\Omega - \int_{\Gamma} \left( u_i^{(n)} T_i^{(n)} - U_i^{(n)} t_i^{(n)} \right) d\Gamma \right]$$  \hspace{1cm} (14)

In Eqn. (14), $C_n$ is a material dependent constant. Displacements obtained from the finite element solution are substituted for $u_i^{(n)}$ in Eqn. (14). The extraction functions $U_i^{(n)}$ and the corresponding tractions $T_i^{(n)}$ vanish on the outer surface (radius $\rho_2$) of the extraction domain $\Omega^e$ (Figure 3) because of the cylindrical cross-section and the cut-off functions Eqn. (13) used. The accuracy of the calculated stress intensity factors thus will mainly depend on a weighted average of the finite element solution inside the extraction domain. This gives the very fast convergence, with increasing $p$, to the exact solution.

**SOLUTION BEHAVIOR CLOSE TO VERTICES**

The stress intensity functions show a complex behavior near the two vertices where the crack fronts intersect the traction free surfaces. The solution behavior in these
regions is important since the maximum values of the stress intensity functions are to be found there. The mathematical theory for the vertex behavior is known in general form. A few details are given here from the theory and we exemplify that our numerical results are very accurate along the entire crack front, i.e. also arbitrarily close to the vertices and in agreement with the basic mathematical findings. An analytical expression, derived from the mathematical theory, describes the near-vertex behavior including the region of maximum stress intensity. The simplicity of this expression makes it possible to store accurate stress intensity factor distributions in very compact form.

In spherical coordinates \((\rho, \omega, \varphi)\), the Cartesian displacements \((u,v,w)\) near a vertex, for a straight crack front, can be written,

\[
\begin{bmatrix}
u(ho, \omega, \varphi) \\
w(ho, \omega, \varphi)
\end{bmatrix} = \sum_{j=1}^{J} B^{(j)} \rho^{\Lambda_j} \begin{bmatrix}
\Theta_1^{(j)}(\omega, \varphi) \\
\Theta_2^{(j)}(\omega, \varphi) \\
\Theta_3^{(j)}(\omega, \varphi)
\end{bmatrix} + \text{higher order terms}
\]

where the scalars \(B^{(j)}\) are so-called vertex intensity factors. In the case of curved crack fronts, the expansion (15) has to be augmented with higher order terms. However, in all the cases studied here the two leading terms in Eqn. (15) stay the same. The functions \(\Theta_i^{(j)}\) have the standard square root singularity for angles \((\omega, \varphi)\) corresponding to points close to the crack front, Andersson et al. [24]. For simplicity, we discuss only the pure mode I case (bending and traction loading). For a Poisson’s ratio of 0.3 and a quarter elliptical crack, we have the universal constants \(\Lambda_1 = 0.54782, \ \Lambda_2 = 1.21826\) which were calculated using the STRIPE-code and a spherical mesh at the vertex. From the definition of the stress intensity factor

\[
K_I = \lim_{r \to 0} \sqrt{\frac{2 \pi r \sigma}{\pi}}
\]

we get, using Eqn. (15), that the stress intensity function near the vertex must satisfy,

\[
K_I(s/a) = S_1(s/a)^{\Lambda_1 - 1/2} + S_2(s/a)^{\Lambda_2 - 1/2} + \text{higher order terms}
\]

where \(s\) is a coordinate along the crack front and \(\Lambda_2 > \Lambda_1, S_1 \text{ and } S_2 \text{ depend on } (a/c, a/t, r/h) \text{ and type of loading. The stress intensity functions, } K_I, \text{ are always zero at the vertex } (s = 0) \text{ since } \Lambda_1 > 1/2. \text{ The very steep gradient in } K_I \text{ close to a vertex is in our case due to the fact that } \Lambda_1 \approx 1/2. \text{ The parameters } S_1 \text{ and } S_2 \text{ can be determined from a linear regression fit to calculated numerical data close to the vertex. We see from Eqn. (16) that for small } s, K_I(s/a)/s^{\Lambda_1 - 1/2} \text{ is a linear function of } s^{\Lambda_2 - \Lambda_1}. \text{ The analytical expression, Eqn. (16) can be used to accurately calculate } K_I \text{ arbitrary close}
to the vertex, or the maximum value of $K_I$ near the vertex. The point $s^*$ where the stress intensity function is maximum is, for $S_2 < 0$ obtained from Eqn. (16), neglecting higher order terms, as,

