LexTech

AFGROW.NET

AFGROW Workshop 2011 - Layton, UT

Lessons Learned While Developing K-Solutions for Pin Loaded Holes

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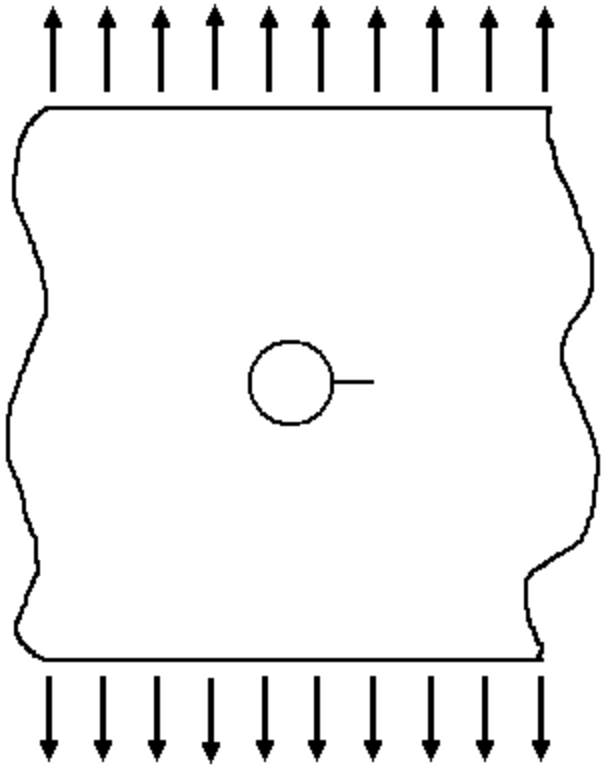
for

LexTech, Inc.

Outline

- Historical Background
  - Closed Form and Handbook K-Solutions
  - Advanced Solution Database
- Compounding/Superposition Methods
- Finite Width Correction for Axial and Bearing Loads
- Offset Hole Correction
  - Axial Loading
  - Bearing Loading
- Near Edge Correction
- Far Edge Correction
- Updated Classic Solution for a Single Through Crack at a Hole Under Bearing Load

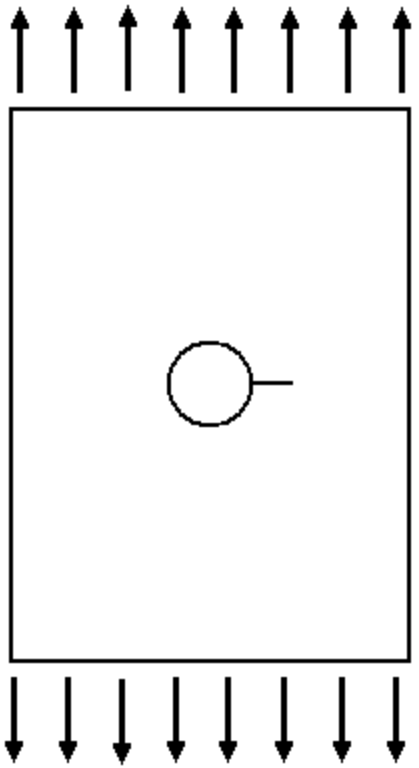
Typical Closed-Form/Handbook Solution



$$\beta_{ref} = 0.7071 + 0.7548\,(R/(R+C)) + 0.3415\,(R/(R+C))^2 + 0.642\,(R/(R+C)^3 + 0.9196\,(R/(R+C)^4$$

Accounts for the effect of the hole on the stress intensity at the crack tip

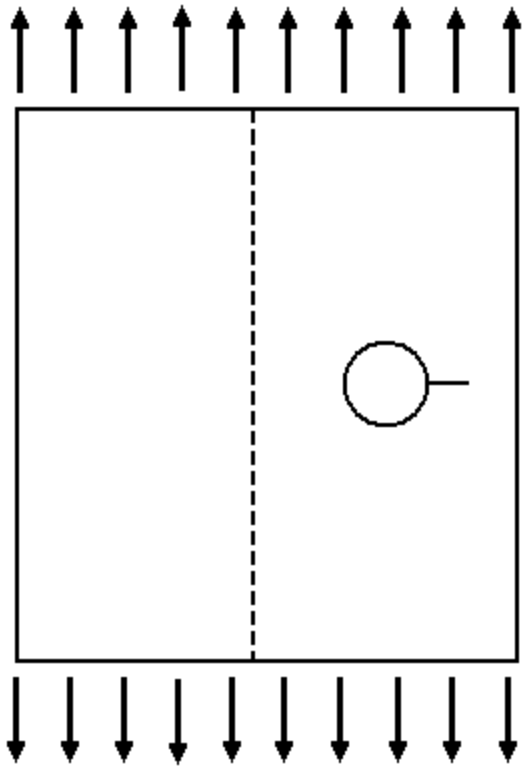
Finite Width Correction



$$\beta = \beta_{ref} \; F_w$$

Accounts for the finite width effect on the stress intensity at the crack tip

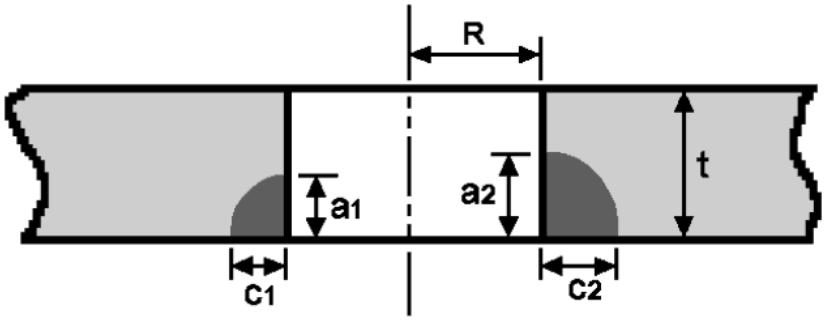
Hole Offset Correction



$$\beta = \beta_{ref} \; F_w \; F_{offset}$$

Accounts for the hole offset effect on the stress intensity at the crack tip

Fawaz Non-Symmetric Corner Crack FEM K-Solution Database



Load Cases: Axial, Bending, & Bearing

W/D : 100 (approximates an infinite plate)

R/t: 0.1, 0.111, 0.125, 0.1428, 0.1667, 0.2, 0.25, 0.333, 0.5, 0.667, 0.75, 0.8, 1.0, 1.25, 1.33, 1.5, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0

a_1/c_1, a_2/c_2: 0.1, 0.111, 0.125, 0.1428, 0.1667, 0.2, 0.25, 0.333, 0.5, 0.667, 0.75, 0.8, 1.0, 1.25, 1.33, 1.5, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0

a_1/t, a_2/t: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 0.99

75,625 models per load case

How Can This Database be Used to Obtain K-Solutions for Finite Plates w/Offset Holes?

COMPOUNDING

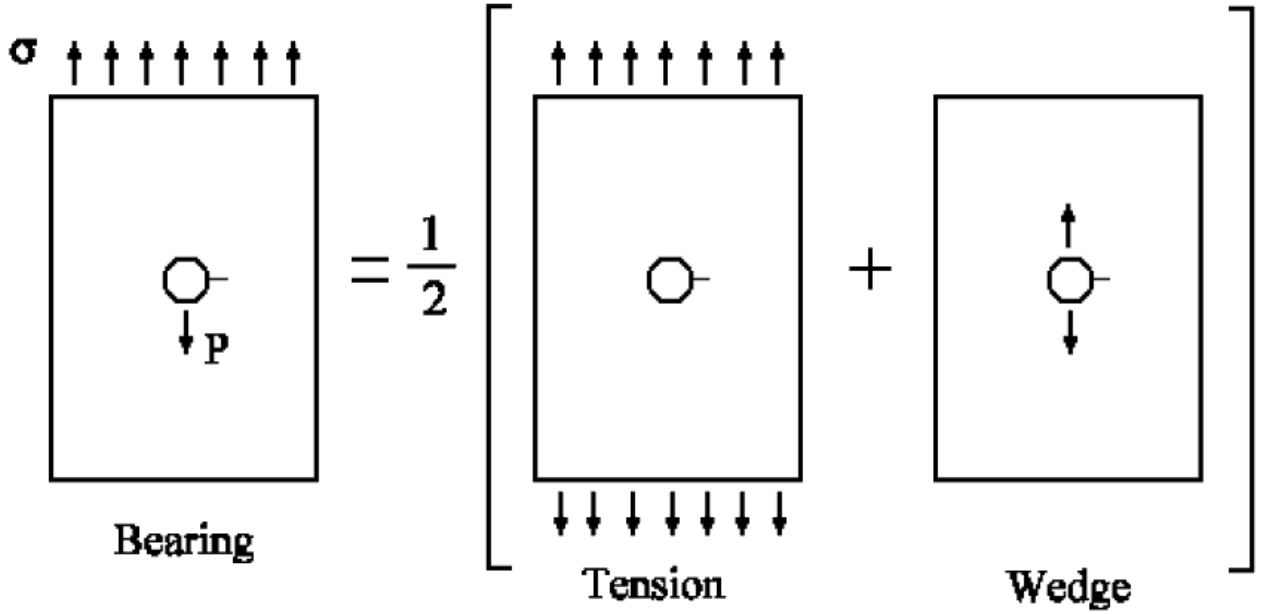
$$\beta = \beta_{ref} \; F_w \; F_{offset}$$

Applicability Issue

Finite width & offset corrections were developed for the axial load case

Are they applicable to the bending and bearing load cases?

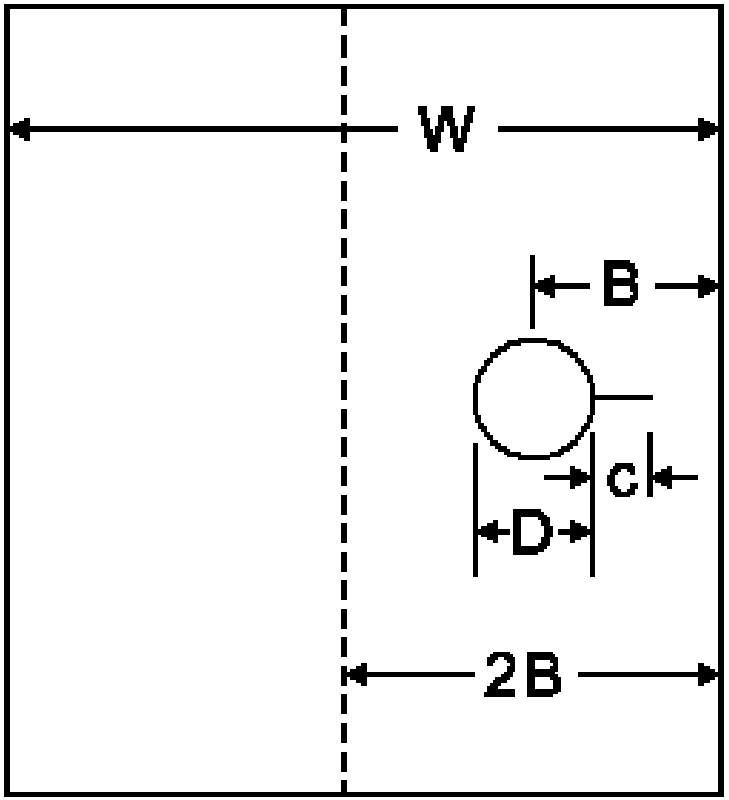
Bearing (Pin) Load Solution by Superposition



$$\beta_{Bearing} = \left(\frac{D}{2W}\right) \beta_{Axial} + \beta_{Bearing(W/D=100)} * F_{wp} \text{ ; (for any given plate width)}$$

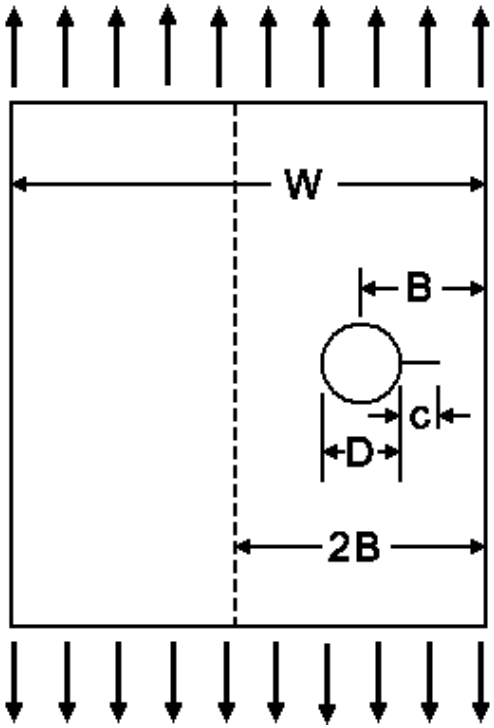
Where, F_{wp} = Finite Plate Correction for the infinite plate bearing solution

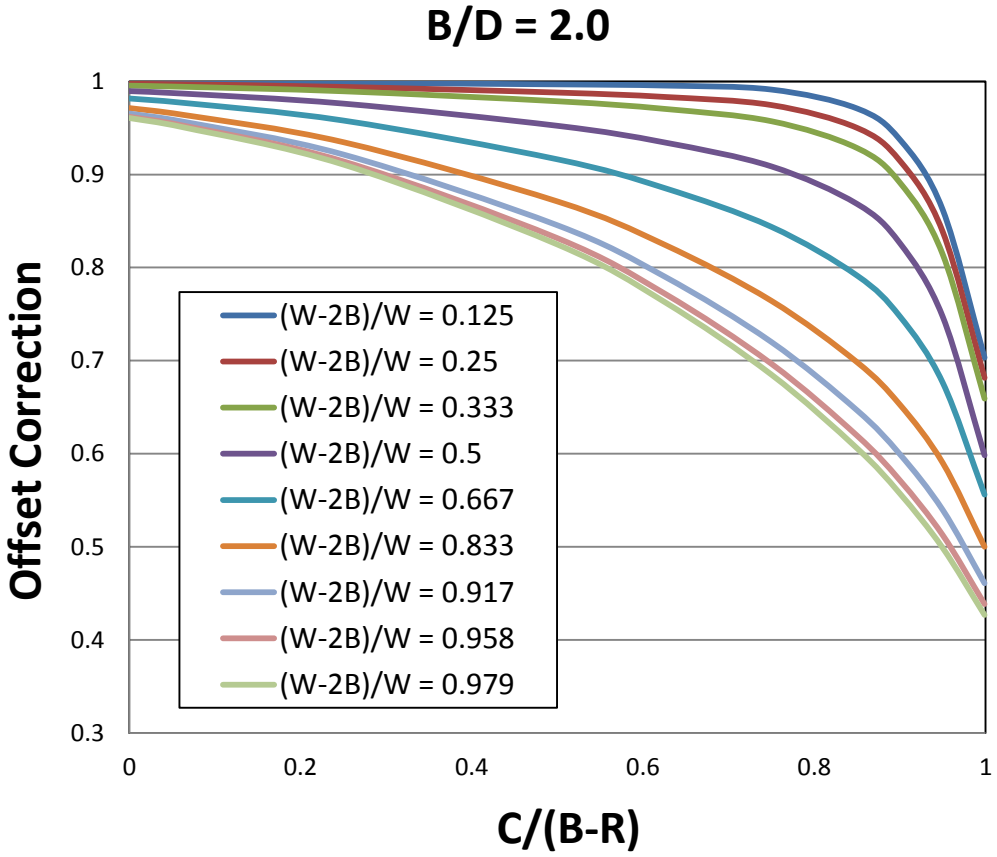
Offset Correction
(Near Edge)



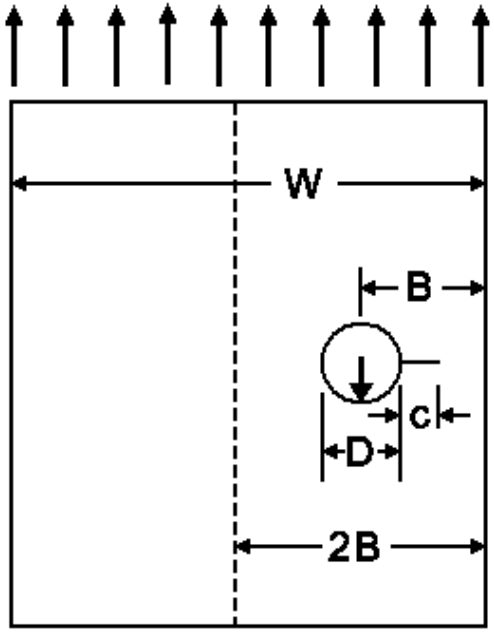
$$F_{Offset} = \frac{K_{Offset}}{K_{Centered\ Hole\ (W=2B)}}$$