$$s^* = a \left( \frac{S_1 (\Lambda_1 - 1/2)}{S_2 (\Lambda_2 - 1/2)} \right)^{\frac{1}{\Lambda_2 - \Lambda_1}}$$  (17)

The corresponding maximum stress intensity factor $K_I$ is obtained after inserting $s = s^*$ into Eqn. (16) and neglecting higher order terms. The two scalars $S_1$ and $S_2$, together with Eqns. (16) and (17) and the universal constants $\Lambda_j$ carry much information and are stored for each of the crack configurations analyzed.

**SPLITTING SCHEME**

A mathematical splitting method was used to efficiently and reliably calculate the 5,672,700 stress intensity factor solutions. A brief overview is given here; however, for a complete discussion of the mathematical theory, see Babuska and Anderson [25], Andersson et al. [26], and Andersson [27]. In the splitting method, the 3D fracture mechanics problem is split into three sub-problems, a complex multi-site cracking scenario is shown in Figure 4 although the current investigation is only considering two cracks at one hole, with the solution obtained by superposition of the sub-problems.

a. Global Crack Free Problem: The solution of the global crack free problem, see Figure 6, is $U_G^{(0)}$ and the only result required from the finite element analysis is the stress distributions on the surfaces where the cracks are to be located. Thus, this sub-problem is independent of the number and size of cracks under consideration.

b. A Set of $M$ Local Problems: A local model is developed for each $a/c$ and $c/r$ to be analyzed (in our case, 275 local models where used to derive the 5,672,700 solutions). The applied load consists of $L$ different normalized crack surface tractions with the solutions denoted as $\{U_{k}^{(m,l)}\} m = 1, 2 \ldots M, l = 1, 2, \ldots L$. The local models contain a single crack. By making the local models large in the thickness direction, only one FE-analysis is needed independently of the $a/t$ ratio of interest. Figure 5 shows a very small part of a local mesh, designed for our $hp$-version of the FE method, for parameters $a/c = 0.8, c/r = 0.125$. Close to the crack front another five cylindrical layers of elements are introduced.
which are not visible in the figure. So-called blended mapping is used in the analysis to describe the exact shape of the crack and the cylindrical hole surface. The results required from the local models are the tractions and displacements on an arbitrarily selected surface, \( \Gamma_i \) (dashed line in Figure 4) which encloses the entire crack in addition to the stress intensity functions \( K_I(s) \), \( K_{II}(s) \), and \( K_{III}(s) \) for all the crack face loadings.

c. A Set of Global Crack Free Problems: The global model in a. is analyzed with prescribed jumps in the tractions and displacements at the surfaces \( \Gamma_i \) used in the local problems. Thus, the output from the local problems is input to the load calculations in the global models. The solutions are denoted as \( \{ \Gamma^{(m,l)} \} \).