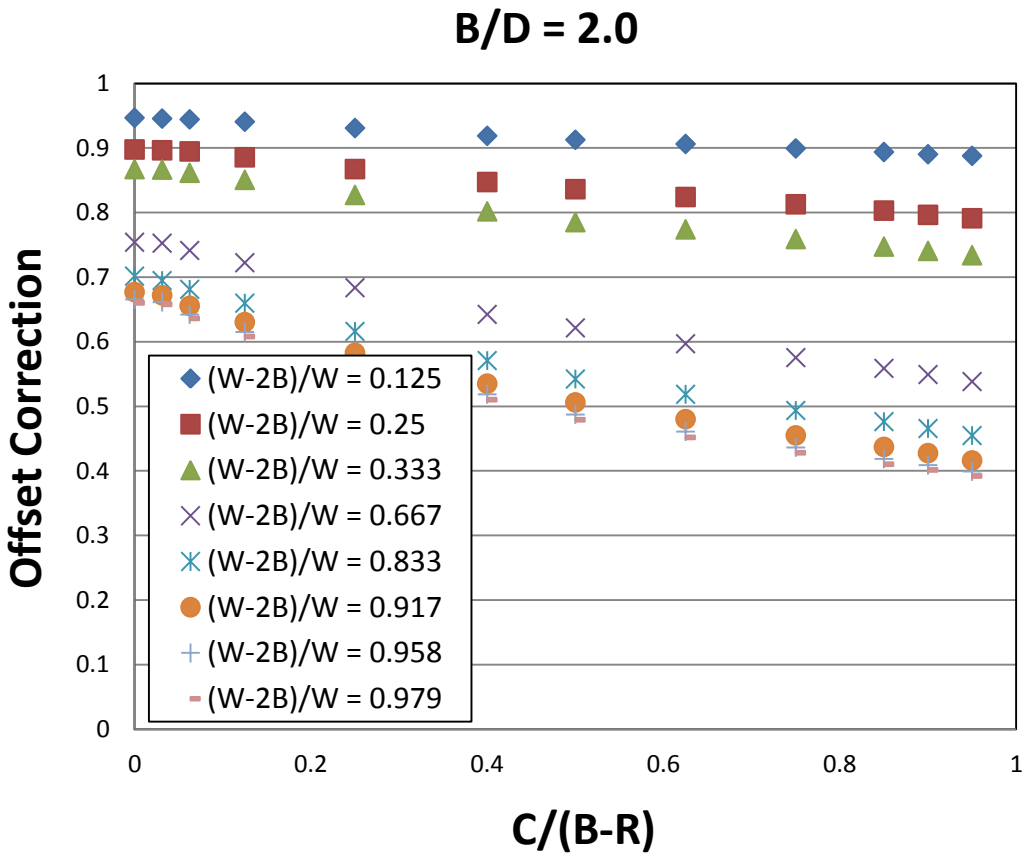
Axial Load Case



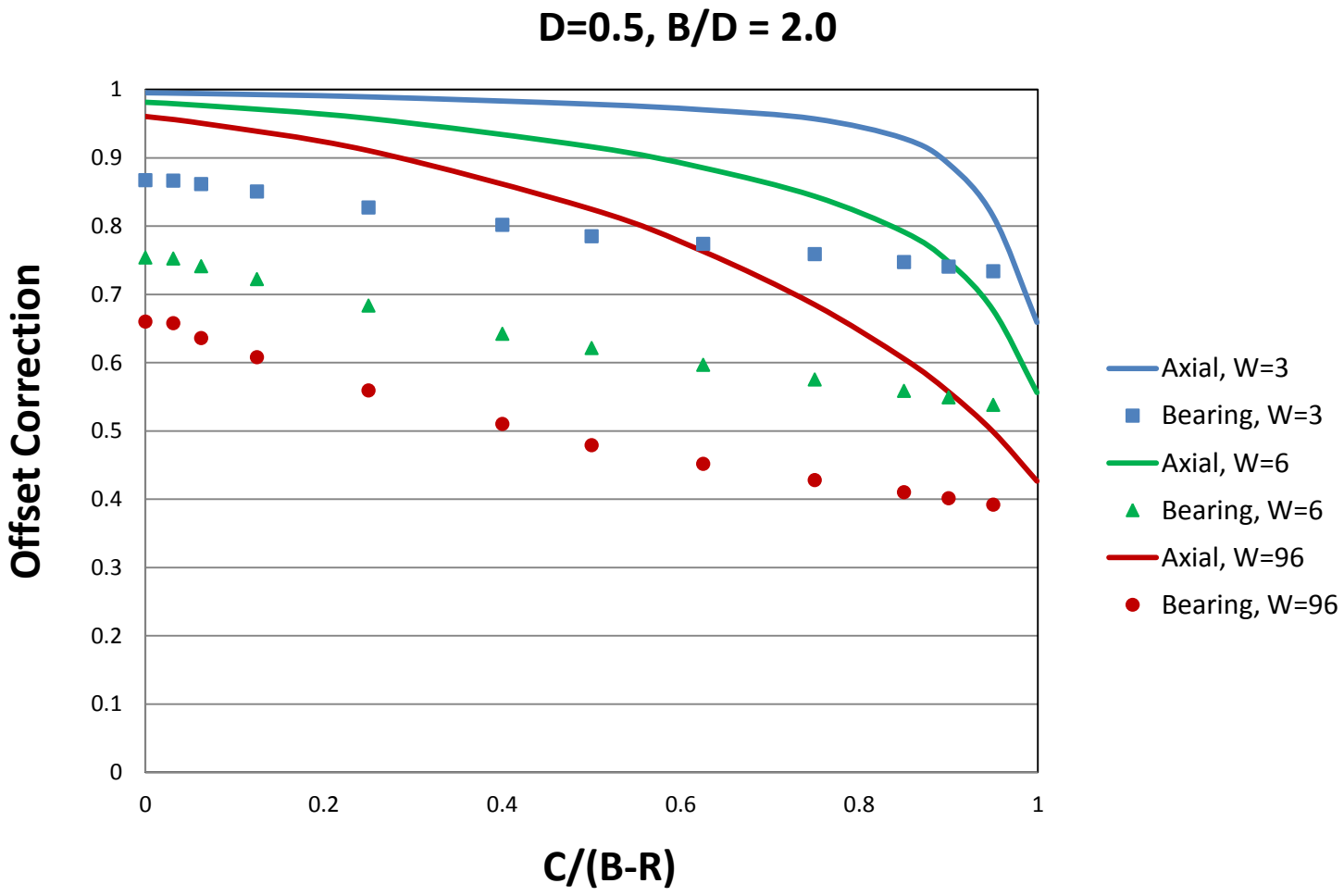


Bearing Load Case



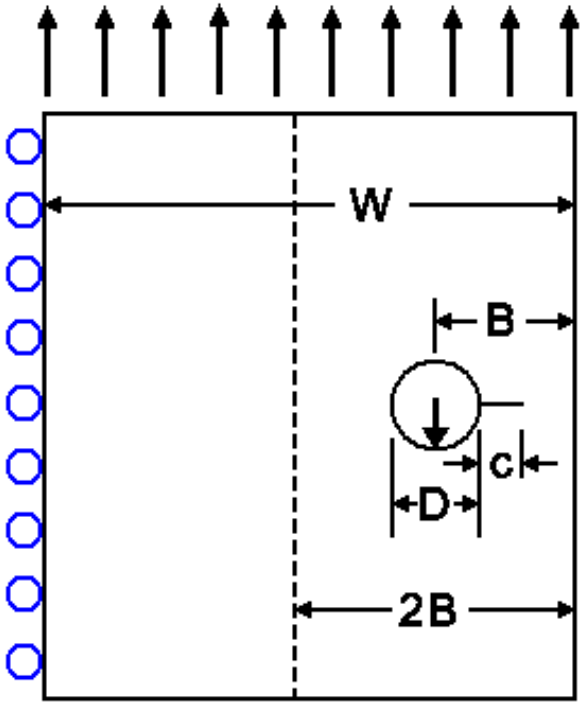


Direct Comparison



Bearing Offset Correction

Near Edge



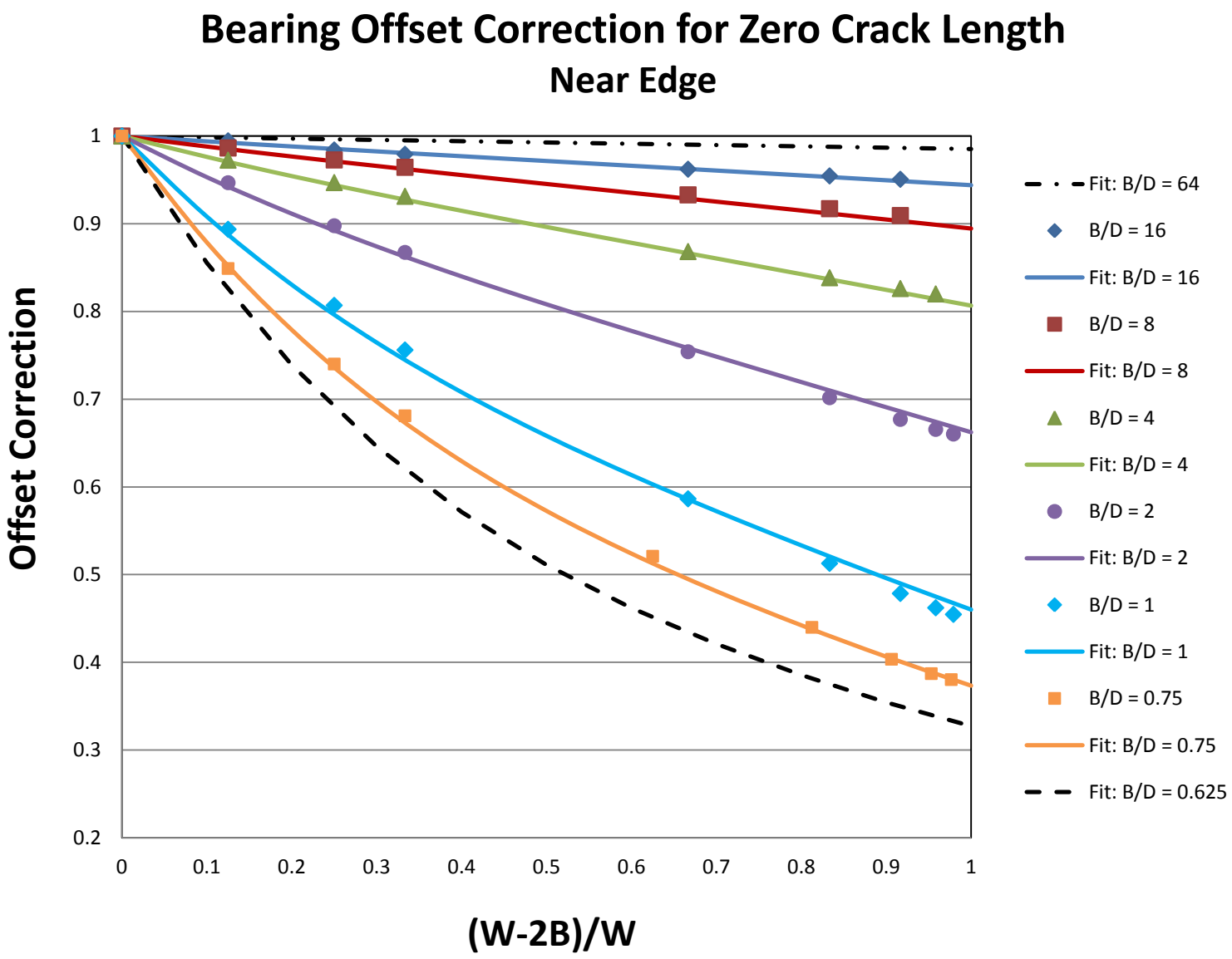
FEM Solutions Completed

B/D = 0.75, 1, 2, 4, 8, 16

(W-2B)/W = 0.125, 0.25, 0.333, 0.667, 0.833, 0.917, 0.958, 0.979

C/(B-R) = 0.0* – 0.95 (depending on B/D)

* Offset correction determined by: $\mathbf{K_{T(offset)}/K_{T(center)}}$


Bearing Offset Correction for Zero Crack Length
Near Edge
Offset Correction
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
(W-2B)/W
Fit: B/D = 64
B/D = 16
Fit: B/D = 16
B/D = 8
Fit: B/D = 8
B/D = 4
Fit: B/D = 4
B/D = 2
Fit: B/D = 2
B/D = 1
Fit: B/D = 1
B/D = 0.75
Fit: B/D = 0.75
Fit: B/D = 0.625

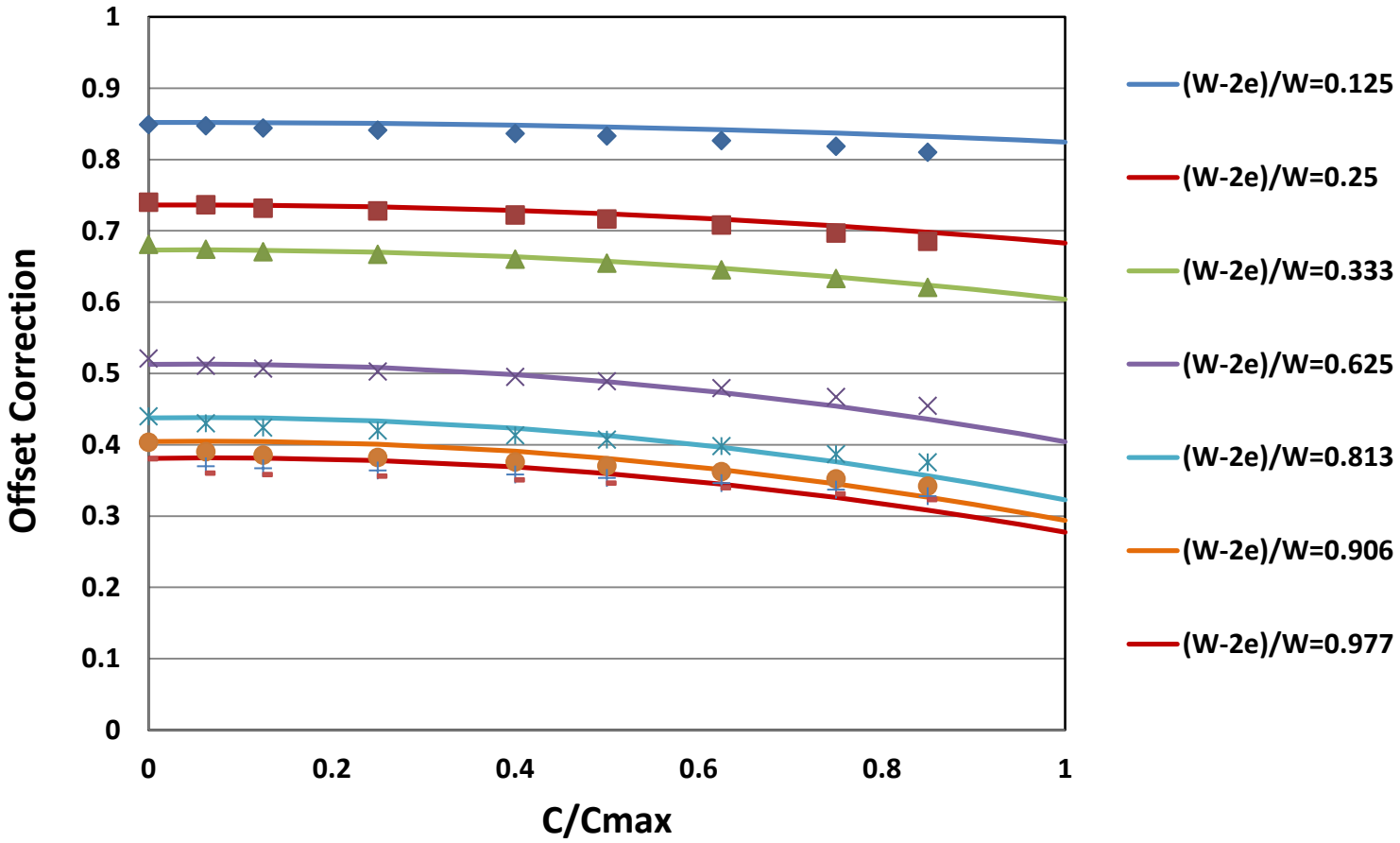
Polynomial Fit

$$F_{0\ Offset} = 1 - (\gamma^{-1})\delta + (0.85\,\gamma^{-1.4})\delta^2 - (0.45\,\gamma^{-1.275})\delta^3 + (0.06\,\gamma^{-1.2})\delta^5$$