Solution of complex, 3D fracture mechanics problems are obtained by proper superposition of sub-problems a–c. The approximate displacement solution, \( \bar{U} \) to the exact 3D solution, \( U \) is written as [28]

\[
\bar{U} = U^{(0)}_G + \sum_{m=1}^{M} \sum_{l=1}^{L} \alpha_{(m,l)} U^{(m,l)}_G - \sum_{m=1}^{M} \sum_{l=1}^{L} \alpha_{(m,l)} U^{(m,l)}_L
\]

(18)

where \( \alpha_{(m,l)} \) are scaling factors determined by solving a small set of linear equations. Thus, with the known \( U^{(0)}_G \), \( U^{(m,l)}_G \), and \( U^{(m,l)}_L \), the solution, \( \bar{U} \) can be calculated with virtually no computational cost/crack configuration. This made it possible to derive over five million basically error free \( K \)-solutions for the three loading cases. The computational efficiency of the strategy devised makes it feasible to use the calculated stress intensity data in Monte Carlo type studies of 3D multiple-site fatigue crack growth [28]. If global geometric non-linearity is significant for the structure being analyzed, sub-problem a. is solved as a fully three-dimensional non-linear problem.

**EXPERIMENTAL INVESTIGATION**

The experimental program was designed to provide an array of crack data; specifically, crack shape \( (a/c) \) and size \( (a/t) \) at various times during the fatigue life, to be used to experimentally verify the \( K \)'s. The test matrix is shown in Table 2. Electro-discharge machined (EDM) notches were used to better control the starter notch geometry. The specimen length and width were chosen to eliminate the finite height effect as reported by Raju and Newman [29] and Tada et al. [30]. All specimens were made from 7075-T651 plate and tested using a blocked fatigue spectrum on a closed loop servohydraulic fatigue machine at 70 MPa and 10 Hz.
The spectrum, shown in Figure 7, was designed to create marker bands, groups of fatigue striations, on the fracture surface. One pass through the spectrum contains 8,170 cycles. *In-situ* crack growth measurement was not used as nearly the entire crack history was reconstructed post-test using a Nikon SMZ645 Stereoscope to detect the marker bands. This technique was successfully demonstrated by Pelloux et al. [31] and in [2] and [3]. The test was halted when the part-through crack grew through the specimen thickness. This was done to reduce the chance the fracture surfaces would come into contact during cycling and damage the marker bands.

**RESULTS AND DISCUSSION**

**FINITE ELEMENT ANALYSIS**

As discussed above, two finite element models, global and local, with $\nu = 0.3$ are used to calculate the stress intensity functions for each crack geometry. The global model, shown in Figure 6, is quite large with $2b = 2h = 200r = 400$ units where $b$ and $h$ are the half width and height, respectively. The hole radius is 2.0 units; thus, $r/b = r/h = 0.01$ resulting in negligible finite width/height effects for all crack lengths considered. The thickness of the global model, $t$, can easily be scaled to obtain $K$ solutions for various $r/t$ ratios. Blended function mapping (Gordon and Hall [32]) is used to ensure the hole has an exact cylindrical surface and the crack front has an exact elliptical shape, see Figure 5. In the $z$-direction, thickness direction, four layers of elements are used with relative thickness of 1, 7, 49, and 343 units, respectively. The mesh is designed for the *hp-version* of the finite element method; thus, seven elements are used along the crack front with relative lengths of 1, 7, 49, 343, 49, 7, and 1 unit. By using this highly graded mesh, the strong variation of the stress intensity function close to the crack vertices are captured with high accuracy. The $K$ variation along the crack front is given in the form of piecewise polynomials as described above. Close to the vertices, an analytical expression based on the mathematical expansion, Eqn. (16), of $K$ close to the vertex is used. Incidentally, the highly graded mesh was designed to accurately calculate $K$’s for cracks that have nearly penetrated the back free surface, $a/t = 0.95$ and 0.99; however, this same mesh design also yields $K$ solutions of the same accuracy for very shallow cracks, $a/c < 0.1$. These low aspect ratio solutions will be most useful in fatigue life prediction for bending dominant problems and corrosion fatigue where shallow surface corrosion damage can begin to propagate in the cycle domain although damage initiated in the time domain.