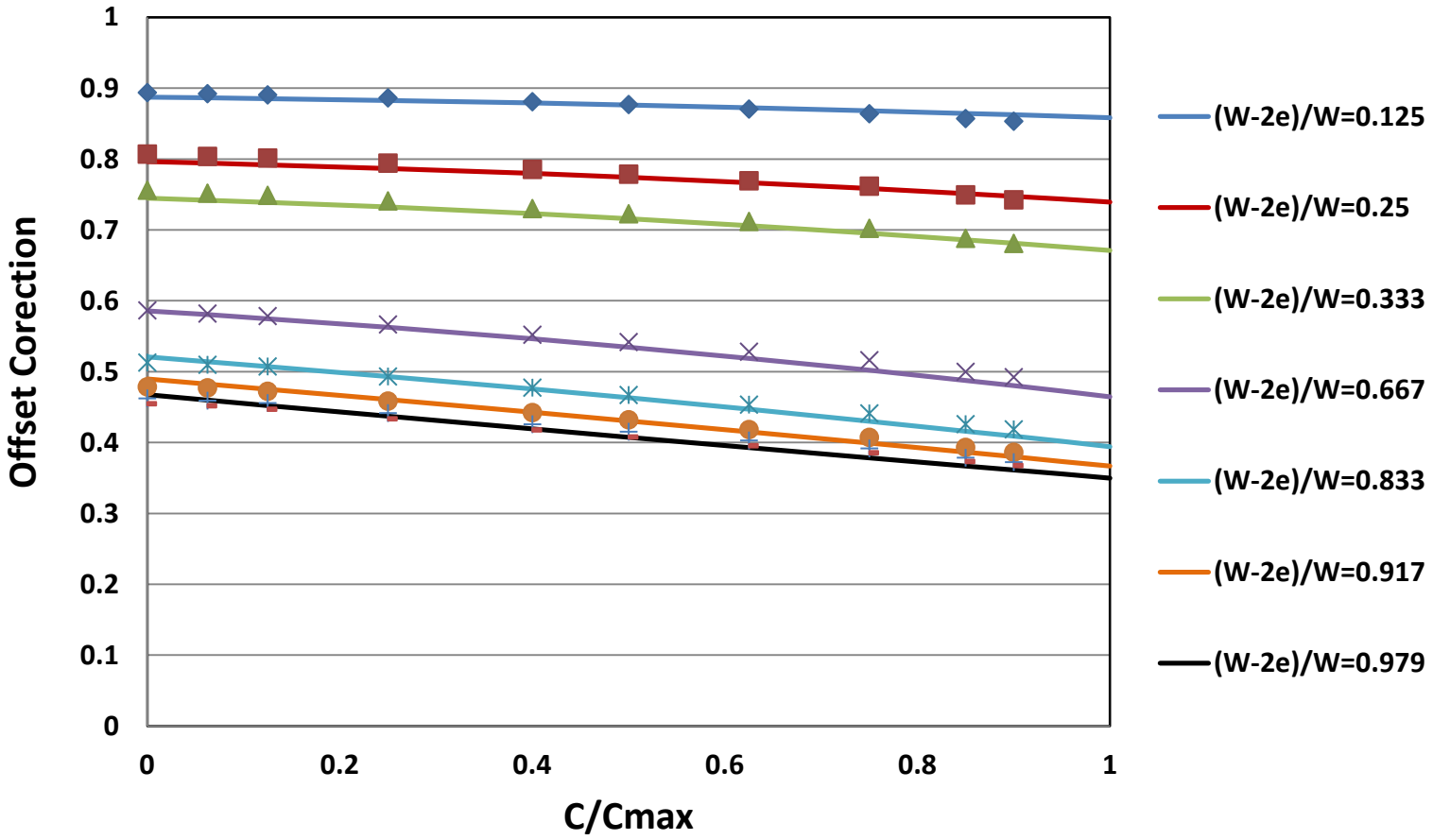
$$\gamma = B/D$$

$$\delta = (W - 2B)/W$$

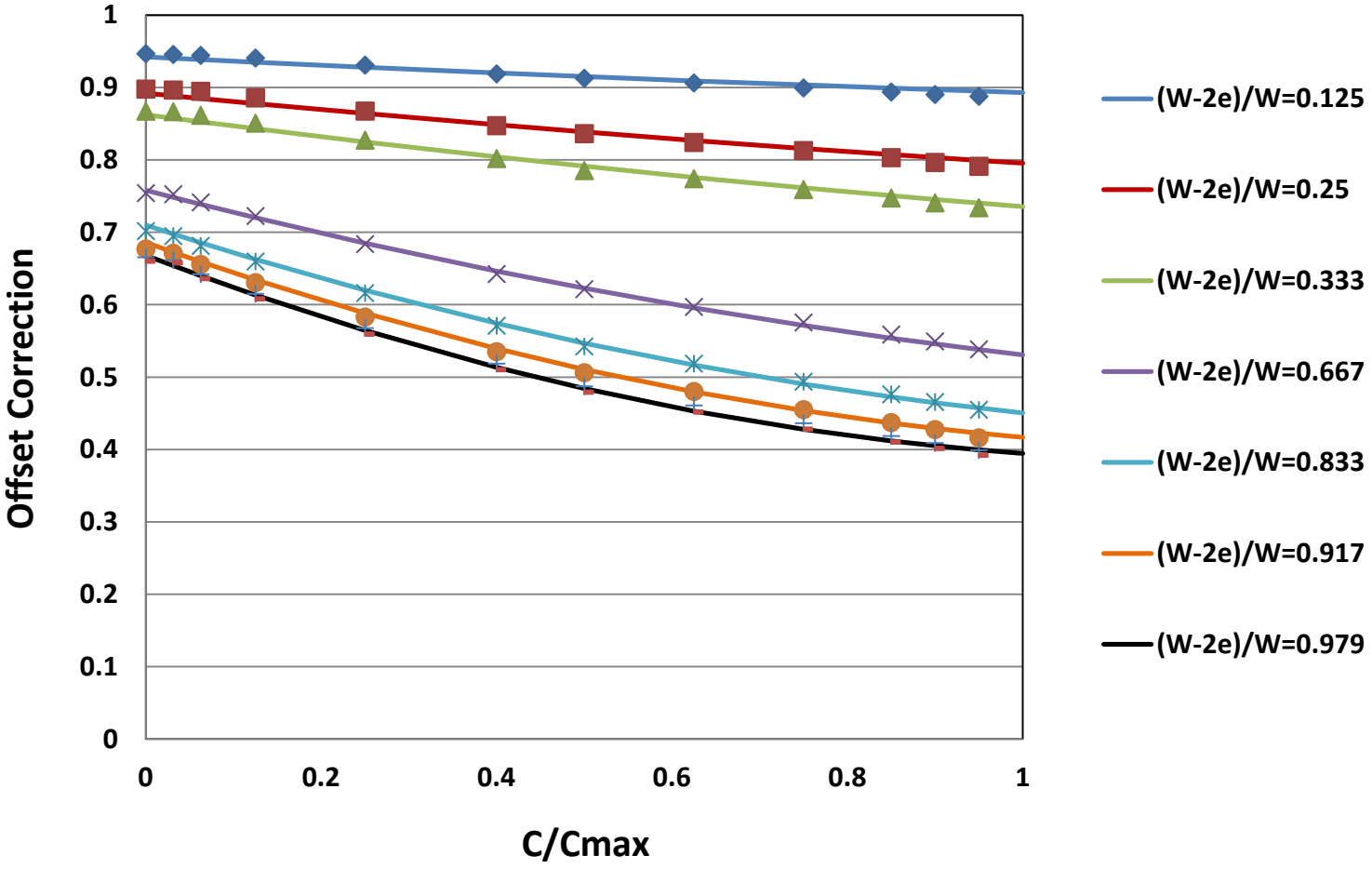
Near Edge Offset Correction
e/D = 0.75



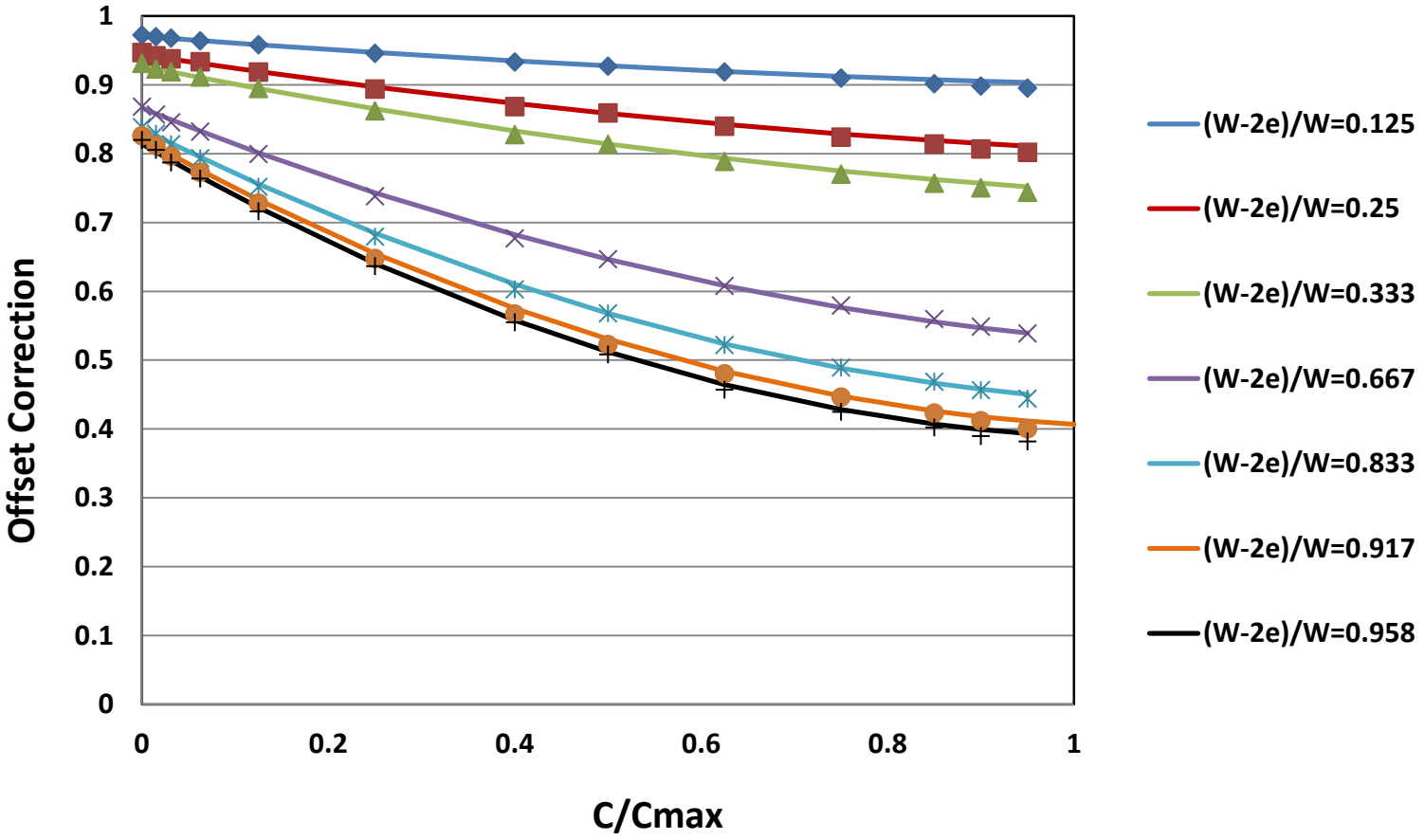
Near Edge Offset Correction
e/D = 1



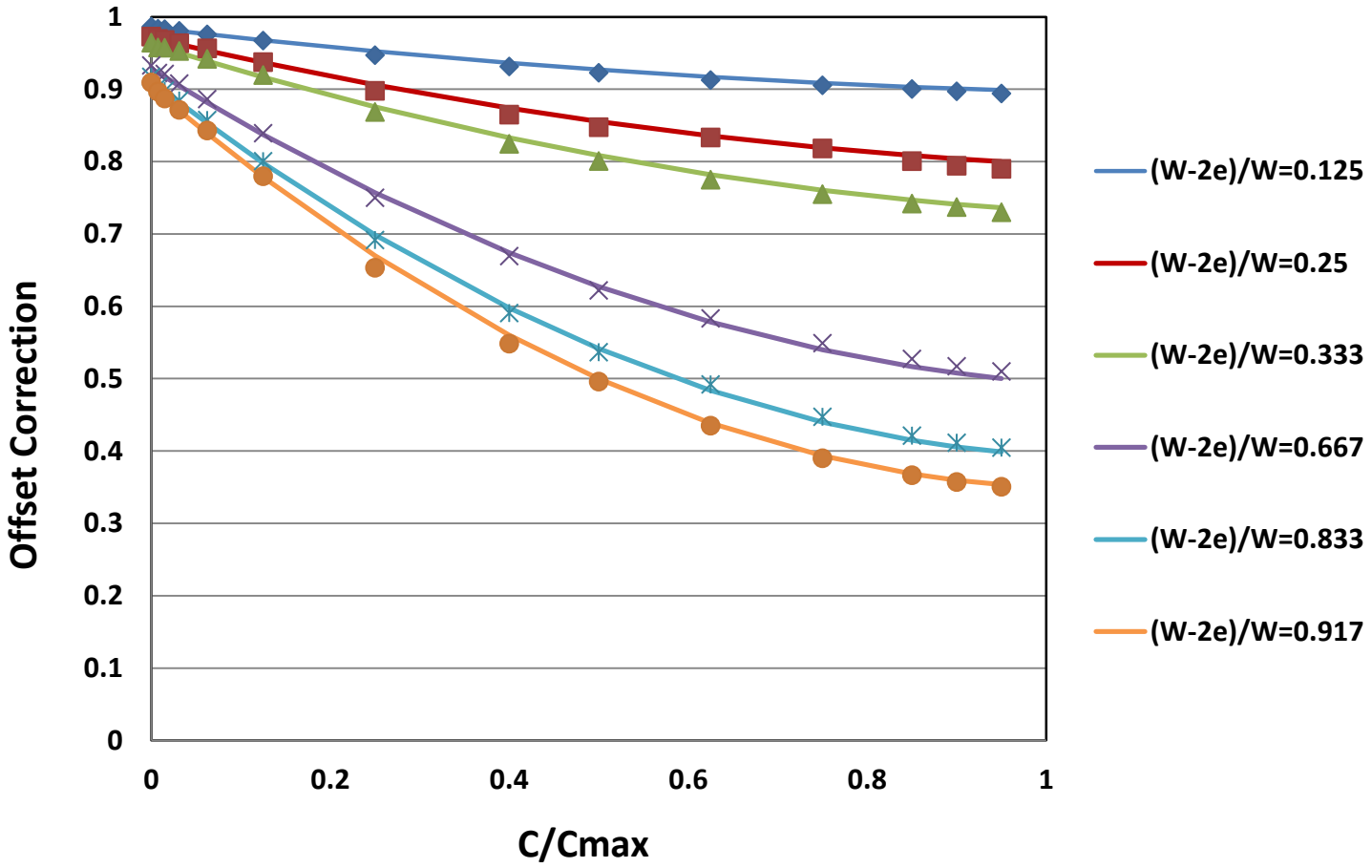
Near Edge Offset Correction
e/D = 2



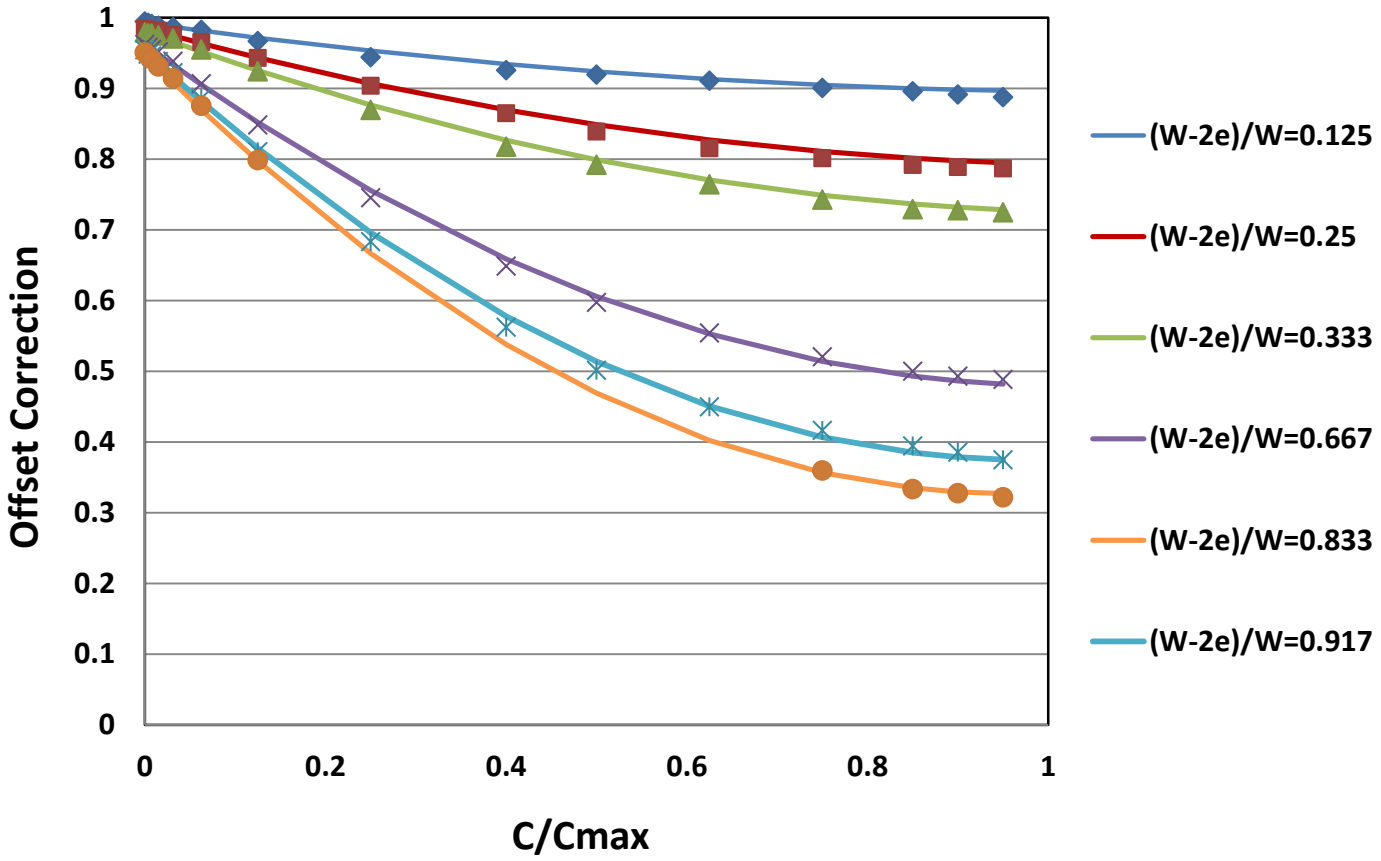
Near Edge Offset Correction
e/D = 4



Near Edge Offset Correction
e/D = 8



Near Edge Offset Correction
e/D = 16



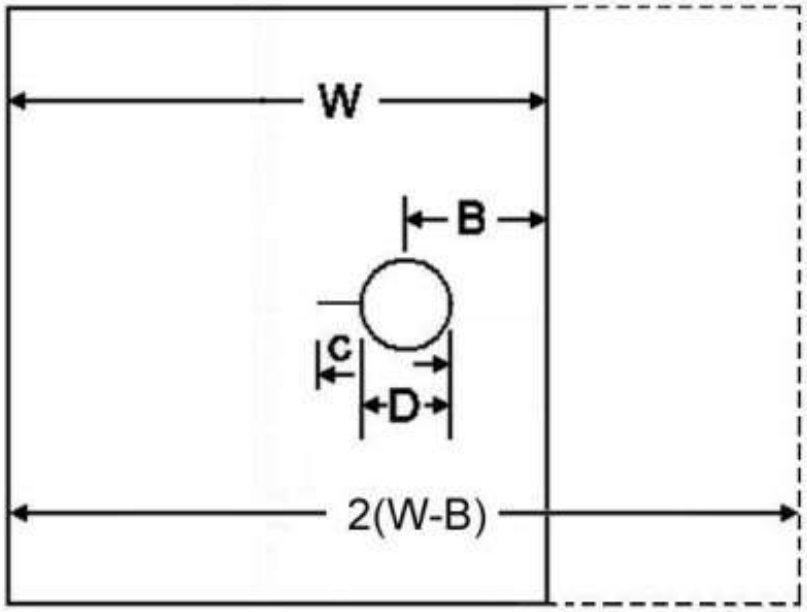
Curve Fit Solution

$$F_{Offset} = F_{0\,Offset} - \delta\left[\frac{\ln(\gamma)}{2} + 0.125 - 0.1\left(1 - e^{-3\left(\frac{\gamma}{16}\right)^5}\right)\right]\left(\frac{C}{C_{max}}\right) + \delta\left[(0.27 + 43e^{-7.3\gamma})ln(\gamma) + 0.12\delta^2 - 0.11\right]\left(\frac{C}{C_{max}}\right)^2$$

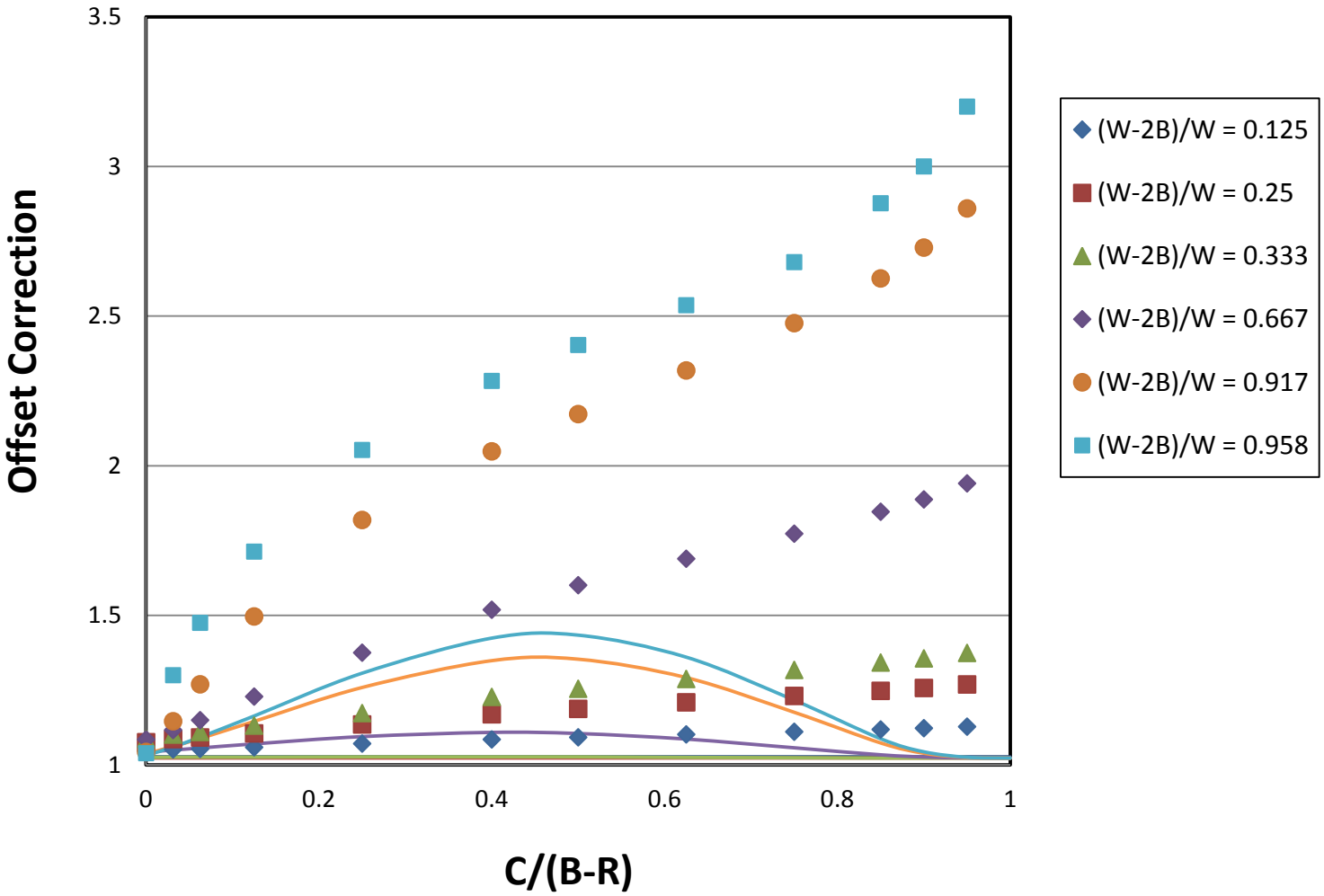
$$\gamma = B/D$$

$$\delta = (W - 2B)/W$$

Offset Correction
(Far Edge)

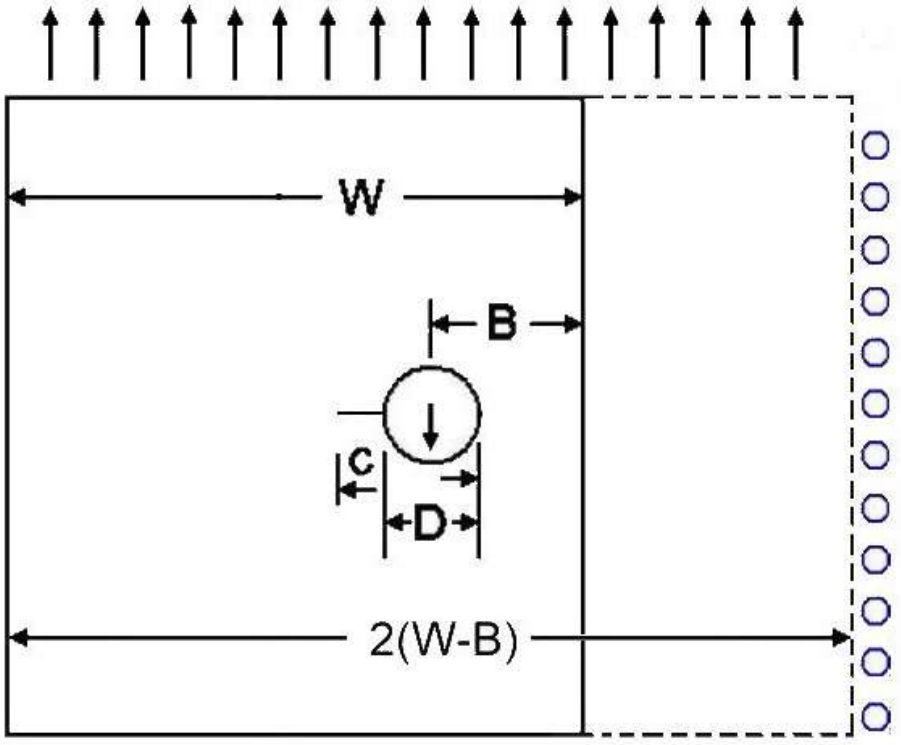


$$F_{Offset} = \frac{K_{Offset}}{K_{Centered\ Hole\ (W=2(W-B))}}$$


Far Side Offset Correction
Bearing vs. Axial Load Case
B/D=2
Offset Correction
3.5
3
2.5
2
1.5
1
0
0.2
0.4
0.6
0.8
1
C/(B-R)
(W-2B)/W = 0.125
(W-2B)/W = 0.25
(W-2B)/W = 0.333
(W-2B)/W = 0.667
(W-2B)/W = 0.917
(W-2B)/W = 0.958