The global model has 600 elements resulting in over 70,000 DoF at $p = 6$. The global mesh is overly refined for all of the crack shapes considered except when $a/t = 0.95$ and 0.99 where careful mesh refinement in the ligament between the crack depth and free surface is required. The local models contains approximately
1800 elements giving 225,000 DoF at $p = 6$. Exponential convergence of the finite element solution is possible when using the *hp-version* as seen in Figure 8 where convergence is obtained already at $p = 3$ for $2\varphi/\pi \leq 0.9$. Higher $p$-levels are required for $2\varphi/\pi > 0.9$ due to the finite thickness effect as discussed by Broek [33] and large stress gradient at the hole edge. For more extreme crack geometries, the most extreme case being shown in Figure 9, higher $p$-levels are required than for all the other solutions. In fact, the error for lower $p$-levels could be increased drastically by using more than $(p-1)^2$ terms in Eqn. (18). However, we have had no difficulties in obtaining very accurate solutions using polynomial order $p = 6$. Although error is much larger for small $p$, the accuracy of the $p = 6$ solution is still less than 0.1% as are all the other solutions.

Stress intensity factor solutions derived in the past using FEA or FEAM have typically used a quarter plate model assuming two symmetry planes, $yz$-plane at $x = 0$ and $xz$-plane at $y = 0$ in Figure 6, to reduce the number of degrees of freedom in the model thereby reducing the solution time. In this effort, no planes of symmetry are employed; thus, $K$’s for each crack configuration, one or two cracks at the hole edge, are explicitly calculated without the need for the well-known Shah correction factor [7] for converting $K$’s for one crack to two cracks at a hole and vice versa.

The results of the Bakuckas round-robin are shown in Figure 10 where the solid lines indicate the upper and lower bounds of all solutions and $\pm 3\%$ deviation from the average solution. Two STRIPE analyses were conducted, one modelling the crack in an infinite plate (open squares) and the other using the same plate dimensions as that used by Bakuckas (open circles). Both of our analyses used a $p$-level of 6 to obtain the converged solution with error control. The $5\%$ maximum difference between our solutions illustrates the well-known finite height/width effect. For this case, we also made an independent check, by modelling half the domain explicitly, and solving the problem for $p = 2, 3…10$, to verify convergence. Stress intensity functions were computed from the basic definition, $K_I = \lim_{r \to a} \sqrt{2\pi \sigma(r)}$ at a distance of $a/5000$ from the crack front. Our previous results were confirmed to the 3rd digits accuracy at all control points. Hence, we did not use the splitting method and the technique described for $K$-extraction, but arrived at identical results. Therefore, we can guarantee that the STRIPE-solution shown in Figure 10 is the exact solution (with actual resolution) to the boundary value problem proposed by Bakuckas. Thus, of the six methods used and for the given crack configuration in the round-robin, the $K$’s calculated using WFM and DIM where the most accurate, less than $2\%$ error compared to the current results. In addition the FEM, FEAM, and BEM are $3\% – 6\%$ over conservative for this particular crack configuration. As discussed by Fawaz and Andersson in [20], the error between the Newman/Raju solutions [16] and the current results can be over $26\%$ for some loading conditions and crack configurations.
FATIGUE CRACK GROWTH EXPERIMENTS

The test matrix was composed of 10 symmetric and 28 unsymmetric crack cases (starter notch geometries) to be used to verify the newly calculated $K$ solutions. In addition, the symmetric cases could also be used to evaluate the accuracy of the $K$ solutions presented in [16]; however, for brevity, those comparisons are not made here.

The first step in the experimental verification of the new $K$ solutions is to extract the crack growth history information from the fracture surface of the test specimens. A typical fracture surface is shown in Figure 11. At higher magnification, the marker bands, as shown in Figure 12, are easily detected. The marker bands are distinct along the entire crack front for most cracks examined as seen in Figure 13. For some specimens, the first set of marker bands were detected at 9 $\mu$m from the starter notch. Each pass of the spectrum has three groups of marker bands in a 10-4-6 sequence. As listed in Table 3, the number of marker band groups per specimen varies from 3 – 51 (number of passes * 3 groups of marks per pass). Five complete crack fronts were reconstructed where possible during fractographic analysis to give an indication of the crack shape development throughout the fatigue life. The procedure used was to measure the first and last marker groups detectable and three more marker groups evenly spaced between the first and last. The crack growth history at the c-tip is derived from the data points closest to the free surface in Figure 13. The resulting crack growth history is shown in Figure 14. The two data sets in Figure 14 are the same except one is measured from the reference origin, the intersection of the bottom sheet surface and hole bore, and the other from the nucleation site on the starter notch. The data such as that shown in Figure 13 and Figure 14 will be used to verify the $K$’s.

Crack Shape Development

The crack (or flaw) shape development in sheet material has been shown by Fawaz and Schijve [34] to change during the fatigue life; however, this conclusion was drawn from examining the fracture surfaces of interrupted tests. In other words, the change in crack shape ($a/c$) was determined from several specimens, not just one. A similar analysis is made here except the change in flaw shape is calculated per specimen. As shown in Table 4, the flaw shape is changing throughout the fatigue life. For most of specimens, the $a/c$ first increases then decreases as the crack grows through the thickness. If the starter notch was large or the marker bands were not detected until just prior to the part-through crack growing through the thickness, the $a/c$ ratio is always decreasing with increase in fatigue cycles. The same results were shown by analysis and experiment in reference [34].
Fatigue Life and Crack Shape Predictions

The predictions are made for both the flaw shape and crack length in time using data as shown in Figure 13 and Figure 14. For both predictions, the initial condition is the crack shape and size at the first fully formed crack front, mark #4 in Figure 13 for example. Not all starter notches allowed for a uniform crack front to form. In Figure 15, the crack nucleated close to the a-tip and propagated along the notch in the “c” direction never developing into a full thickness crack before the test was halted. Four nucleation sites were found along the starter notch shown in Figure 16 and the crack front did not become uniform until after three full passes of the spectrum (approximately 24,510 cycles) had been applied. The K solutions developed here are not applicable for the non-uniform crack cases; thus these crack growth histories cannot be used.

Crack growth rate data for this batch of material was developed in accordance with ASTM E647 for stress ratios of 0.1, 0.4, and 0.6 as shown in Figure 17. A prediction of the crack history of specimen 2-2 is shown in Figure 18. The predictions using the newly developed K’s were for the exact unsymmetric crack geometry at mark 7 for both cracks. Specifically, the initial crack sizes were \( a_1 = 2.411 \) mm, \( c_1 = 1.867 \) mm and \( a_2 = 2.528 \) mm, \( c_2 = 1.790 \) mm. Only symmetric cracks can be used with the Newman/Raju solutions, \( a_1 = a_2 = 2.411 \) mm, \( c_1 = c_2 = 2.528 \) mm for this analysis, and in this instance assuming the cracks are symmetric is unconservative (overestimating the fatigue life).

In Figure 19 and Figure 20, predictions of the flaw shape are shown. The initial condition of the prediction is the c- and a-tip dimensions at mark 7. The crack is grown using the maximum K in the vicinity of the vertex and the location of the maximum K. Typically, when using the Newman/Raju equations, the crack is propagated close to the vertex at \( 0^\circ \leq \phi_c \leq 10^\circ \) at the c-tip and \( 80^\circ \leq \phi_a \leq 90^\circ \) at the a-tip where \( \phi_i \) is the parametric angle of the ellipse. The crack is grown from mark 7 and halted at final crack front at mark 12.