Bearing Offset Correction

Far Edge



FEM Solutions Completed

B/D = 0.75, 1, 2, 4, 8, 16

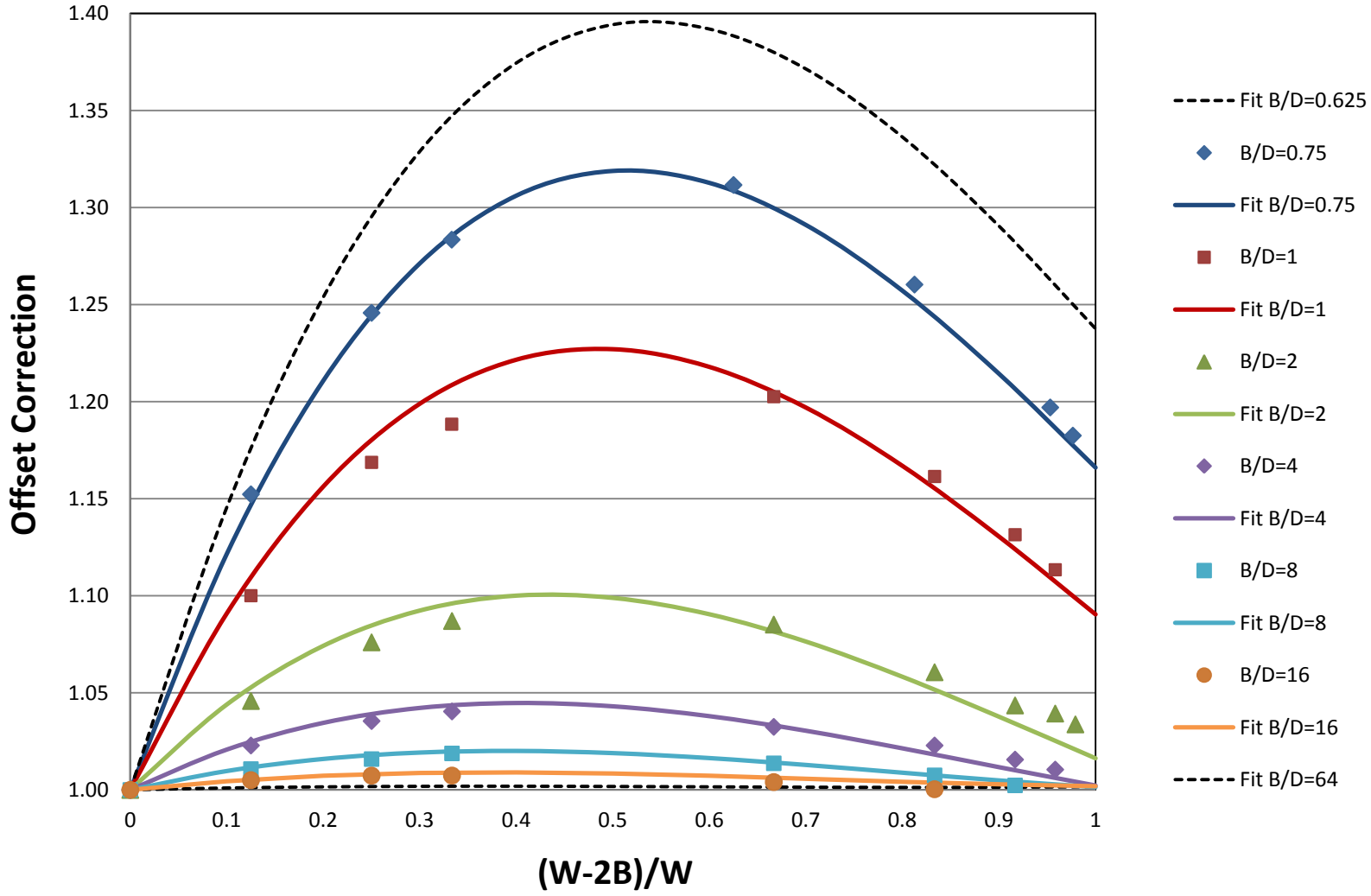
(W-2B)/W = 0.125, 0.25, 0.333, 0.667, 0.833, 0.917, 0.958, 0.979

C/(B-R) = 0.0* – 0.95 (depending on B/D)

* Offset correction determined by: $\mathbf{K_{T(offset)}/K_{T(center)}}$

Offset Correction Fit for Zero Crack Length

Far Edge



Polynomial Fit

$$\boldsymbol{F_0}_{Offset} = 1 + \left[(1.16(0.1+\gamma)^{-1.1})\delta - \left(\frac{1.776}{(0.25+\gamma)^{1.04}}\right)\delta^2 + (0.555(0.25+\gamma)^{-0.9})\delta^3\right]$$

$$\gamma = B/D$$

$$\delta = (W - 2B)/W$$

Meanwhile…….

Some discrepancies were discovered in the current AFGROW Classic Solution for a single through crack at a hole under bearing loading.

Classic Single Through Cracked Hole Bearing Solution

	W/D										
Norm C	1.3	1.5	2	2.5	3	4	8	16	40	100	1000
0	4.5150	3.2000	2.3000	1.9050	1.7000	1.5260	1.1970	1.0363	0.8898	0.8787	0.8700
0.025	4.4348	3.1133	2.1487	1.7332	1.5689	1.3885	1.1186	0.9769	0.8433	0.8287	0.8201
0.05	4.3594	3.0331	2.0200	1.6001	1.4500	1.2794	1.0493	0.9186	0.7898	0.7738	0.7651
0.1	4.2414	2.9165	1.8701	1.4564	1.2750	1.1216	0.9293	0.8080	0.6813	0.6637	0.6552
0.15	4.1913	2.8540	1.7914	1.3565	1.1614	0.9983	0.8245	0.7082	0.5855	0.5691	0.5612
0.2	4.1908	2.8098	1.7202	1.2757	1.0821	0.9020	0.7298	0.6222	0.5110	0.4947	0.4873
0.25	4.2037	2.7745	1.6356	1.2060	1.0158	0.8239	0.6424	0.5387	0.4457	0.4292	0.4223
0.3	4.2198	2.7460	1.5612	1.1429	0.9465	0.7532	0.5636	0.4691	0.3908	0.3748	0.3683
0.35	4.2455	2.7228	1.4990	1.0854	0.8795	0.6861	0.4960	0.4131	0.3415	0.3236	0.3173
0.4	4.2876	2.7054	1.4435	1.0333	0.8222	0.6250	0.4380	0.3594	0.2958	0.2791	0.2733
0.45	4.3526	2.6950	1.3959	0.9860	0.7720	0.5729	0.3843	0.3101	0.2549	0.2370	0.2314
0.5	4.4414	2.6935	1.3562	0.9430	0.7266	0.5268	0.3358	0.2673	0.2178	0.1983	0.1933
0.55	4.5460	2.7046	1.3248	0.9037	0.6867	0.4883	0.2928	0.2265	0.1849	0.1679	0.1633
0.6	4.6575	2.7330	1.3025	0.8691	0.6520	0.4560	0.2553	0.1885	0.1539	0.1377	0.1333
0.65	4.7670	2.7829	1.2912	0.8417	0.6227	0.4226	0.2222	0.1572	0.1274	0.1137	0.1103
0.7	4.8895	2.8581	1.2936	0.8257	0.5997	0.3981	0.1954	0.1320	0.1069	0.0946	0.0913
0.75	5.1337	2.9620	1.3151	0.8200	0.5884	0.3798	0.1751	0.1117	0.0873	0.0755	0.0723
0.8	5.4893	3.1043	1.3652	0.8259	0.5800	0.3630	0.1540	0.0920	0.0695	0.0583	0.0553
0.825	5.6639	3.2000	1.4069	0.8399	0.5837	0.3582	0.1445	0.0832	0.0616	0.0500	0.0470
0.85	6.0283	3.3351	1.4654	0.8640	0.5933	0.3586	0.1355	0.0759	0.0539	0.0420	0.0392
0.875	6.8300	3.5963	1.5459	0.8800	0.6113	0.3666	0.1269	0.0700	0.0459	0.0348	0.0326
0.9	8.1821	4.0870	1.6728	0.9026	0.6427	0.3810	0.1205	0.0639	0.0378	0.0284	0.0262
0.925	9.9887	4.8792	1.8506	1.1291	0.6979	0.3970	0.1191	0.0555	0.0316	0.0228	0.0192
0.95	12.1363	5.9209	2.2067	1.6711	0.7968	0.4400	0.1145	0.0470	0.0255	0.0177	0.0122
0.975	14.5111	7.1285	2.9777	2.4350	1.0919	0.6033	0.1473	0.0734	0.0235	0.0148	0.0062
1	16.9997	8.4200	3.9000	3.3100	1.6000	0.8510	0.2920	0.1380	0.0670	0.0310	0.0000

Values were normalized as: $\frac{C/(C+R)}{(1-D/W)}$

Difference Between Current and Updated Solution (%)