Thus far in this effort, we solved two ordinary differential equations, Eqn. (1), by simple Euler integration. In (1), the \( \Delta K \)’s should be taken as representative values of the stress intensity factor range \( \Delta K(\phi) \) “near” vertices a and c. In the present analyses, the maximum value of the stress intensity factor function range near the two vertices were used for \( \Delta K^{(a)}(a,c), \Delta K^{(c)}(a,c) \). In Figure 21, which shows virtually exact values of the function \( \Delta K(\phi) \) for a crack size of interest \( (a/t = 0.95 \) and \( a/c = 1.5) \), clearly shows that the two peak values of \( \Delta K(\phi) \) are much larger than \( \Delta K(\phi) \) over the major part of the crack front which indicates that the crack growth rate might be overestimated if maximum K values are used in the fatigue analysis.
A more consistent approach for considering the $\Delta K(\phi)$ variation along the crack front is to use an integrated $K$ measure. Although this can be done in various ways, we suggest, and are in the process of implementing, the following approach. The corner crack is assumed to be quarter-elliptical throughout the crack growth process. In each load cycle, the crack front propagates a distance $\delta s$ perpendicular to the existing crack front characterized by $a_o$ and $c_o$. The local crack front advancement $ds$ is given by,

$$ds(\phi) = C(\Delta K_f(\phi))^n$$

Thus, with known $\Delta K(\phi,a_o,c_o)$, we can compute the crack front increment from Eqn. (19) at arbitrary points $0 \leq \phi \leq \pi/2$. In a second step, a new elliptical crack front, characterized by $a_1$ and $c_1$, is fitted by least squares linear regression to the crack shape obtained from (19) and $(a_o,c_o)$. By using the simplified approach for the crack case shown in Figure 21, Eqn. (1), the crack advance at the $a$-tip is 2.5 times larger than that at the $c$-tip. Using the more consistent approach, Eqn. (19), the crack advance at $a$ is the same as $c$.

**CONCLUSIONS**

The $p$-version finite element method was used to calculate over 5.6 million $K$ solutions for two diametrically opposed, unsymmetric, corner cracks at a hole in an infinite sheet subject to tension, bending, and bearing loading. The experimental program yielded crack shape and crack growth information used to verify the new analytical solutions. Good agreement was found in both the crack shape and crack growth history. Efforts are currently underway to increase the correlation between experiment and prediction by growing the crack at 32 points along the crack front and not just at the $a$- and $c$-vertices.

**ACKNOWLEDGEMENT**

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REFERENCES


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**Symmetric Cracks**

**Unsymmetric Cracks**

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**EN = Electrode Number**

All dimensions in mm
Table 3 Fatigue Test Results

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EN = Electrode Number

Table 4 Crack Shape Development

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* Test stopped before marker sequence
** Crack transitioned to through the thickness crack
Figure 1 Parameter definition for two unequal corner cracks at centrally located hole in a finite width sheet subject to general loading

Figure 2 Local Domain
Figure 3 Domain $\Omega^6$ used for extraction of edge intensity functions

Figure 4 Splitting scheme sub-problems
Figure 5: Part of local mesh: $a/c = 0.8$, $a/t = 0.2$. Note that only the polyhedral shape is depicted. In the FE-analysis, the hole surface has an exact cylindrical shape and the crack front has an exact elliptical shape.

Figure 6: Global mesh used for all crack configurations and applied loads.
Figure 7 Blocked Marker Spectrum

Figure 8 Convergence study for a single shallow corner crack at a hole subject to remote tension
Figure 9 Convergence study for a single deep corner crack at a hole subject to remote tension

Figure 10 Comparison of published results [17] vs. STRIPE
Figure 11 Starter notch and fracture surface detail

Figure 12 Marker Bands
Figure 13 Crack fronts reconstructed from marker bands

Figure 14 c-tip crack history, left crack, specimen 2-2
Figure 15 Nonuniform crack nucleation in specimen 5-13A left

Figure 16 Multiple crack nucleation sites and non-uniform crack front formation in specimen 3-3 left
Figure 17 Crack growth rate data for 7075-T651 plate

Figure 18 Prediction of crack growth history for specimen 2-2
Figure 19 Crack shape prediction, left crack, specimen 2-2

Figure 20 Crack shape prediction, right crack, specimen 2-2
Figure 21 $K$ variation along crack front