	W/D										
Norm C	1.3	1.5	2	2.5	3	4	8	16	40	100	1000
0	-20.5	-18.9	-6.8	-3.4	-3.1	0.8	-2.0	-5.3	-13.0	-10.8	-10.9
0.025	-2.0	-5.7	-3.4	-4.4	-1.8	1.3	2.2	0.3	-6.0	-4.3	-4.5
0.05	-0.2	-3.4	-3.9	-5.5	-2.2	1.4	5.3	4.6	-3.2	-1.8	-2.1
0.1	-0.7	-3.1	-3.5	-4.9	-3.5	1.7	10.0	9.4	0.2	1.0	0.5
0.15	-0.7	-2.7	-1.6	-3.8	-3.2	1.3	12.0	11.7	1.2	2.1	1.6
0.2	-0.3	-1.8	-0.1	-2.7	-1.7	1.3	12.5	12.9	2.5	3.4	2.9
0.25	-0.1	-1.5	-0.1	-1.6	-0.2	1.6	11.5	11.6	3.1	3.9	3.3
0.3	-0.2	-0.9	-0.2	-0.7	0.2	1.6	9.7	10.7	4.0	4.8	4.2
0.35	-0.4	-0.5	-0.1	0.0	-0.1	0.9	8.1	10.9	4.5	4.5	3.9
0.4	-0.5	-0.2	-0.1	0.6	-0.2	-0.2	6.8	9.8	4.3	4.5	3.9
0.45	-0.5	-0.1	-0.1	1.0	-0.1	-0.7	4.8	8.1	4.1	3.4	2.6
0.5	-0.3	0.0	-0.1	1.3	-0.1	-1.2	2.5	6.7	3.6	1.5	0.8
0.55	-0.3	-0.1	-0.1	1.3	-0.1	-1.2	0.2	4.0	3.3	1.8	1.1
0.6	-0.7	-0.1	-0.1	1.0	-0.1	-0.7	-1.9	0.4	2.1	0.5	-0.3
0.65	-2.0	-0.3	-0.1	0.8	-0.1	-1.3	-3.9	-2.2	1.8	1.5	1.2
0.7	-4.0	-0.6	-0.1	0.8	-0.2	-0.7	-4.8	-3.0	4.8	5.8	5.6
0.75	-5.1	-1.5	-0.1	0.8	0.5	0.2	-3.0	-2.0	7.6	9.2	9.1
0.8	-6.3	-3.4	-0.1	0.1	-0.1	0.2	-3.6	-2.1	11.3	14.1	14.3
0.825	-8.0	-4.7	-0.1	0.1	0.1	-0.2	-3.3	-1.6	14.2	16.6	16.8
0.85	-7.9	-5.9	-0.1	0.2	0.3	0.4	-4.7	-0.7	17.2	19.2	20.0
0.875	-3.1	-5.2	-0.2	-2.2	-0.1	1.7	-6.1	2.0	18.9	24.8	28.6
0.9	5.7	-1.4	0.2	-5.7	-0.1	2.9	-7.3	4.7	18.7	30.6	35.3
0.925	13.8	4.3	-0.6	7.3	-0.1	1.2	-6.7	0.0	22.6	42.0	39.7
0.95	15.1	5.8	0.4	36.2	-0.7	0.1	-12.8	-8.3	24.3	59.2	37.2
0.975	-0.7	-7.7	-0.5	47.9	2.8	6.6	-4.2	41.1	41.3	109.4	39.5
1	-96.0	-96.2	-94.9	-91.1	-93.0	-92.4	-88.3	-82.1	-58.1	-16.2	0.0

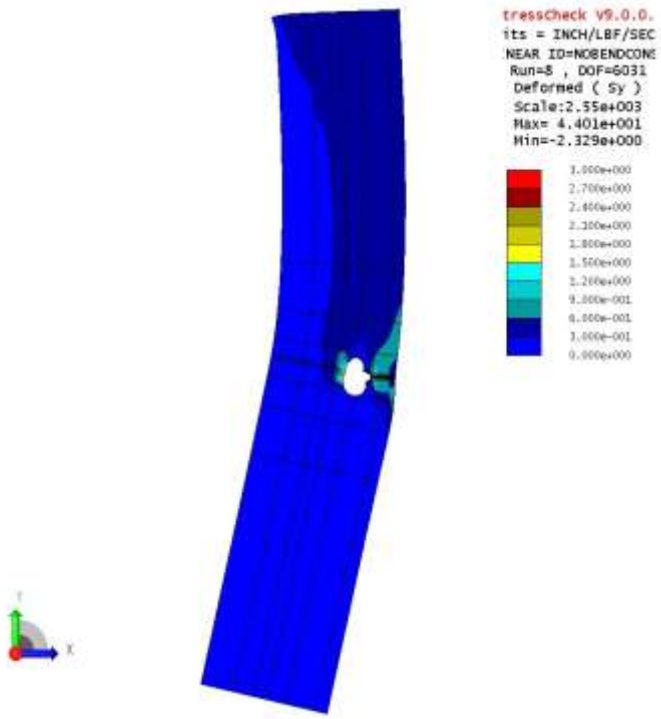
Updated/Expanded Solution Matrix

	W/D													
Norm C	1.3	1.5	2	2.5	3	4	6	8	12	16	24	40	100	1000
0.000	5.6807	3.9450	2.4684	1.9725	1.7537	1.5136	1.3149	1.2218	1.1355	1.0946	1.0550	1.0229	0.9850	0.9760
0.025	4.5248	3.3026	2.2250	1.8121	1.5970	1.3708	1.1786	1.0946	1.0127	0.9735	0.9403	0.8969	0.8657	0.8583
0.050	4.3660	3.1414	2.1028	1.6932	1.4824	1.2620	1.0779	0.9963	0.9160	0.8783	0.8455	0.8157	0.7880	0.7814
0.100	4.2695	3.0086	1.9382	1.5314	1.3207	1.1023	0.9236	0.8451	0.7728	0.7386	0.7086	0.6799	0.6574	0.6520
0.150	4.2221	2.9319	1.8204	1.4104	1.1999	0.9854	0.8115	0.7359	0.6666	0.6341	0.6027	0.5785	0.5572	0.5522
0.200	4.2040	2.8627	1.7222	1.3107	1.1013	0.8908	0.7217	0.6490	0.5824	0.5513	0.5216	0.4987	0.4785	0.4737
0.250	4.2081	2.8156	1.6379	1.2251	1.0178	0.8109	0.6463	0.5762	0.5125	0.4827	0.4543	0.4324	0.4133	0.4087
0.300	4.2275	2.7719	1.5647	1.1509	0.9449	0.7414	0.5813	0.5136	0.4523	0.4239	0.3968	0.3759	0.3577	0.3534
0.350	4.2637	2.7356	1.5007	1.0852	0.8807	0.6802	0.5243	0.4587	0.3998	0.3725	0.3468	0.3269	0.3095	0.3054
0.400	4.3110	2.7116	1.4453	1.0271	0.8235	0.6260	0.4736	0.4101	0.3534	0.3273	0.3025	0.2836	0.2670	0.2631
0.450	4.3760	2.6971	1.3976	0.9760	0.7727	0.5772	0.4281	0.3666	0.3119	0.2868	0.2631	0.2450	0.2292	0.2255
0.500	4.4560	2.6940	1.3579	0.9310	0.7274	0.5333	0.3874	0.3275	0.2747	0.2505	0.2277	0.2103	0.1954	0.1918
0.550	4.5599	2.7062	1.3266	0.8925	0.6874	0.4941	0.3503	0.2922	0.2410	0.2178	0.1957	0.1790	0.1649	0.1616
0.600	4.6921	2.7367	1.3041	0.8603	0.6527	0.4591	0.3170	0.2602	0.2103	0.1877	0.1666	0.1507	0.1370	0.1338
0.650	4.8652	2.7915	1.2925	0.8353	0.6235	0.4281	0.2864	0.2312	0.1828	0.1607	0.1404	0.1251	0.1121	0.1090
0.700	5.0953	2.8767	1.2950	0.8191	0.6007	0.4009	0.2592	0.2052	0.1571	0.1361	0.1166	0.1020	0.0894	0.0865
0.750	5.4100	3.0086	1.3164	0.8138	0.5855	0.3789	0.2353	0.1805	0.1343	0.1140	0.0951	0.0811	0.0691	0.0663
0.800	5.8580	3.2126	1.3671	0.8247	0.5805	0.3623	0.2150	0.1597	0.1139	0.0940	0.0759	0.0625	0.0511	0.0484
0.825	6.1594	3.3567	1.4083	0.8389	0.5832	0.3587	0.2065	0.1494	0.1045	0.0846	0.0671	0.0539	0.0429	0.0402
0.850	6.5448	3.5430	1.4663	0.8626	0.5913	0.3573	0.1994	0.1421	0.0957	0.0764	0.0598	0.0460	0.0352	0.0327
0.875	7.0477	3.7937	1.5489	0.8993	0.6118	0.3603	0.1944	0.1352	0.0882	0.0686	0.0512	0.0386	0.0279	0.0253
0.900	7.7421	4.1451	1.6702	0.9573	0.6432	0.3702	0.1923	0.1299	0.0814	0.0610	0.0442	0.0318	0.0218	0.0194
0.925	8.7775	4.6773	1.8614	1.0524	0.6983	0.3922	0.1954	0.1276	0.0761	0.0555	0.0383	0.0258	0.0161	0.0137
0.950	10.5440	5.5943	2.1977	1.2266	0.8026	0.4397	0.2092	0.1314	0.0736	0.0513	0.0334	0.0205	0.0111	0.0089
0.975	14.6097	7.7203	2.9934	1.6465	1.0621	0.5659	0.2562	0.1538	0.0796	0.0520	0.0302	0.0166	0.0071	0.0044
0.980	16.2647	8.5868	3.3183	1.8196	1.1700	0.6199	0.2777	0.1650	0.0840	0.0540	0.0305	0.0162	0.0065	0.0036
0.990	22.7857	12.0056	4.6033	2.5040	1.5978	0.8350	0.3655	0.2129	0.1044	0.0649	0.0346	0.0168	0.0056	0.0018
0.998	50.4029	26.4211	10.0095	5.3665	3.3766	1.7286	0.7359	0.4198	0.1993	0.1205	0.0609	0.0268	0.0068	0.0004
0.9995	98.7111	52.3990	19.5979	10.5311	6.4648	3.2991	1.3648	0.7715	0.3617	0.2168	0.1085	0.0470	0.0112	0.0001
0.9999	215.000	120.000	42.5315	22.9476	13.9094	6.8283	2.8176	1.5036	0.7284	0.4342	0.2154	0.0925	0.0218	0.0000
1.0000	420.000	220.000	77.0000	37.0000	23.0000	11.2500	4.7500	2.5000	1.2750	0.7700	0.3720	0.1600	0.0370	0.0000

Work to be Completed

- Finish Curve Fit Solution for the Far Edge Crack as a Function of Crack Length
- Implement the Updated “Classic” Solution for a Single Crack at an Offset Hole
- Incorporate the Above Cases in the Upcoming AFGROW Release
- Update the “Classic” Solution for a Double, Symmetric Crack at an Offset Hole

Off the Record



Boundary Condition Issues Can be Very Problematic for the Bearing Offset Correction